Electromagnetics Theory

Magnetostatics (part 2)

Outline:

1. Magnetic Forces and Torques
2. The Biot-Savart Law
3. Magnetic Force between Two Parallel Conductors
4. Maxwell's Magnetostatic Equations
5. **Vector Magnetic Potential**
6. **Magnetic Boundary Conditions**
7. **Inductance**

Vector Magnetic Potential

Recall electrostatic potential V

$$\mathbf{E} = -\nabla V \quad \text{(V/m)}$$

To find **E**, determine V by ⇨ $V = \frac{1}{4\pi\varepsilon}\int_v \frac{\rho_v}{R}\,dv \quad \text{(V)}$ volume distribution

Vector magnetic potential **A**

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (\text{Wb/m}^2)$$

To find **B**, determine **A** by

$$\mathbf{A} = \frac{\mu}{4\pi}\int_v \frac{\mathbf{J}}{R}\,dv$$ volume distribution

$$\mathbf{A} = \frac{\mu}{4\pi}\int_s \frac{\mathbf{J}_s}{R}\,ds$$ surface distribution

$$\mathbf{A} = \frac{\mu}{4\pi}\int_l \frac{I\,d\mathbf{l}}{R}$$ line distribution

SI unit for **A** is (Wb/m)

Three approaches to compute B

- **Biot-Savart Law**

Common approach which need to perform integration given by

$$\mathbf{H} = \frac{I}{4\pi}\oint_l \frac{d\mathbf{l} \times \hat{\mathbf{R}}}{R^2} \quad \text{(A/m)}$$

- **Ampere's law**

-Only for simple current distributions with symmetrical geometries
-Limited application

- **Vector magnetic potential**

Preferred approach compared to Biot-Savart law.
Reason: Easier integration given by

$$\mathbf{A} = \frac{\mu}{4\pi}\int_v \frac{\mathbf{J}}{R}\,dv \quad \text{(Wb/m)}$$

Magnetic Flux Φ

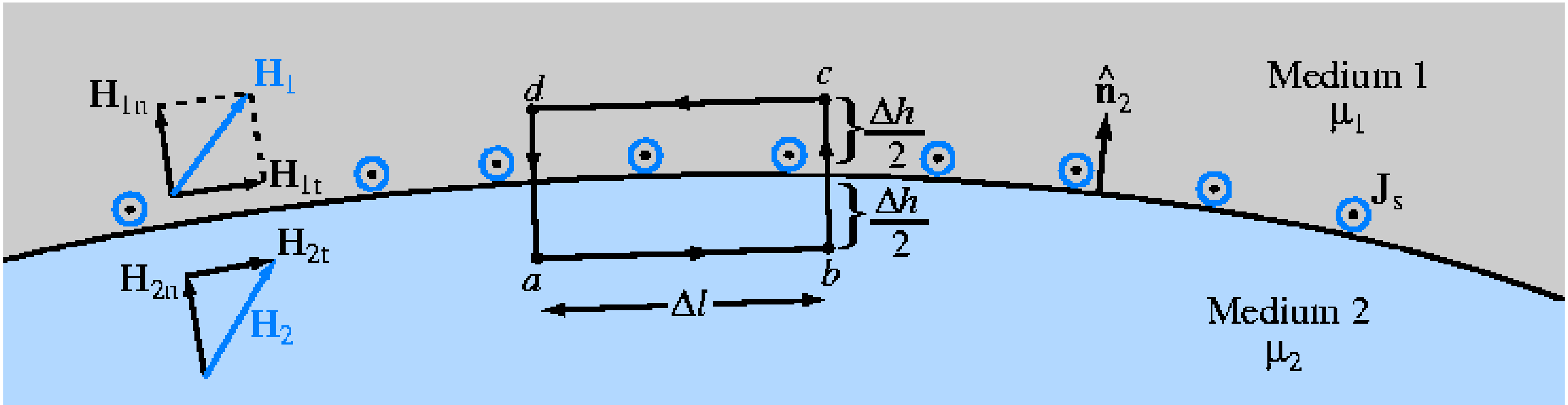
The *magnetic flux* Φ linking a surface S is defined as the total magnetic flux density (**B**) passing through surface S

