Electromagnetics Theory

Magnetostatics (part 1)

Outline:

1. **Magnetic Forces and Torques**
2. **The Biot-Savart Law**
3. **Magnetic Force between Two Parallel Conductors**
4. **Maxwell's Magnetostatic Equations**
5. Vector Magnetic Potential
6. Magnetic Boundary Conditions
7. Inductance

Overview

Electrostatics

$$\nabla \cdot \mathbf{D} = \rho_v$$
$$\nabla \times \mathbf{E} = 0$$

$$\mathbf{D} = \varepsilon \mathbf{E}$$

Magnetostatics

$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{H} = \mathbf{J}$$

$$\mathbf{B} = \mu \mathbf{H}$$

The Maxwell's equations separated into two uncoupled pairs

B ≡ Magnetic flux density; **H** ≡ Magnetic field intensity

E-Static *vs* M-Static

Attribute	**Electrostatics**	**Magnetostatics**
Sources	Stationary charges	Steady currents
Constitutive parameter (s)	ε and σ	μ
Equations *Differential form* *Integral form*	$\nabla \cdot \mathbf{D} = \rho_v$; $\nabla \times \mathbf{E} = 0$ $\oint_S \mathbf{D} \cdot d\mathbf{s} = Q$; $\oint_C \mathbf{E} \cdot d\mathbf{l} = 0$	$\nabla \cdot \mathbf{B} = 0$; $\nabla \times \mathbf{H} = J$ $\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$; $\oint_C \mathbf{H} \cdot d\mathbf{l} = I$
Potential	$\mathbf{E} = -\nabla V$	$\mathbf{B} = \nabla \times \mathbf{A}$
Energy density	$w_e = ½\, \varepsilon E^2$	$w_m = ½\, \mu H^2$
Force on charge q	$\mathbf{F}_\mathrm{e} = q\mathbf{E}$	$\mathbf{F}_\mathrm{m} = q\mathbf{u} \times \mathbf{B}$
Circuit element (s)	C and R	L

Magnetic Forces

What will be the magnetic force ($\mathbf{F}_m$) acting on a charged particle moving in magnetic fields?

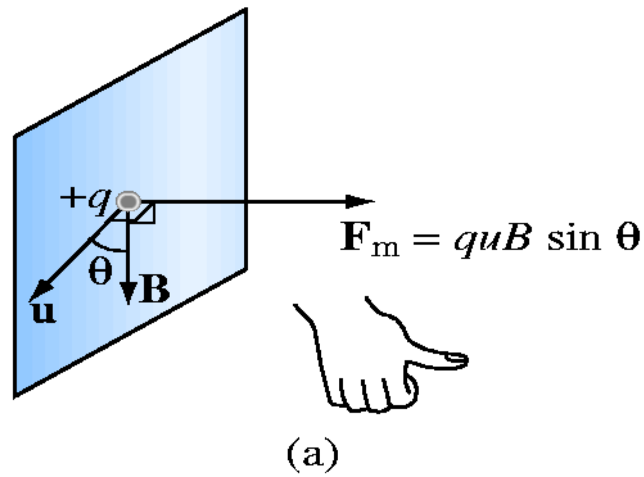
Based on the experiments,

u ≡ velocity

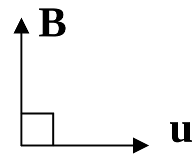
$\theta \equiv \angle(\mathbf{u} \text{ and } \mathbf{B})$

$$\mathbf{F}_m = q\mathbf{u} \times \mathbf{B}$$

$$F_m = q\,u\,B \sin\theta$$



F_m max. *when*



F_m min. *when* ???

Will the polarity of q change the direction of $\boldsymbol{F}_m$?

Magnetic Forces (cont'd)

What will be forces acting on the charged particle, if a charged particle is moving in the presence of both **E** and **B** ?

Electromagnetic force

$$\mathbf{F} = \mathbf{F}_e + \mathbf{F}_m$$

$$= q\mathbf{E} + q\mathbf{u} \times \mathbf{B}$$

$$= q(\mathbf{E} + \mathbf{u} \times \mathbf{B})$$

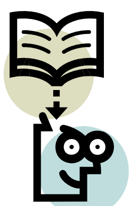
⇐ *Lorentz force*

Any differences between the electric force and magnetic force?

Magnetic Forces (cont'ed)

Some important differences:

- F_e is always in direction of electric field **E**, F_m is always perpendicular to magnetic field **B**
- F_e is acted to stationary and moving charge, F_m only when its move
- F_e gives work to a particle, F_m does not give work, only change the direction of a particle



Can the magnetic field change the speed of a moving charged particle?

Exercise

1. An electron moving in the negative *x*-direction perpendicular to a magnetic field experiences a deflection in the negative *z*-direction. What is the direction of the magnetic field?

2. A proton moving with speed of 2×10^6 m/s through a magnetic field with magnetic flux density of 2.5T experiences a magnetic force of magnitude 4×10^{-13} N. What is the angle between the magnetic field and the proton's velocity?

