Electromagnetics Theory

Maxwell's Equation for Time-Varying Fields

Outline:

5.1 Faraday's Law

5.2 Stationary Loop on a Time-varying magnetic field

5.3 The ideal transformer

5.1 Faraday's Law

Table 5-1: Maxwell's equations.

Reference	Differential Form	Integral Form	
Gauss's law	$\nabla \cdot \mathbf{D} = \rho_v$	$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q$	(5.1)
Faraday's law	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$	(5.2)†
No magnetic charges (Gauss's law for magnetism)	$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$	(5.3)
Ampère's law	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$	$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}\right) \cdot d\mathbf{s}$	(5.4)

†For a stationary surface S.

• In dynamic cases, we have to deal with the coupling that exists between electric fields and magnetic fields express by eq (2) and eq (4) Maxwell equation in differential form.

•From eq (2):

"A time-varying magnetic field give rises to an electric field" ⇓

FARADAY'S LAW

•From eq (4):

"A time-varying electric field give rises to an magnetic field" ⇓

AMPERE'S LAW

“ Magnetic fields can produce an electric current in a closed loop, but only if the magnetic flux linking the surface area of the loop changes with time”

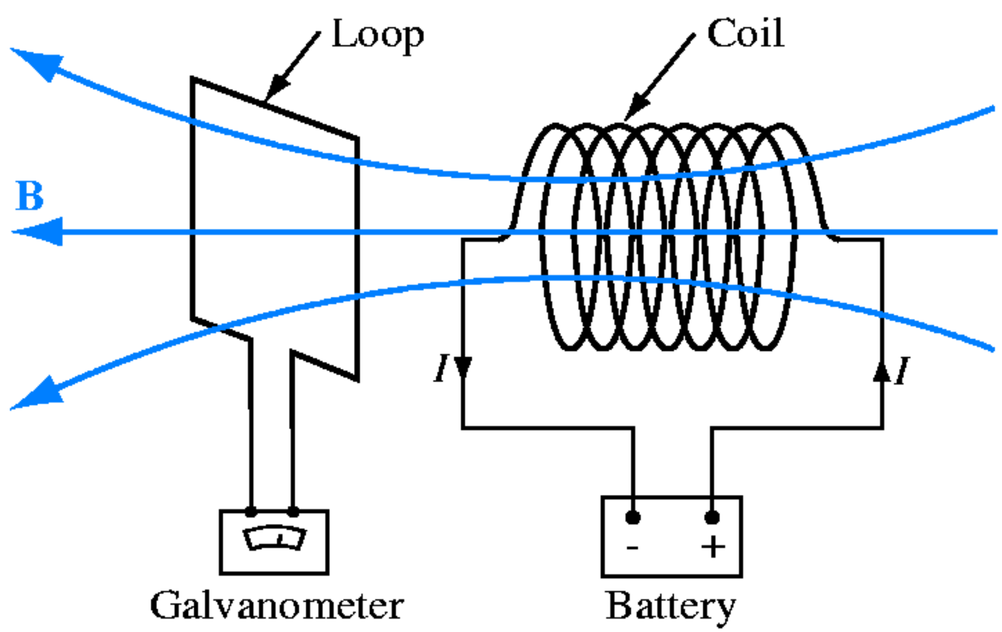
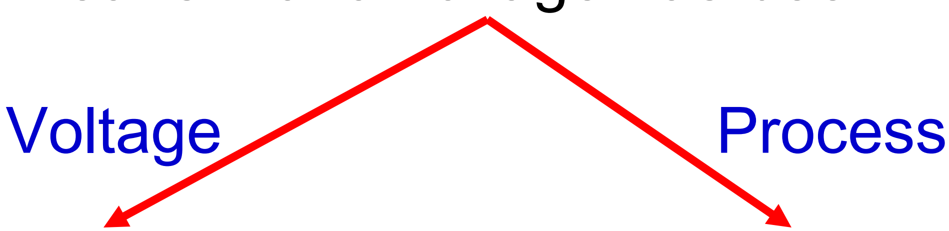


Figure 5-1

- Magnetic flux ϕ passing through a loop as the integral of the normal component of the magnetic flux density over the surface area of the loop S;

$$\phi = \int_{s} \vec{B} \bullet d\vec{s} \left(Wb \right)$$

- When galvanometer detects the flow of current through the coil, it means that a voltage has been induced.