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s} \qquad \text{(Wb)}$$

From $\mathbf{B} = \nabla \times \mathbf{A}$ and invoke Stoke's theorem

$$\Phi = \int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{s} = \oint_C \mathbf{A} \cdot d\mathbf{l} \quad \text{(Wb)}$$

Magnetic Boundary Conditions



Normal Components

$$\oint_S \mathbf{D} \cdot \mathbf{d}s = Q \quad \Rightarrow \quad D_{1n} - D_{2n} = \rho_s$$

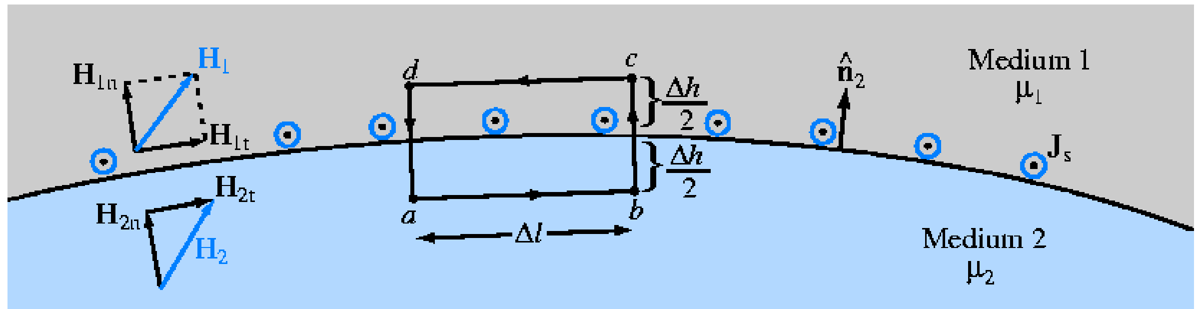
$$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0 \quad \Rightarrow \quad B_{1n} = B_{2n}$$

Gauss's Law for Magnetism

Magnetic Boundary Conditions (cont'ed)

Ampere's Law

Tangential Components



$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_a^b \mathbf{H}_2 \cdot d\mathbf{l} + \int_c^d \mathbf{H}_1 \cdot d\mathbf{l} = I$$