F_m on a Current-Carrying Conductor

$$\hat{\mathbf{B}} = -\hat{\mathbf{x}}$$
$$\hat{\mathbf{u}} = \hat{\mathbf{z}}$$

Charged particles drifting through the wire

↓

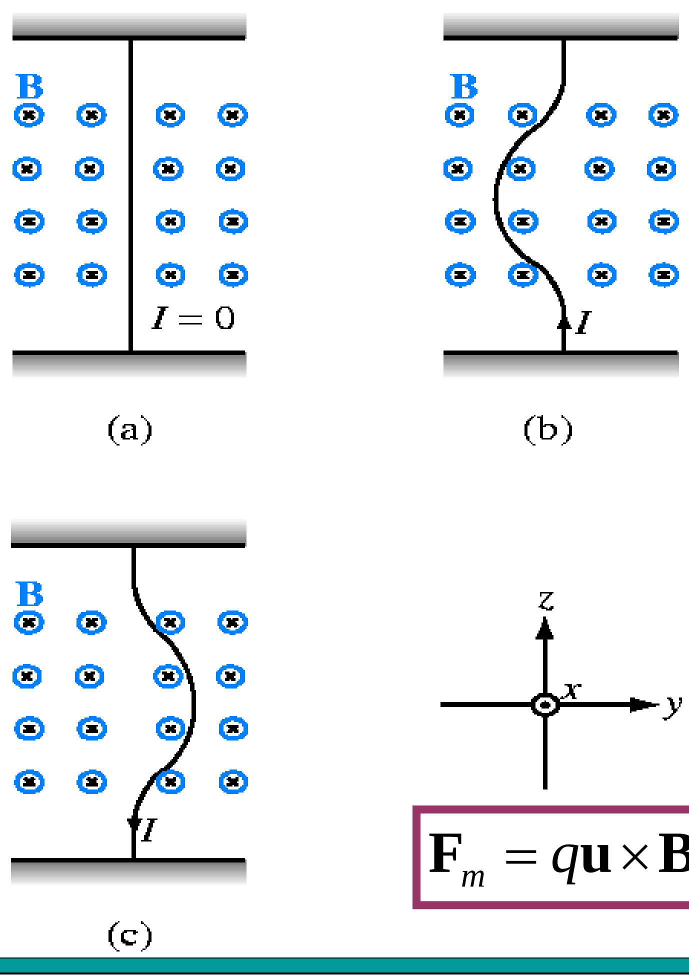
Current Flow

↓ Wire placed in **B**

Wire experience $\mathbf{F_m}$

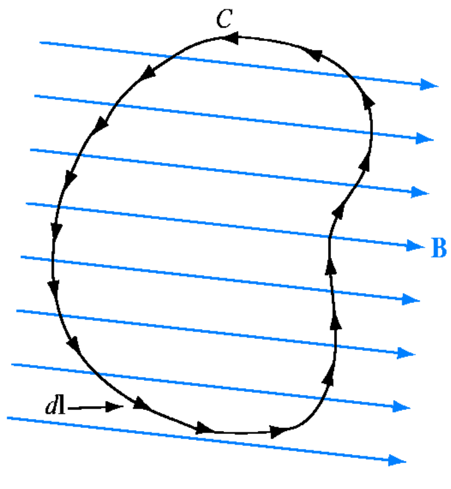
↓

$$\mathbf{F}_m = I\oint_C d\mathbf{l}\times\mathbf{B} \quad \text{(N)}$$



$$\mathbf{F}_m = q\mathbf{u}\times\mathbf{B}$$

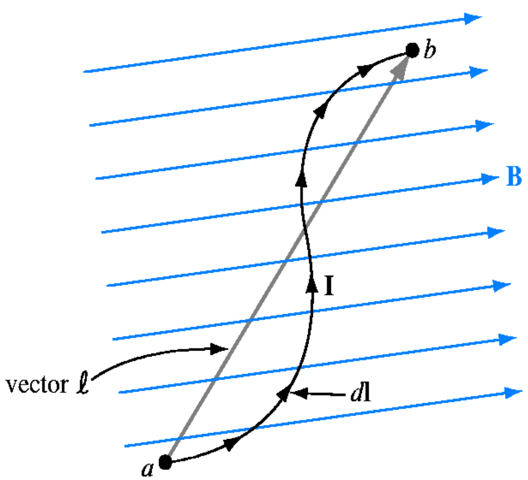
$\mathbf{F}_m$ for Closed loop and Line Path



(a)

$$\mathbf{F}_m = I(\oint_C d\mathbf{l}) \times \mathbf{B} = 0$$

The total magnetic force on any closed current loop in a uniform magnetic field is zero



$$\mathbf{F}_m = I(\int_a^b d\mathbf{l}) \times \mathbf{B} = I(l) \times \mathbf{B}$$

Example (Ulaby, pp 123)

The semicircular conductor shown in Figure lies in the *x*-*y* plane and carries a current *I*. The closed circuit is exposed to a uniform magnetic field $\mathbf{B} = \hat{\mathbf{y}} B_0$. Determine

(a) The magnetic force $\mathbf{F}_1$ on the straight section of the wire

(b) The force $\mathbf{F}_2$ on the curved section.