Electromotive force (EMF)   Electromagnetic induction

$$V_{emf} = -N\frac{d\phi}{dt} = -N\frac{d}{dt}\int_s \vec{B} \bullet d\vec{s} \qquad (\mathrm{V})$$

- An EMF (electromagnetic force) can be generated in a closed conducting loop under any of the following 3 condition:

1) A time varying magnetic field linking a stationary loop, V_{emf}^{tr}
2) A moving loop with a time-varying area in a static field $\mathbf{B}$, V_{emf}^{m}
3) A moving loop in a time-varying field $\mathbf{B}$

The total emf is given by:

$$V_{emf} = V_{emf}^{tr} + V_{emf}^{m}$$

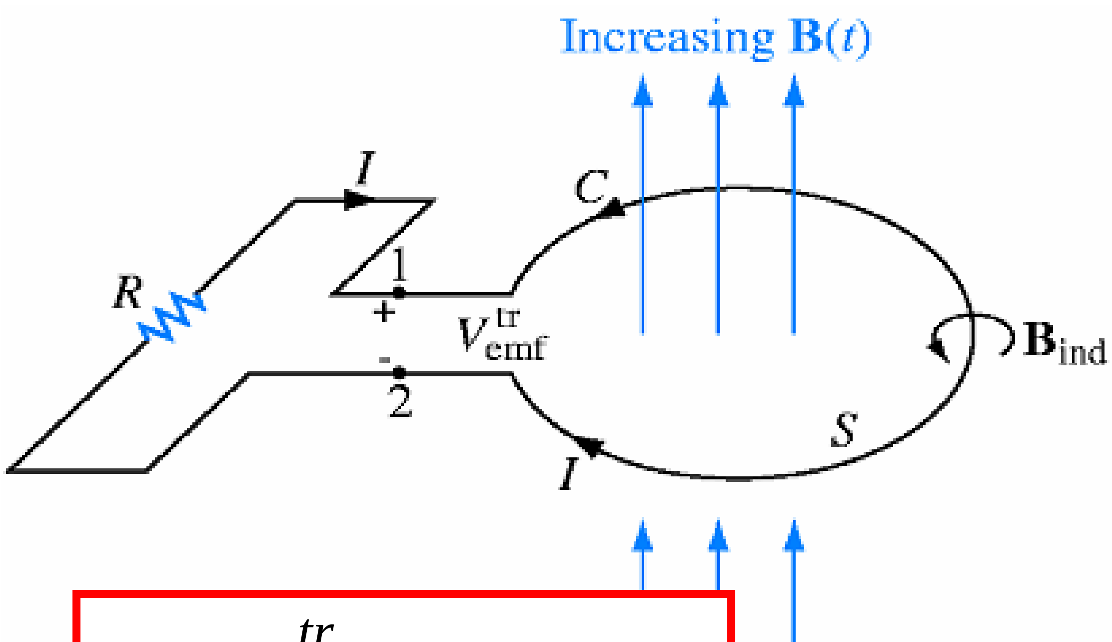
Conclusion:

1) If loop is stationary: $V_{emf} = V_{emf}^{tr}$

2) If **B** is static: $V_{emf} = V_{emf}^{m}$

3) If loop is moving & **B** is time-varying: $V_{emf} = V_{emf}^{tr} + V_{emf}^{m}$

5.2 Stationary Loop on a Time-varying magnetic field

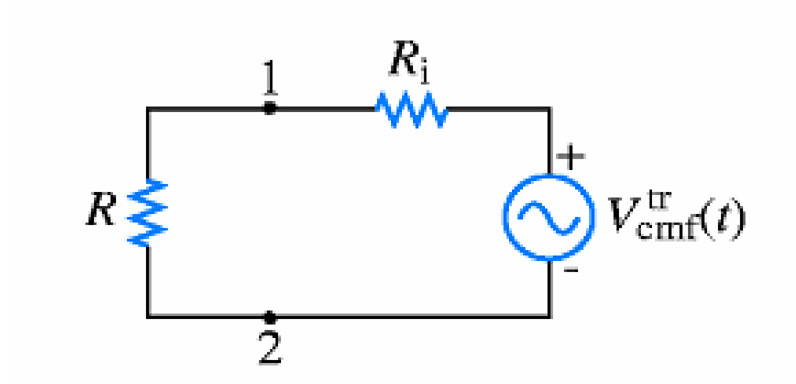


Induced emf called **Transformer EMF**, V_{emf}^{tr}

$$V_{emf} = V_{emf}^{tr} = -N\frac{d\phi}{dt} = -N\frac{d}{dt}\int_s \vec{B} \bullet d\vec{s} \quad \text{(V)}$$