$$H_{2t}\Delta l - H_{1t}\Delta l = J_s \Delta l$$

$$H_{2t} - H_{1t} = J_s$$

*In perfect conductor

$$H_{1t} - H_{2t} = 0$$

To incorporates directional relationship

$$\hat{\mathbf{n}}_2 \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s$$

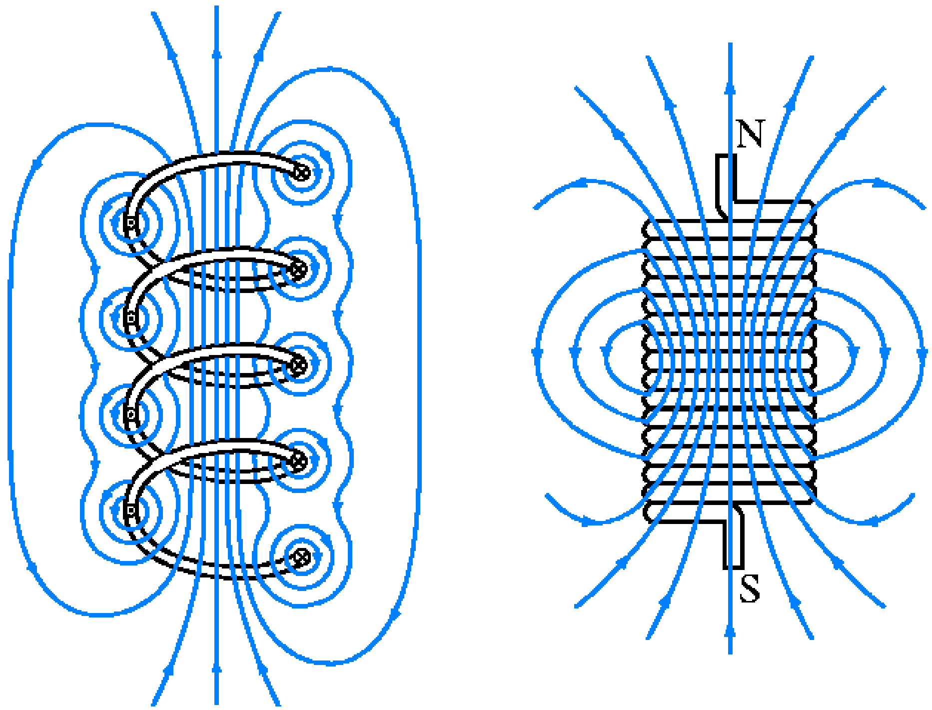
Exercise

With reference to Figure in previous slide, determine the angle between $\mathbf{H_1}$ and $\hat{\mathbf{n}}_2 = \hat{\mathbf{z}}$ if $\mathbf{H_2} = (\hat{\mathbf{x}}3 + \hat{\mathbf{z}}2)$ (A/m), $\mu_{r1} = 2$, and $\mu_{r2} = 8$, and $\mathbf{J}_s = 0$

$\theta = 20.6^{o}$

Inductance

- An inductor is the magnetic analogue of an electrical capacitor
- A tightly wound solenoid resembles permanent magnet



(a) Loosely wound solenoid

(b) Tightly wound solenoid

Magnetic Field in a Solenoid

Objective: To get ***Self-inductance*** (L)

L = magnetic flux linkage / current flow

$$L = \frac{\Lambda}{I}$$

⇧

magnetic flux linkage

$$\Lambda = N\Phi$$

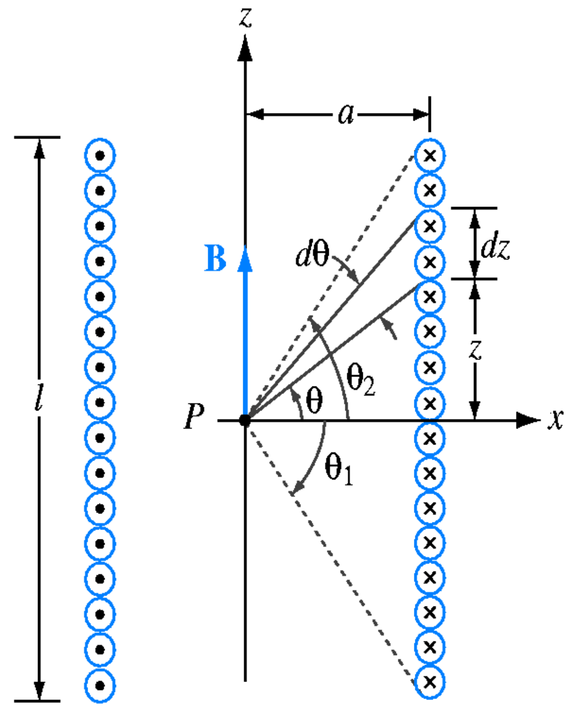
⇧

Magnetic flux linking a surface

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s}$$

⇧

B = ?? in a solenoid



$$\mathbf{B} = \hat{\mathbf{z}}\frac{\mu nI}{2}(\sin\theta_2 - \sin\theta_1)$$

$$= \hat{\mathbf{z}}\mu nI = \hat{\mathbf{z}}\frac{\mu NI}{l} \text{ (long solenoid)}$$

Self-inductance

Magnetic flux linkage is defined as the total magnetic flux linking a given circuit or conducting structure.