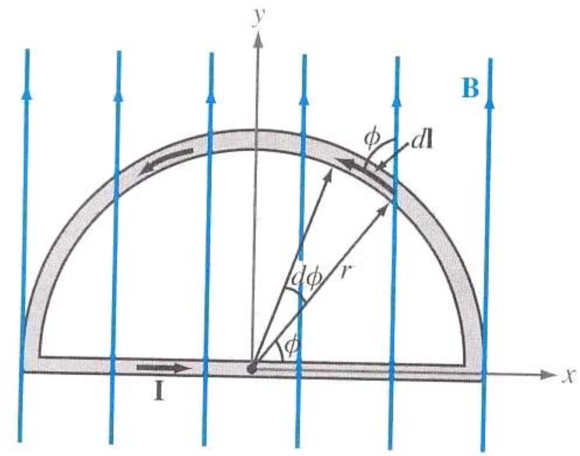


Figure 4-4: Semicircular conductor in a uniform field (Example 4-1).

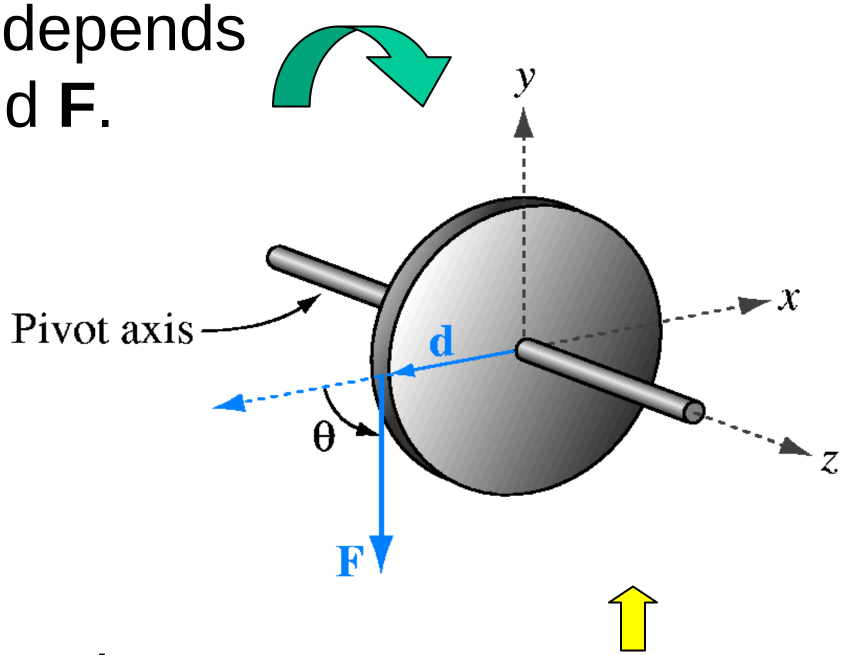
Magnetic Torque

- The strength of the rotation depends on the cross product of **d** and **F**.

Torque:

$$\mathbf{T} = \mathbf{d} \times \mathbf{F} \quad \text{(N.m)}$$

where: **d** ≡ moment arm



$$\mathbf{T} = \hat{\mathbf{z}} rF \sin\theta$$

- Torque does not represent work or energy, even the unit is the same.
- Positive *T* represented by the counterclockwise rotation, while negative *T* for clockwise rotation.

Magnetic Torque (Cont'ed)

From eqn.

$$\mathbf{F}_m = I\left(\int_a^b d\mathbf{l}\right)\times\mathbf{B} = I(l)\times\mathbf{B}$$

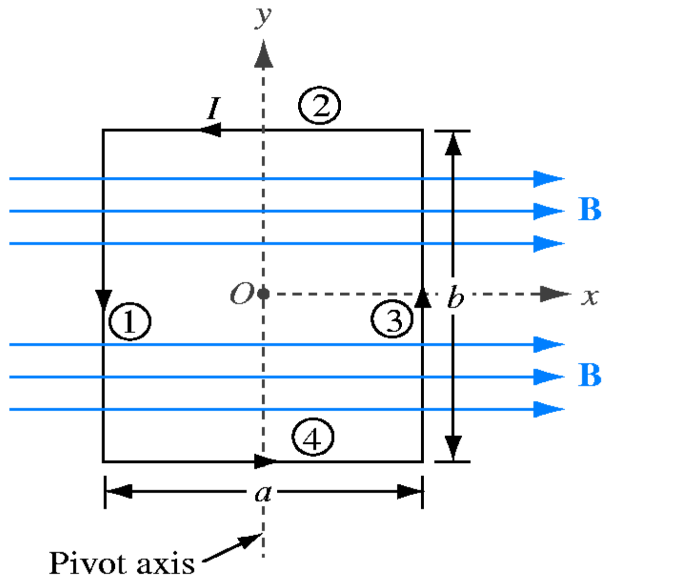
$$\mathbf{F}_2 = \mathbf{F}_4 = 0$$ *Why?*

$$\mathbf{F}_1 = I(-\hat{\mathbf{y}}b)\times\hat{\mathbf{x}}B_0 = \hat{\mathbf{z}}IbB_0$$

