

Electromagnetics Theory

Electrostatic (Part 3)

Outline:

- 3.1 Maxwell equation
- 3.2 Charge & current distribution
- 3.3 Coulomb's Law
- 3.4 Gauss Law
- 3.5 Electric Scalar Potential
- 3.6 Conductors**
- 3.7 Dielectrics**
- 3.8 Electric Boundary conditions**
- 3.9 Capacitance**

3.6 Conductors

3.6.1 Resistance

- $R = \frac{V}{I}$

where we know,

$$V = -\int \vec{E} \cdot d\vec{l} \quad (1)$$

Current flow a
cross section A



$$I = \int_s \vec{J} \cdot d\vec{s} \quad (2)$$

$$\vec{J} = \sigma \vec{E}$$

Thus

$$R = \frac{V}{I} = \frac{-\int_l \vec{E} \cdot d\vec{l}}{\int_s \vec{J} \cdot d\vec{s}} = \frac{-\int_l \vec{E} \cdot d\vec{l}}{\int_s \sigma \vec{E} \cdot d\vec{s}} (\Omega)$$

Resistance (cont.)

Where G known as conductance is given by

$$G = \frac{1}{R} \quad (\Omega^{-1}) \text{ or } (\text{S}) \textit{siemens}$$

$$G = \frac{\sigma A}{l}$$

3.7 Dielectrics

- Dielectric strength , E_{ds} - highest magnitude of \vec{E} that material can sustain without breakdown.

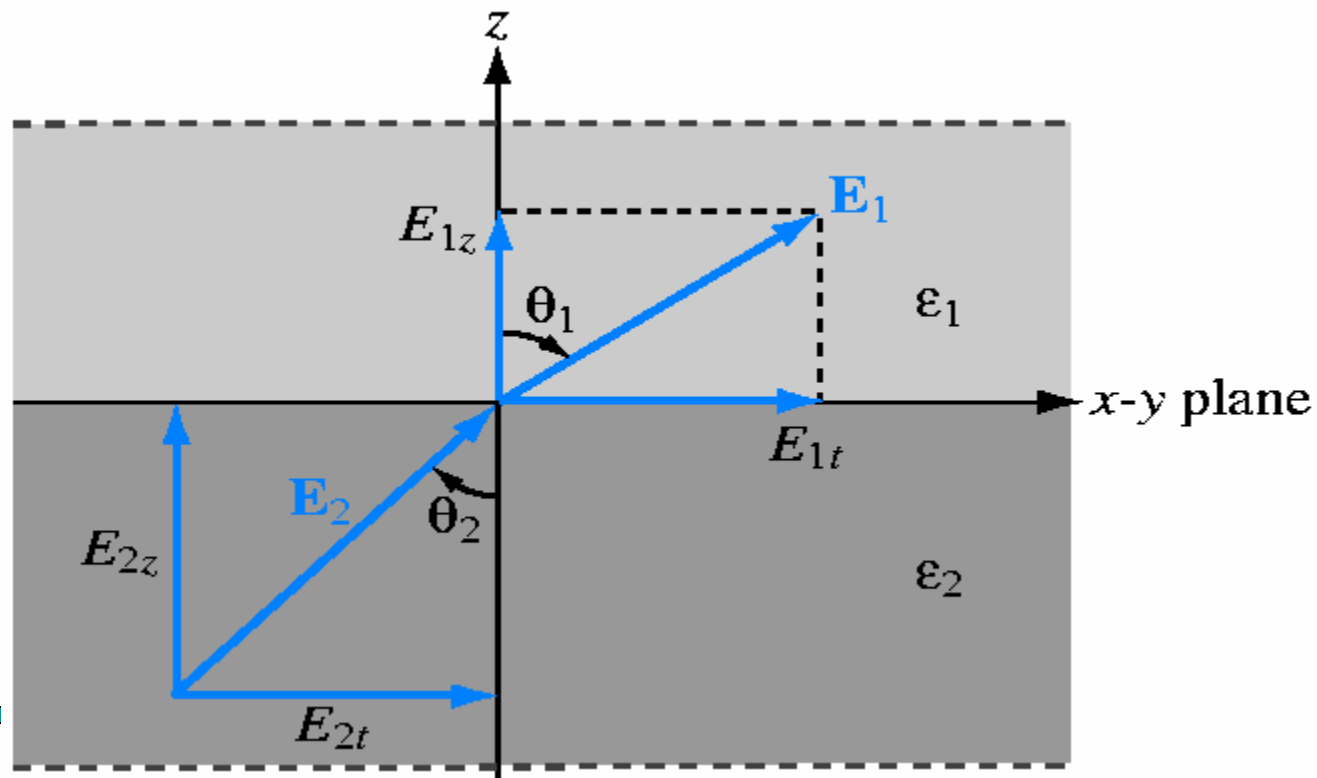
- Breakdown voltage, V_{br} :