$$\therefore V_{emf}^{tr} = V_{12}$$

•If loop has an internal resistance, R_i the equivalent circuit is:



$$V_{emf}^{tr} = I(R + R_i)$$

$$\therefore I = \frac{V_{emf}^{tr}}{(R + R_i)}$$

good conductor: $Ri \leq R$
(R_i is small)

$$\therefore I = \frac{V_{emf}^{tr}}{R}$$

Example 5.1 – Inductor in a changing magnetic field

An inductor is formed by winding N turns of a thin conducting wire into a circular loop of radius a. The inductor loop is in the x-y plane with its center at the origin, and it is connected to a resistor R as shown above. In the presence of a magnetic field given by , where is the angular frequency. Find

a) The magnetic flux linking a single turn of the inductor.
b) the transformer emf, given that N=10, Bo=0.2T, a=10cm and w=103rad/s
c) polarity of at t=0
d) the induced current in the circuit for R=1k(assume wire resistance is too small)

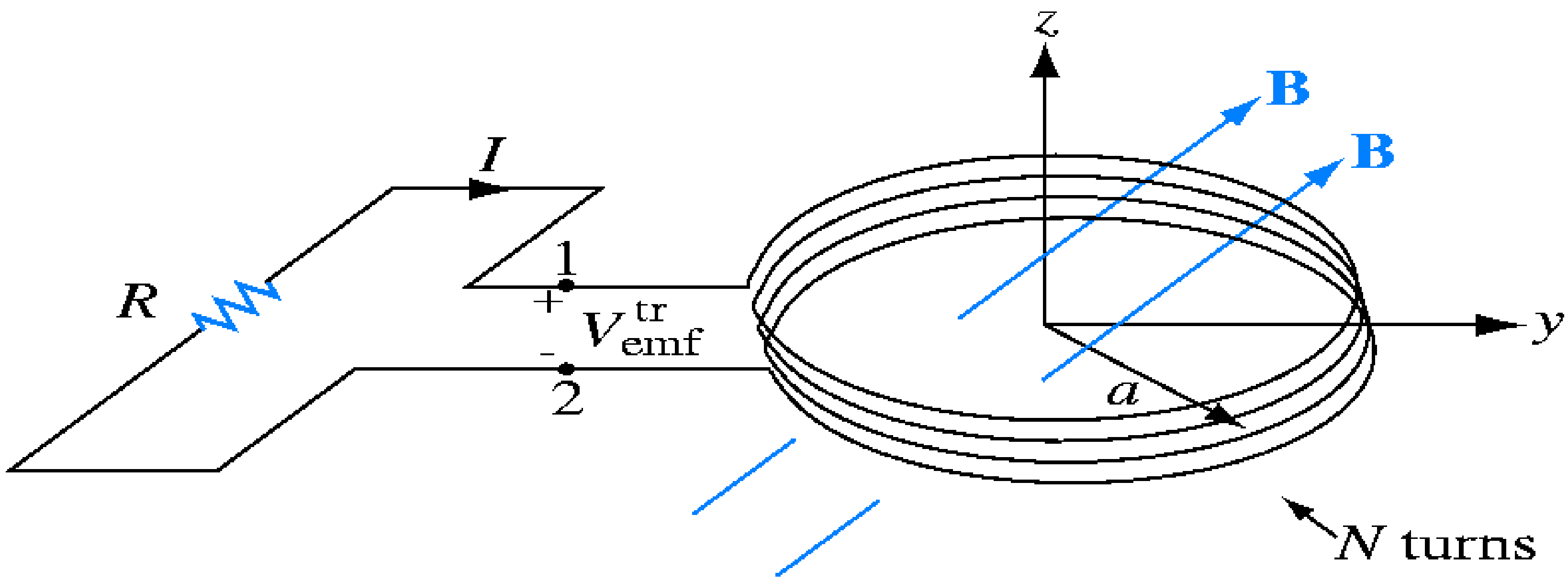
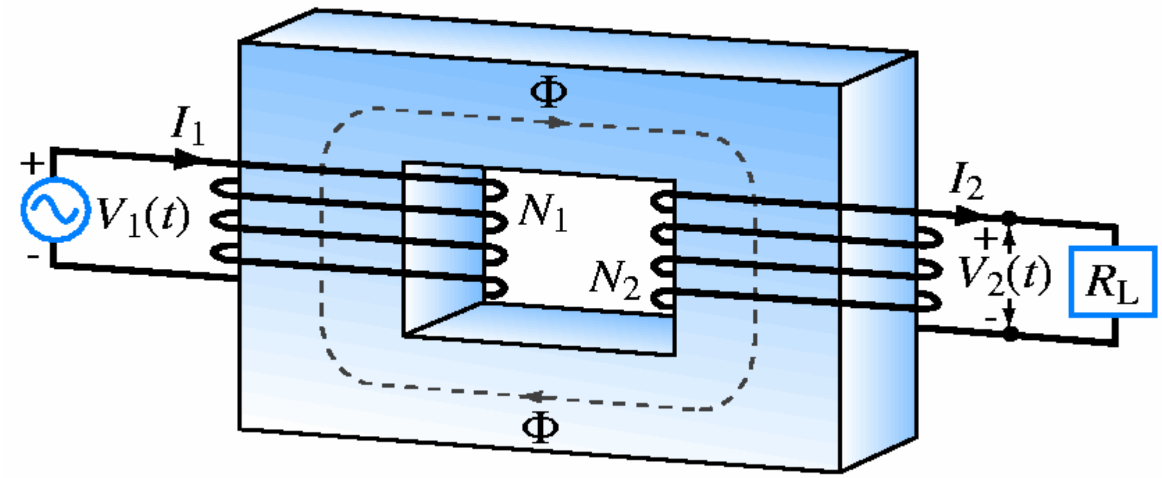