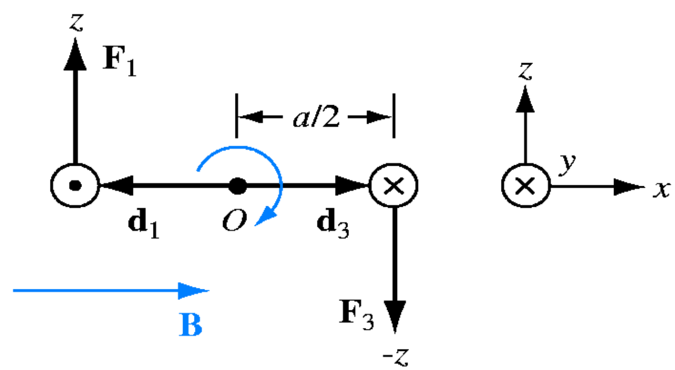
$$\mathbf{F}_3 = I(\hat{\mathbf{y}}b)\times\hat{\mathbf{x}}B_0 = -\hat{\mathbf{z}}IbB_0$$

$$\mathbf{T} = \mathbf{d}_1\times\mathbf{F}_1 + \mathbf{d}_3\times\mathbf{F}_3 = \hat{\mathbf{y}}IAB_0$$

★ Only valid for **B**//loop plane

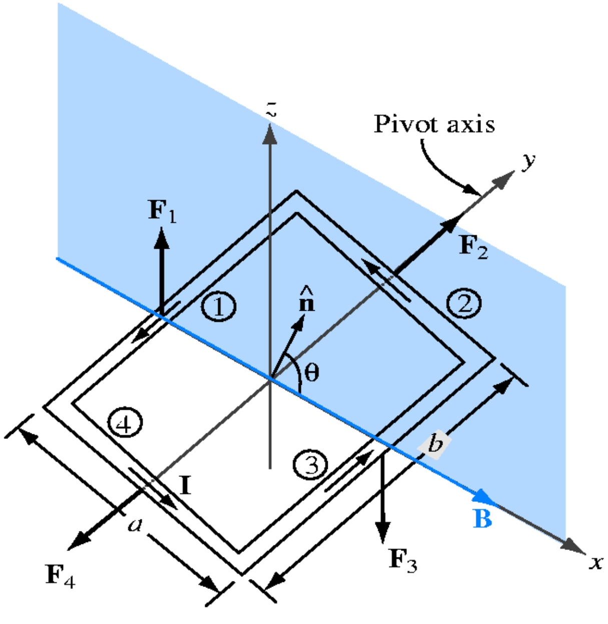


(a)



(b)

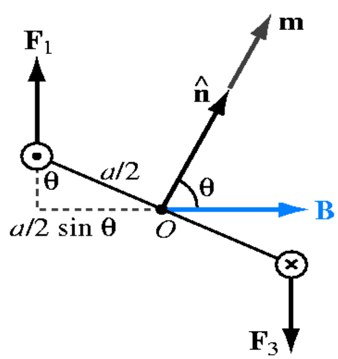
Magnetic Torque (Cont'ed)



(a)

Loop plane starts to rotate.

$$T = IAB_0 \sin\theta$$



(b)

Moment arm, **d** = (*a*/2)sin θ

Exercise

A square coil of 200 turns and 0.5-m-long sides is in a region with a uniform magnetic flux density of 0.2 T. If the maximum magnetic torque exerted on the coil is 4×10^{-2} (N.m), what is the current flowing in the coil?

I = 4 mA

The Biot-Savart Law

In 1820, Hans Oersted demonstrates that an electric current in a wire causes a compass needle to orient itself perpendicular to the wire.

