Electromagnetics Theory

Electrostatic (Part 2)

Outline:

3.1 Maxwell equation

3.2 Charge & current distribution

3.3 Coulomb's Law

3.4 Gauss Law

3.5 Electric Scalar Potential

3.6 Conductors

3.7 Dielectrics

3.8 Electric Boundary conditions

3.9 Capacitance

3.5 Electric Scalar Potential

3.5.1 Electric Potential as a Function of Electric Scalar

•When there's $\vec{F}_e$, thus work is produce, W .

• $\vec{F}_e$ produced by charge q which is moving a distance of dl by the presence of field .

$$\vec{F}_e = q\vec{E} \quad (\mathrm{N})$$

•If we move charge q against the force $\vec{F}_e$, we need to provide external forces to contract $\vec{F}_e$.

•Therefore

$$\vec{F}_{ext} = -\vec{F}_e = -q\vec{E}$$

Electric Scalar Potential (cont.)

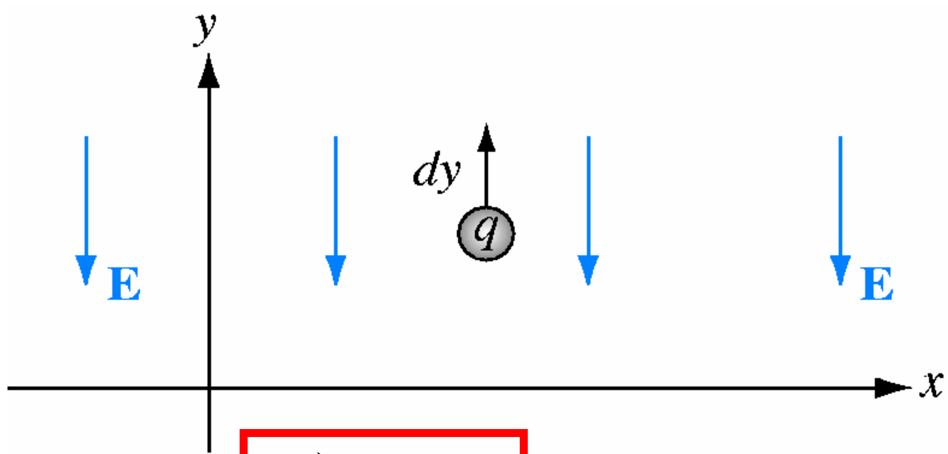
•Where function for *work done* or *energy expended* with a distance of dl under influence of $\vec{F}_{ext}$ is

$$dW = \vec{F}_{ext} \bullet d\vec{l}$$
$$= -q\vec{E} \bullet d\vec{l} \quad (\mathrm{J}) \quad \ldots\ldots\ldots\ldots\ldots(1)$$

Thus

$$W = -\int_a^b q\vec{E} \bullet d\vec{l} \quad (\mathrm{J}) \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Electric Scalar Potential (cont.)



From fig above,

$$d\vec{l} = dy\hat{y}$$

$$\vec{E} = -\vec{E}\hat{y}$$

Thus

$$\begin{aligned} dW &= -q\vec{E} \bullet d\vec{l} \\ &= -q\left(-\vec{E}\hat{y}\right) \bullet \left(dy\hat{y}\right) \\ &= -q\vec{E}dy \end{aligned}$$

Electric Scalar Potential (cont.)

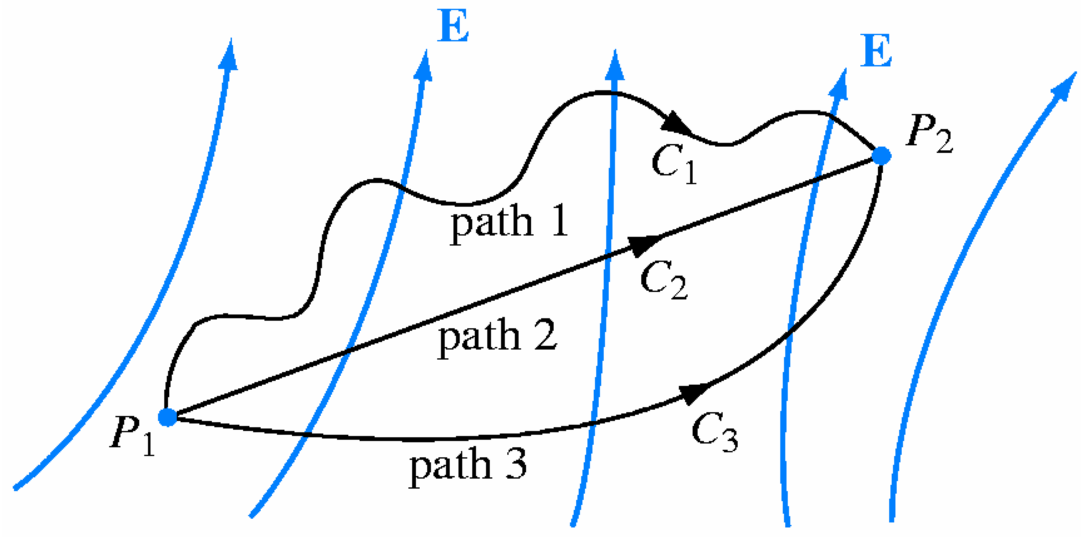
The differential electric potential or Differential voltage, dv is given by

$$dv = \frac{dW}{q}$$

$$= -\vec{E} \bullet d\vec{l} \; (\text{J/c or V}) \; ....(3)$$

3.5.2 Potential diff between 2 point charge

- Potential difference between two points P_2 & P_1 are shown in fig. below



- By integrating eq (3)

$$\int_a^b dv = \int_a^b \frac{dW}{q}$$

$$= -\int_{a/p_1}^{b/p_2} \vec{E} \bullet d\vec{l}$$

$$= V_{21} = V_2 - V_1 \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Potential diff between 2 point charge (cont.)