$$V_{br} = E_{ds} d$$

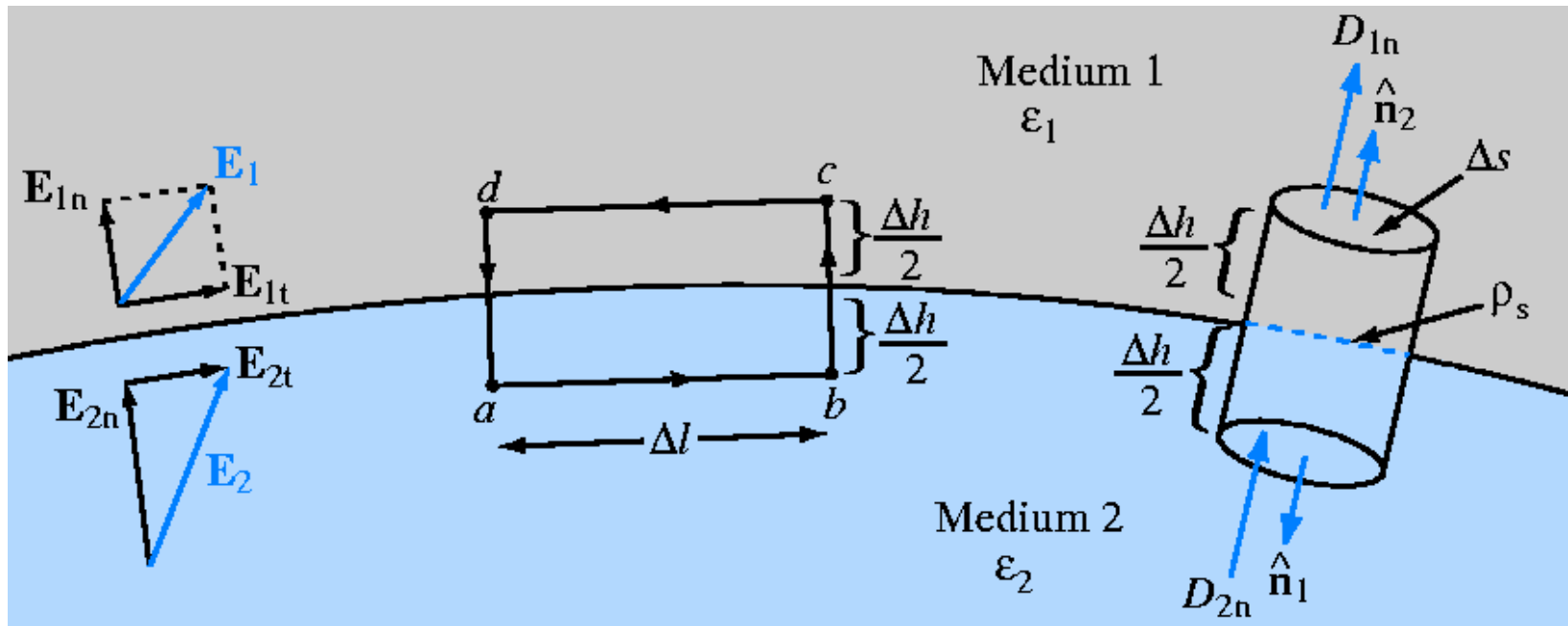
where d = distance

3.8 Electric Boundary Condition

- Previously, we have discuss \vec{E} & \vec{D} in free space or 1 medium.
- So, what happen when \vec{E} go into 2 medium?



Electric Boundary Condition (cont.)