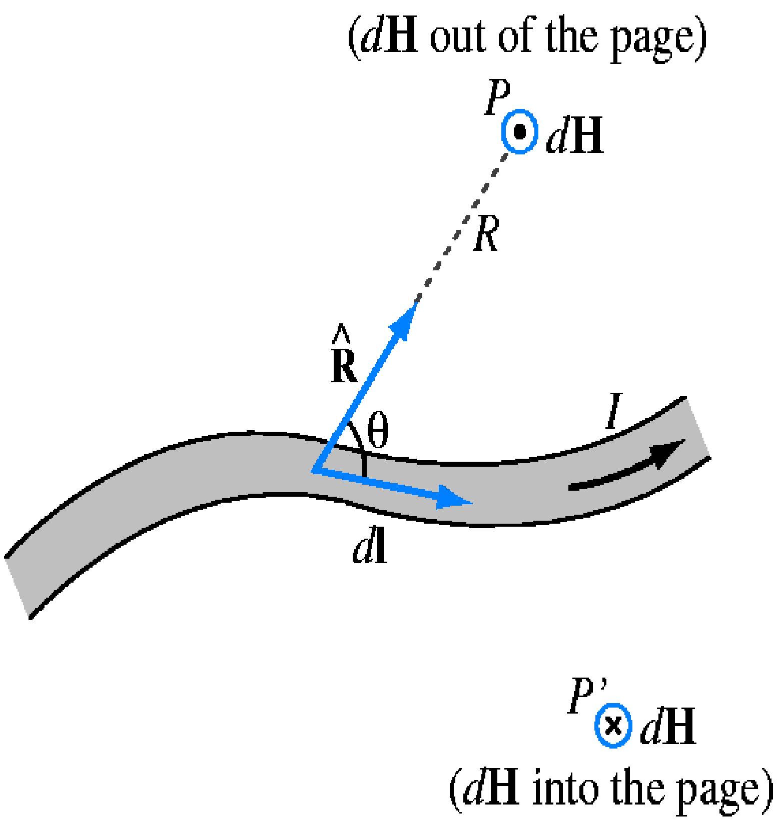
⇩

At the same year, Jean Biot and Felix Savart develop the **Biot-Savart law** relating the magnetic field induced by a wire segment to the current flowing through it.

⇩

$$d\mathbf{H} = \frac{I}{4\pi} \frac{d\mathbf{l} \times \hat{\mathbf{R}}}{R^2} \quad \text{(A/m)}$$

Biot-Savart Law (cont'ed)



$$d\mathbf{H} = \frac{I}{4\pi}\frac{d\mathbf{l}\times\hat{\mathbf{R}}}{R^2} \quad \text{(A/m)}$$

R ≡ distance vector between ***dl*** and point *P*

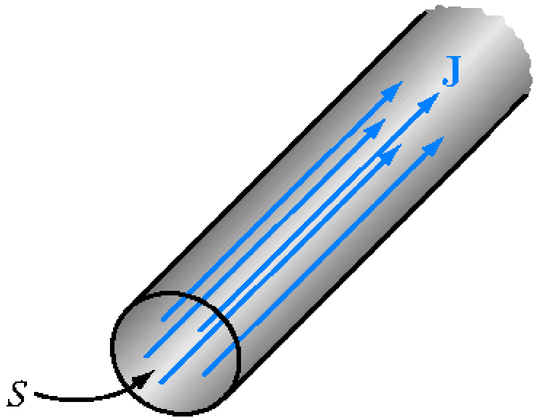
$d\mathbf{l}$ ≡ along the direction of the current I

$\hat{\mathbf{R}}$ ≡points *from* the current elements *to* point *P*.

To get total **H**, the Biot-Savart law becomes

$$\mathbf{H} = \frac{I}{4\pi}\int_l \frac{d\mathbf{l}\times\hat{\mathbf{R}}}{R^2} \quad \text{(A/m)}$$

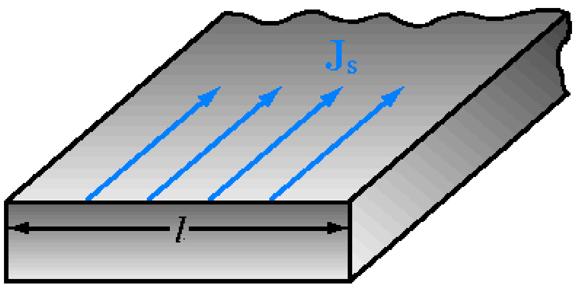
Biot-Savart Law (cont'ed)



(a) Volume current density **J** in (A/m^2)

Volume current density **J** [A/m^2]

$$\mathbf{H} = \frac{1}{4\pi}\int_v \frac{\mathbf{J}\times\hat{\mathbf{R}}}{R^2}\,dv \qquad \text{(A/m)}$$



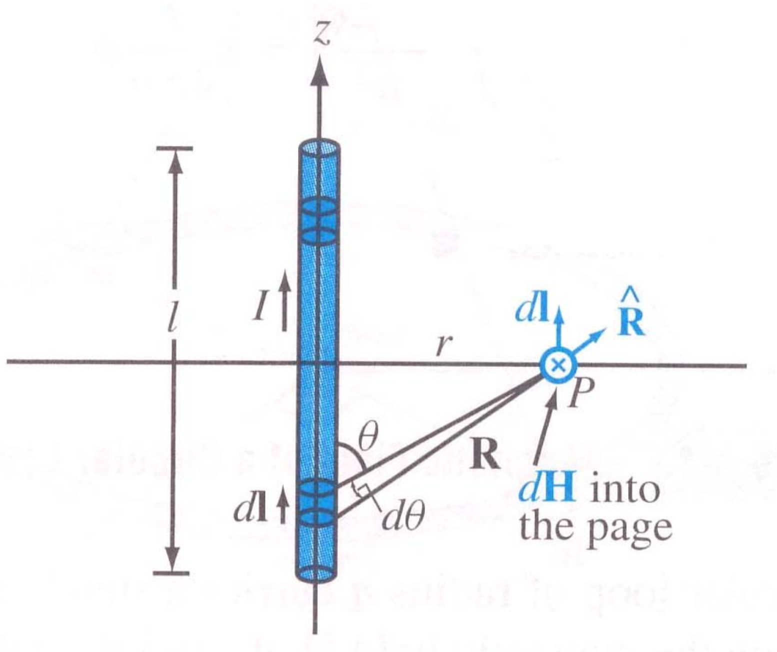
(b) Surface current density $\mathbf{J}_s$ in (A/m)