Figure 5-3

5.3 The ideal transformer



- Tx consists of 2 coils wound around a common magnetic core
- Core of primary circuit:
  - has N_1 turns
  - connected to A-C voltage source, V_1 (t)
- Core of secondary circuit:
  - has N_2 turns
  - Connected to load resistor, R_L

- In Ideal Tx:

1) Core has infinite permeability ($\mu = \infty$)
2) Magnetic flux confined within core
3) There's no power list in core (P_1=P_2)

- V_1 generate current I_1 in primary coil, which establish a flux ϕ in the magnetic core:

$$V_1 = -N_1 \frac{d\phi}{dt} \quad \cdots\cdots\cdots \quad (1)$$

- On secondary side:

$$V_2 = -N_2 \frac{d\phi}{dt} \quad \cdots\cdots\cdots \quad (2)$$

- Combine eq (1) & (2):

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

V proportional to turns ratio

- where

$$P_1 = I_1 V_1$$

$$P_2 = I_2 V_2$$

$$\therefore \frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{N_2}{N_1}$$

$$I_1 V_1 = I_2 V_2$$

In Secondary side: $V_2 = I_2 R_L$

In primary side:

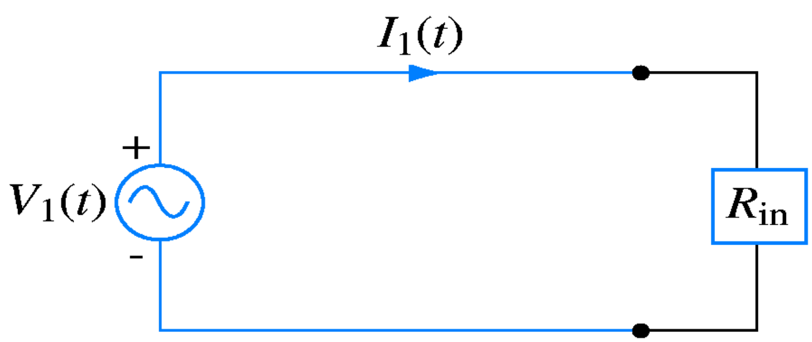


Figure 5-6

$$V_1 = I_1 R_{in}$$

Thus;

$$R_{in} = R_L \left(\frac{N_1}{N_2} \right)^2$$

Where load is an impedance Z_L

$$\therefore Z_{in} = Z_L \left(\frac{N_1}{N_2} \right)^2$$