Tangen comp: $\vec{E}_{1t} = \vec{E}_{2t}$

Normal comp: $D_{1n} - D_{2n} = \rho_s$

where $\vec{D} = \epsilon \vec{E}$

$$\frac{\vec{D}_{1t}}{\epsilon_1} = \frac{\vec{D}_{2t}}{\epsilon_2}$$

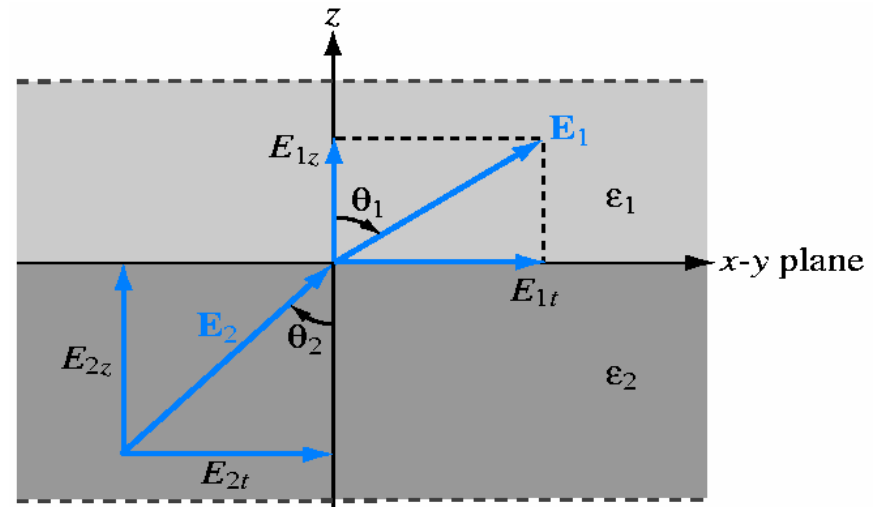
$$\epsilon_1 \vec{E}_{1n} = \epsilon_2 \vec{E}_{2n} = \rho_s$$

Example 3.7 - Electric Boundary Condition

(pg 96)

The x-y plane is a charge-free boundary separating two dielectric media with permittivities ϵ_1 and ϵ_2 as shown in fig. If the electric field in medium 1 is

$$\vec{E}_1 = E_{1x}\hat{x} + E_{1y}\hat{y} + E_{1z}\hat{z}$$



Find:

- The electric field \vec{E}_2 in medium 2.
- The angles θ_1 and θ_2 .

Exercise (Electric Boundary Condition)

With reference to fig in Example 3.7, find \vec{E}_1 if

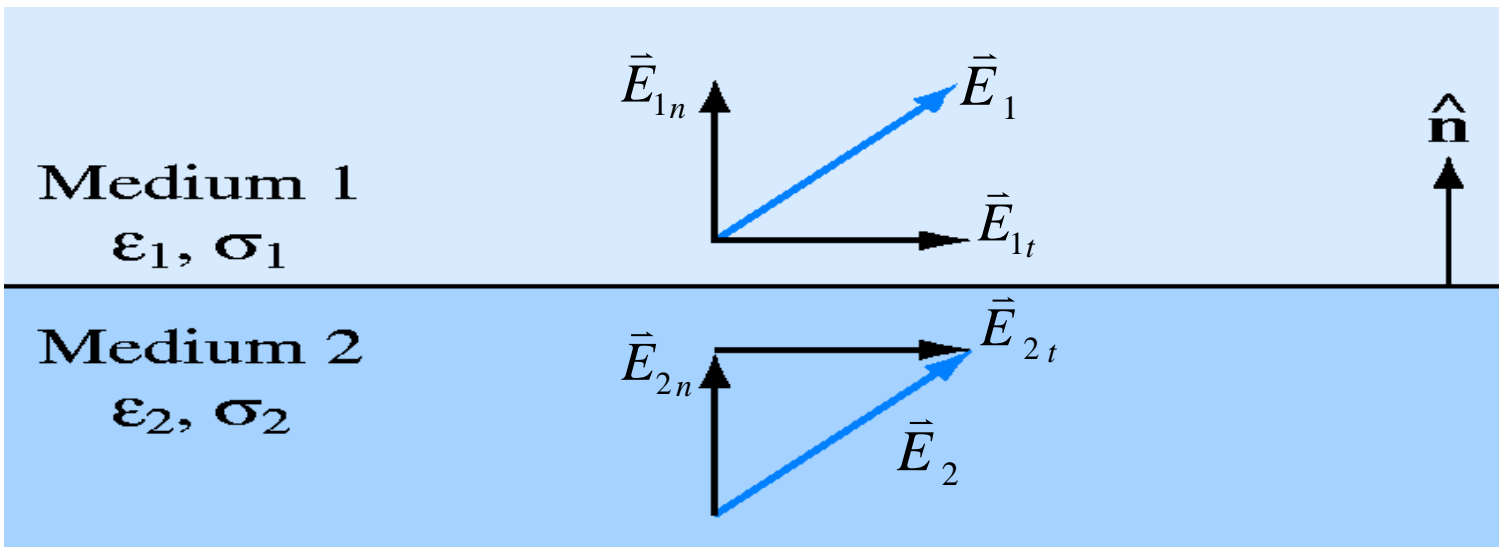
$$\vec{E}_2 = 4\hat{x} - 3\hat{y} + 3\hat{z} \text{ (V / m)}$$

where $\epsilon_1 = 2\epsilon_0$

$$\epsilon_2 = 8\epsilon_0$$

Assume the boundary to be charge free.