Surface current density $\mathbf{J}_s$ [A/m]

$$\mathbf{H} = \frac{I}{4\pi}\int_s \frac{\mathbf{J}_s\times\hat{\mathbf{R}}}{R^2}\,ds \qquad \text{(A/m)}$$

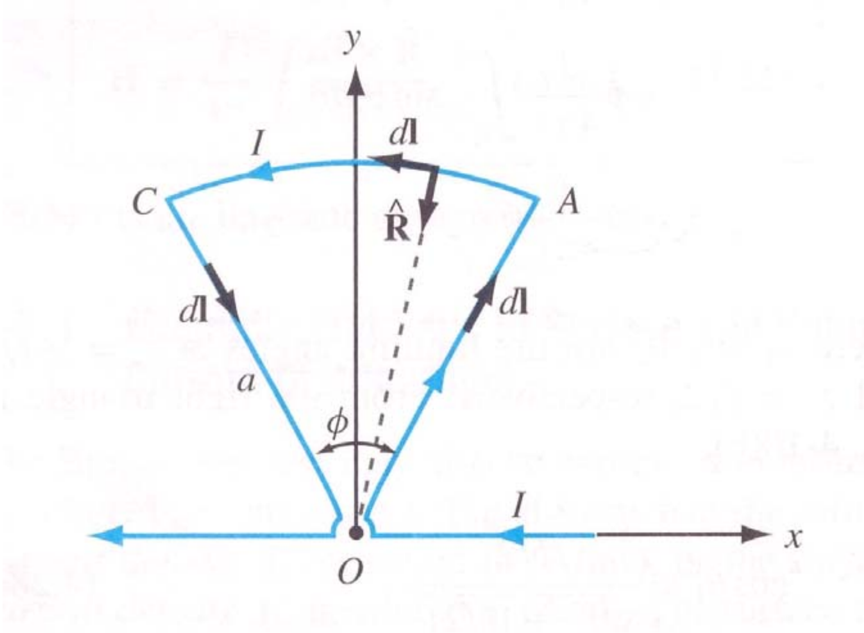
Example (Ulaby pp. 129)

A linear conductor of length *l* and carrying a current *I* is placed along the *z*-axis as shown in the figure. Determine the magnetic flux density **B** at a point *P* located at a distance *r* in the *x*-*y* plane in free space.



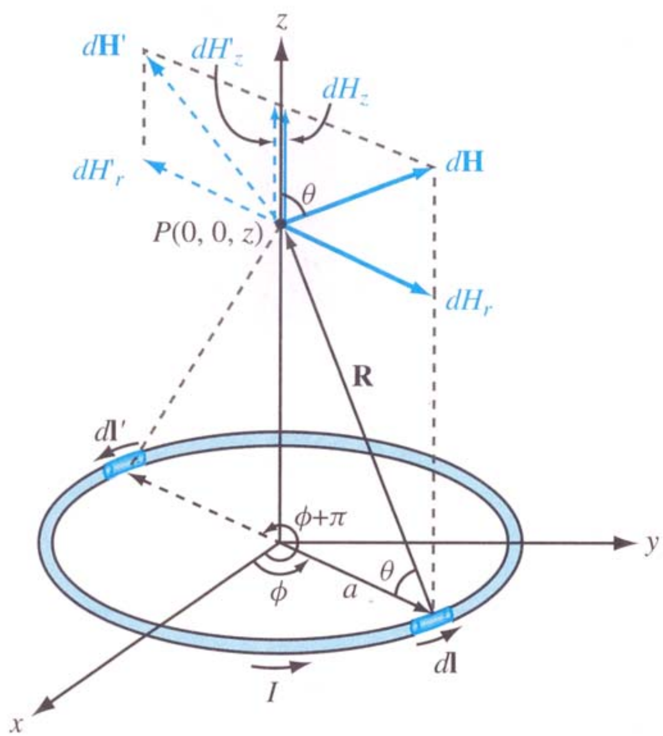
Example (Ulaby pp. 130)

Determine the magnetic field at the apex *O* of the pie-shaped loop shown in the figure. Ignore the contributions to the field due to the current in the small arcs near *O*.