For a solenoid with *N* turns

$$\Lambda = N\Phi$$

$$= \mu \frac{N^2}{l} IS \text{ (wb)}$$

Self-inductance

$$L = \frac{\text{magnetic flux linkage}}{\text{current flow}}$$

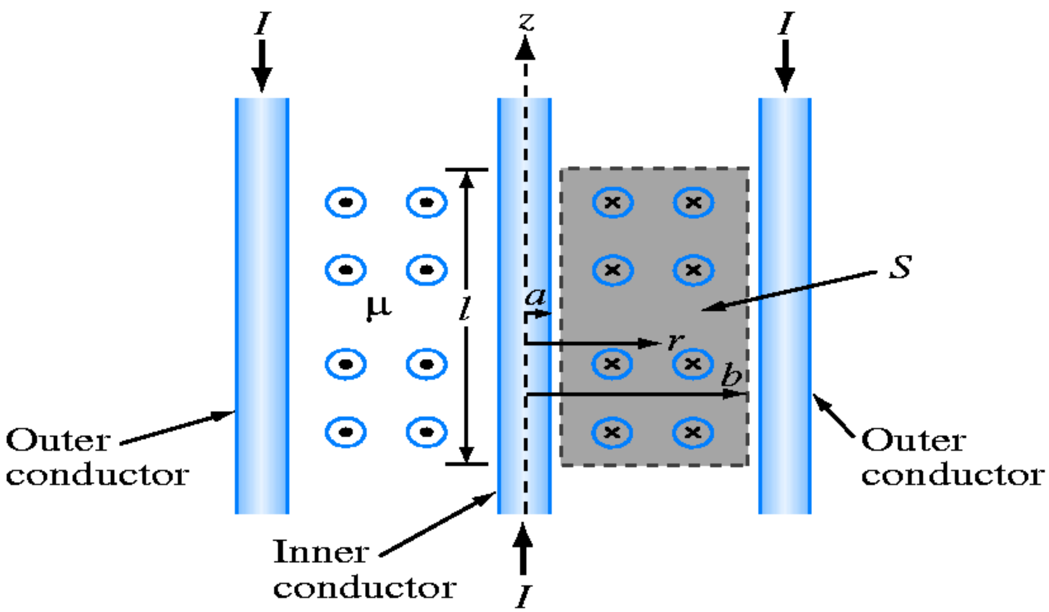
$$L = \frac{\Lambda}{I}$$

***L* for solenoid**

$$L = \mu \frac{N^2}{l} S$$

Exercise

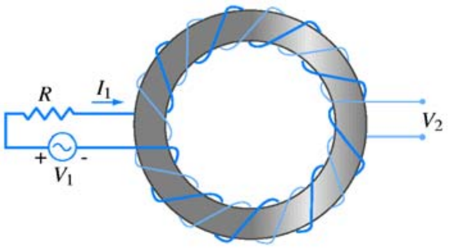
Develop an expression for the inductance for unit length of a coaxial transmission line. The conductors have radii *a* and *b*, as shown in Figure below, and the insulating material has the linear permeability of μ.

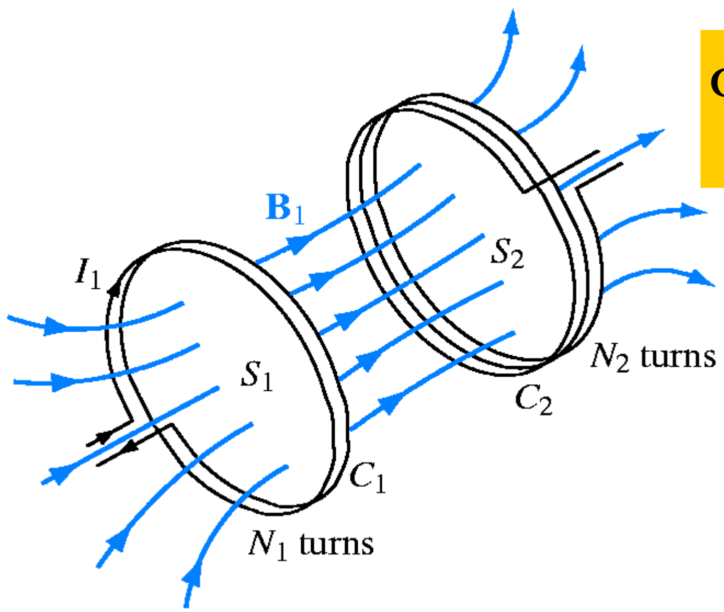


$$L' = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right)$$

Mutual-inductance

Mutual Inductance- describe the magnetic coupling between two different conducting structures.





Flux through loop 2 by $\mathbf{B}_1$

$$\Phi_{12} = \int_{S2} \mathbf{B}_1 \cdot d\mathbf{s}$$

$$\Lambda_{12} = N_2 \Phi_{12}$$

$$L_{12} = \frac{\Lambda_{12}}{I_1} = \frac{N_2}{I_1} \int_{S_2} \mathbf{B}_1 \cdot d\mathbf{s}$$

Thank You