3.8.1 Dielectric-Conductor Boundary



- At conductor surface $\vec{D}_1 = \epsilon_1 \vec{E}_{1n} = \rho_s \hat{n}$
- For $+ve \rho_s$: \vec{E} points away from conductor surface
- For $-ve \rho_s$: \vec{E} points directly toward conductor surface

3.8.2 Conductor-Conductor Boundary

$$D_{1n} - D_{2n} = \rho_s$$

where $\vec{D} = \epsilon \vec{E}$

Thus;

$$\epsilon_1 \vec{E}_{1n} = \epsilon_2 \vec{E}_{2n} = \rho_s$$

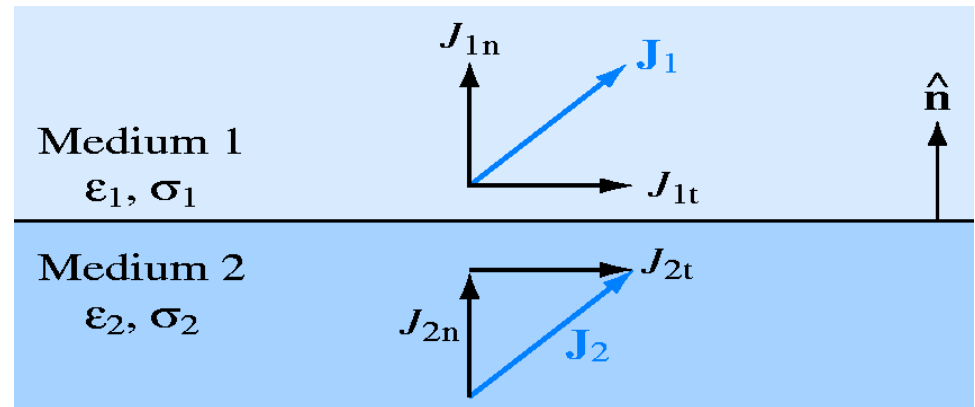
Since, we are conductor medium \vec{E} give rises to \vec{J}
(Current density)

$$\vec{J} = \sigma \vec{E}$$

Thus;

$$\vec{J}_1 = \sigma_1 \vec{E}_1$$

$$\vec{J}_2 = \sigma_2 \vec{E}_2$$



Conductor-Conductor Boundary (cont.)

Tangen comp:

$$\vec{J}_{1t} = \sigma_1 \vec{E}_{1t}$$

$$\vec{J}_{2t} = \sigma_2 \vec{E}_{2t}$$

$$E_{1t} = E_{2t}$$

$$\therefore \frac{J_{1t}}{\sigma_1} = \frac{J_{2t}}{\sigma_2}$$

Normal comp:

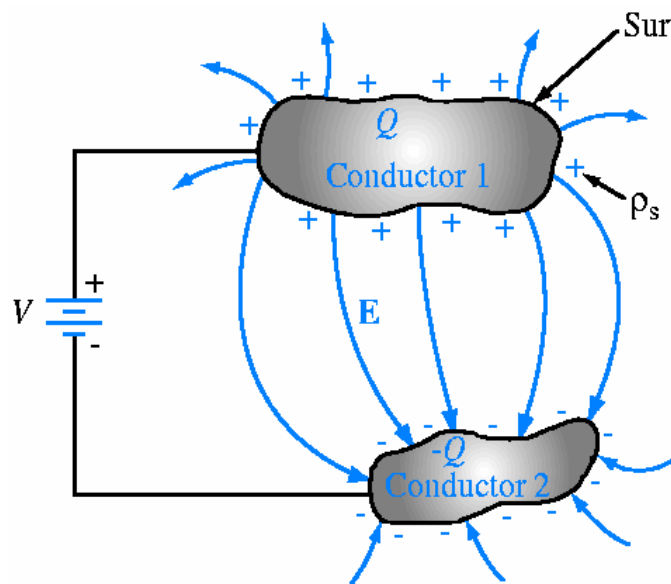
$$D_{1n} = D_{2n}$$

$$\frac{\epsilon_1 J_{1n}}{\sigma_1} - \frac{\epsilon_2 J_{2n}}{\sigma_2} = \rho_s$$