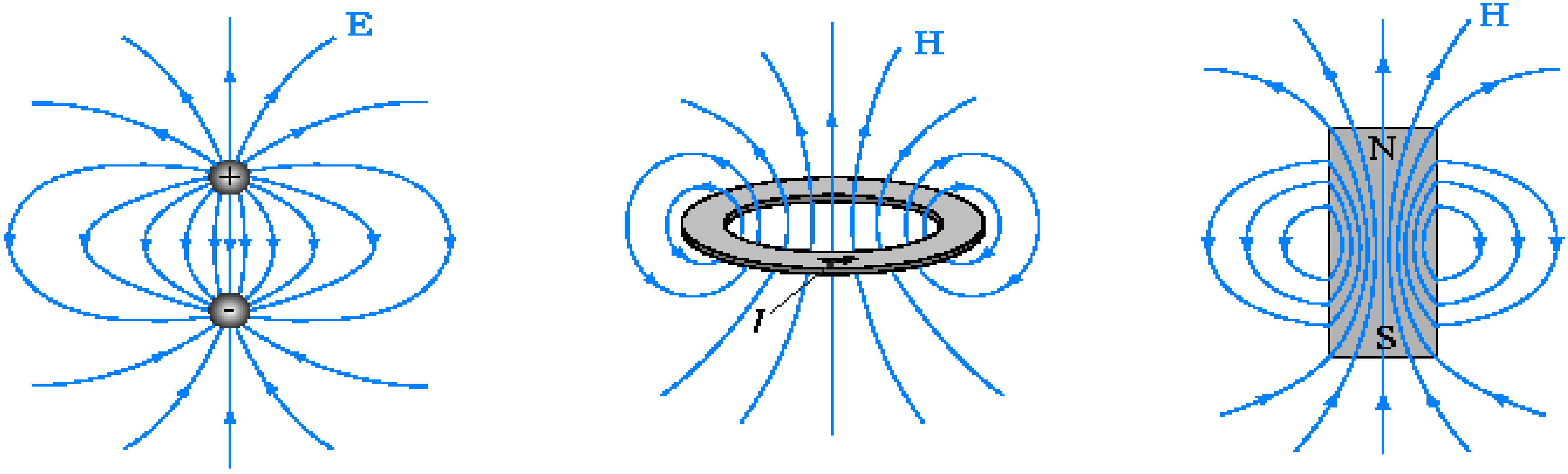


Example (Ulaby pp. 130-131)

A circular loop of radius *a* carries a steady current *I*. Determine the magnetic field **H** at a point on the axis of the loop.