3.9 Capacitance

• Where capacitance, C define as

$$C = \frac{Q}{V} \text{ (C/V or F)} \quad \dots(1)$$



$$Q = \int \rho_s d_s \quad \dots(2)$$

and

$$V = -\int \vec{E} \cdot d\vec{l} \quad \dots(3)$$

$$\therefore C = \frac{Q}{V} = \frac{\int \rho_s d_s}{-\int \vec{E} \cdot d\vec{l}} \quad \dots(4)$$

• Normal component of \vec{E} at any point of conductor surface given by

$$E_n = \vec{E} \cdot \hat{n} = \frac{\rho_s}{\epsilon} \quad \dots(5)$$

Capacitance (cont.)

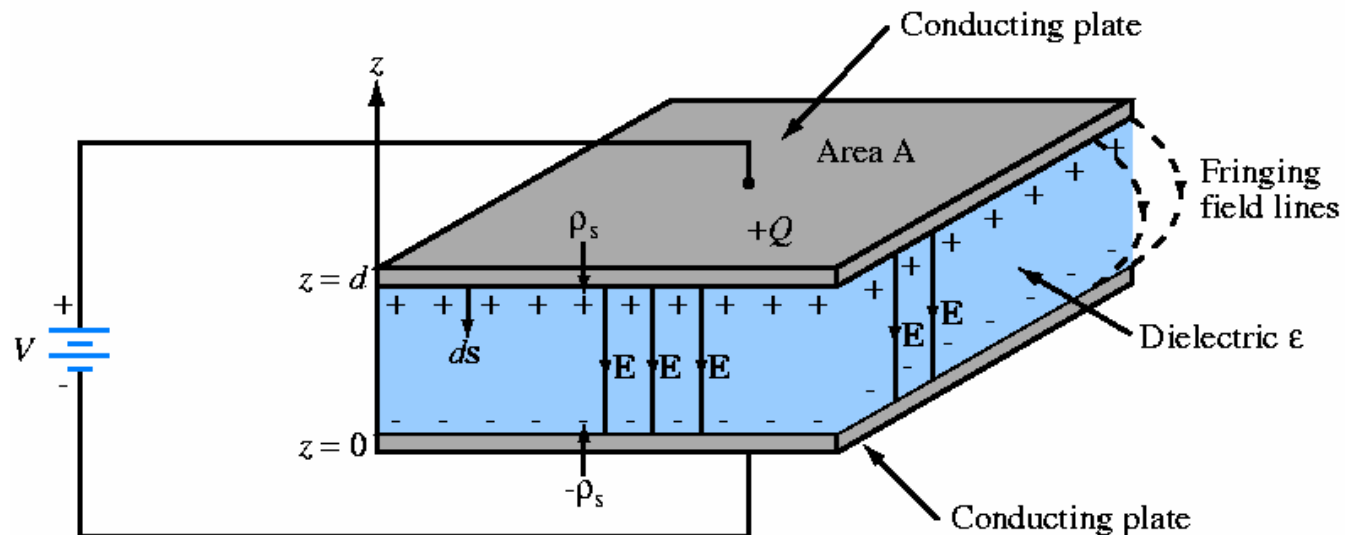
•Substitute (5) into (4)

$$C = \frac{Q}{V}$$
$$= \frac{\int \rho_s d_s}{-\int \vec{E} \cdot d\vec{l}}$$
$$= \frac{\int \epsilon \vec{E} \cdot \hat{n} d_s}{-\int \vec{E} \cdot d\vec{l}}$$

$$\therefore C = \frac{\int \epsilon \vec{E} \cdot d_s}{-\int_l \vec{E} \cdot d\vec{l}} \quad (\text{F})$$

Example 3.8 - Capacitance (pg 100)

- Obtain the expression for the capacitance C of a parallel plate capacitor comprised of two parallel plates each of surface area A and separated by distance d . the capacitor is filled with dielectric material with permittivity ϵ .
- Find the breakdown voltage if $d = 1\text{cm}$ and the dielectric material is quartz.



End of Chapter 3
Thank you