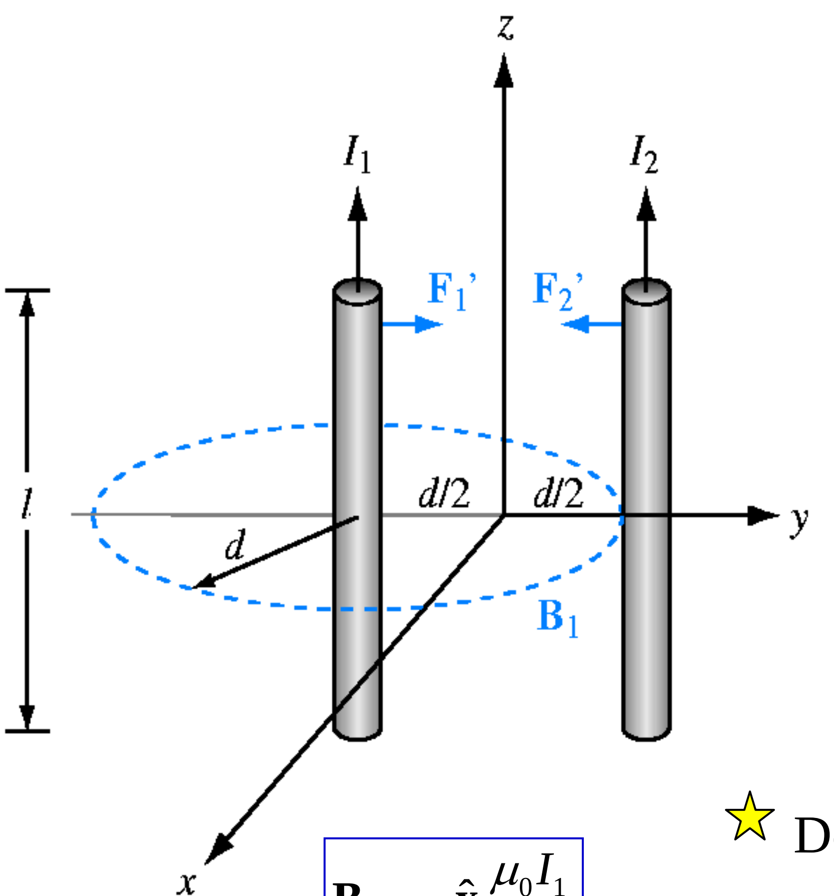
Magnetic Dipole



(a) Electric dipole (b) Magnetic dipole (c) Bar magnet

Their field patterns are similar in all three cases

Magnetic Force between Two // Conductor



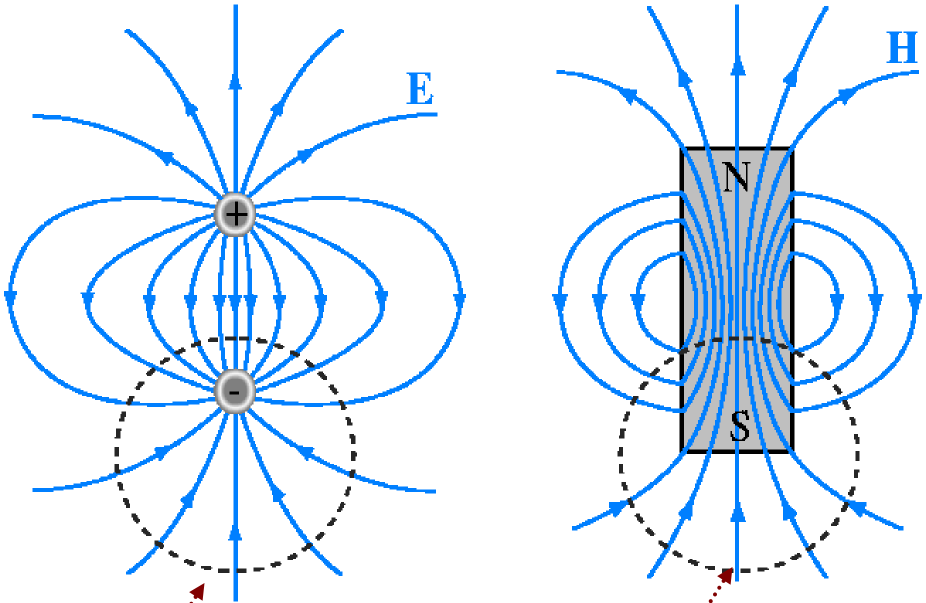
$$\mathbf{B}_1 = -\hat{\mathbf{x}}\frac{\mu_0 I_1}{2\pi d}$$

The two wires attracts each other with equal force. (I_1 and I_2 same direction)

When the currents are in opposite directions, the wires would repel each other with equal force

☆ Determine $\mathbf{B}_2$, $\mathbf{F}_1$ and $\mathbf{F}_2$

Gauss's Law for Magnetism



(a) Electric dipole (b) Bar magnet

Net electric flux ≠0

$$\oint_S \mathbf{D}\cdot d\mathbf{s} = Q$$

Net magnetic flux=0

$$\oint_S \mathbf{B}\cdot d\mathbf{s} = 0$$

$$\nabla\cdot\mathbf{B} = 0 \Leftrightarrow \oint_S \mathbf{B}\cdot d\mathbf{s} = 0$$

It is "the law of nonexistence of isolated monopoles" and "the law of conservation of magnetic flux"

Do you know magnetic poles always occur in pairs?

Ampere's Law

$$\nabla \times \mathbf{H} = \mathbf{J}$$

Maxwell's eqn

$$\int_S (\nabla \times \mathbf{H}) \cdot d\mathbf{s} = \int_S \mathbf{J} \cdot d\mathbf{s}$$

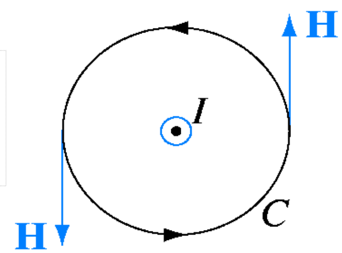
Surface integration

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = I$$

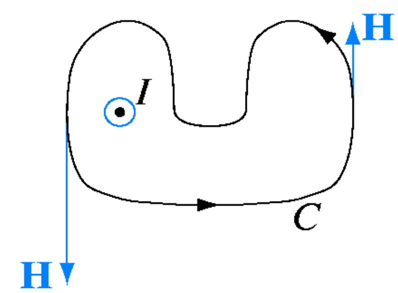
Applying Stoke's theorem

Ampere's Law

$I \equiv steady\ current$

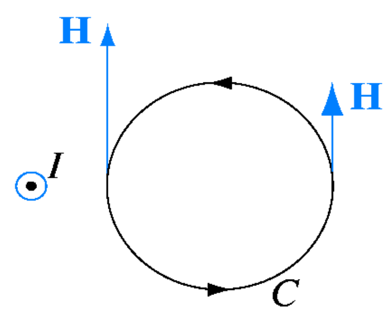


(a)



(b)

For figure (c) the $\oint_C \mathbf{H} \cdot d\mathbf{l} = 0$, therefore the choice of contour is important



(c)

Example (Ulaby pp. 135)

A long (practically infinite) straight wire of radius *a* carries a steady current *I* that is uniformly distributed over the cross section of the wire. Determine the magnetic field **H** at a distance *r* from the axis of the wire both

(a) Inside the wire ($r \le$ a) and

(b) Outside the wire ($r \ge$ a)

(c) Plot *H* versus *r*

(a) $\mathbf{H}_1 = \hat{\phi}\frac{r_1}{2\pi a^2}I$

(b) $\mathbf{H}_2 = \hat{\phi}\frac{I}{2\pi r_2{}^2}$

…To be continued… (part 2)

Thank You