- In fact, the line integral of the electrostatic field around any closed contour C is zero

$$\oint_c \vec{E} \bullet d\vec{l} = 0 \quad \ldots\ldots\ldots\ldots\ldots(5)$$

- From the 2nd Maxwell equation; if $\frac{\partial}{\partial t} = 0$,then

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\Downarrow$$

$$\nabla \times \vec{E} = 0 \quad \ldots\ldots\ldots(6)$$

Potential diff between 2 point charge (cont.)

- If we take the surface integral of $\nabla \times \vec{E}$ over an open surface S & by applying Stoke's theorem:

$$\int_s \left(\nabla \times \vec{B}\right) \bullet d\vec{s} = \oint_c \vec{B} \bullet d\vec{l}$$

- Thus we get

$$\int_s \left(\nabla \times \vec{E}\right) \bullet d\vec{s} = \oint_c \vec{E} \bullet d\vec{l} = 0 \quad ....(7)$$

Potential diff between 2 point charge (cont.)

- Usually, reference point V_1 is at ∞ , thus from eq (4), we assume that V_1=0 at P_1 is at ∞ .

- Therefore the electric potential V at any point P is given by

$$V = -\int_{\infty}^{P} \vec{E} \bullet d\vec{l} \quad \ldots\ldots\ldots\ldots(8)$$

Exercise (Potential Difference)

A spherical shell of radius R has a uniform surfaces charge density ρ_s . Determine the electric potential at the center of the shell.

$$V = \frac{\rho_s R}{\varepsilon} \quad V$$

3.5.2 Electric Potential due to point charge

- From last discussion given: $$\vec{E}_1 = \frac{q_1}{4\pi\varepsilon R^2}\hat{R} \quad \text{V/m}$$

- We know $$V = -\int_{\infty}^{R} \vec{E} \bullet d\vec{l}$$

$$= -\int_{\infty}^{R} \left(\frac{q}{4\pi\varepsilon R^2}\hat{R} \right) \bullet dR\hat{R}$$

$$V = \frac{q}{4\pi\varepsilon R} \quad (\mathrm{V}) \quad ....(1)$$

Where R – distance of

$$dl - dR\hat{R}$$

Electric Potential due to point charge (cont.)

- If q is not at origin, but at $\vec{R}_1$, then V at observation become

$$V\left(\vec{R}\right)=\frac{q}{4\pi\varepsilon\left|\vec{R}-\vec{R}_1\right|}\quad(\mathrm{V})\quad\ldots\ldots\ldots(2)$$

- General eq for V:

$$V\left(\vec{R}\right)=\frac{1}{4\pi\varepsilon}\sum_{i=1}^{N}\frac{q_i}{\left|\vec{R}-\vec{R}_1\right|}\quad(\mathrm{V})\quad\ldots\ldots\ldots(3)$$

3.5.3 Electric Potential due to continuous distribution

•Replace qi with $\rho_l d_l, \rho_s d_s$ & $\rho_v d_v$.

•Convert sum to integration.

•Define $R = \left|\vec{R} - \vec{R}_1\right|$ as distance between integration point & observation point.

$$\therefore \quad V(\vec{R}) = \frac{1}{4\pi\varepsilon}\int_l \frac{\rho_l d_l}{R} \text{ (V)}$$

$$V(\vec{R}) = \frac{1}{4\pi\varepsilon}\int_s \frac{\rho_s d_s}{R} \text{ (V)}$$

$$V(\vec{R}) = \frac{1}{4\pi\varepsilon}\int_v \frac{\rho_v d_v}{R} \text{ (V)}$$

3.5.4 Electric Potential as a function of electric Potential

- Previously

$$dV = -\vec{E} \bullet d\vec{l} \cdots\cdots\cdots (1)$$

- Scalar function V, given by

$$dV = \nabla V \bullet d\vec{l} \cdots\cdots\cdots (2)$$

- By comparing (1) & (2)

$$\vec{E} = -\nabla V$$

3.5.5 Poison's Equation

- In $\vec{E}$ as function of V, we have

$$\vec{E} = -\nabla V \qquad \cdots\cdots(1)$$

& from 1st Maxwell eq:

$$\nabla \bullet \vec{D} = \rho_v \quad \& \quad \vec{D} = \varepsilon \vec{E}$$

- Thus

$$\nabla \bullet \vec{E} = \frac{\rho_v}{\varepsilon} \qquad \cdots\cdots(2)$$

Poison's Equation (cont.)

- Substitute (1) into (2)

$$\nabla \bullet (-\nabla V) = \frac{\rho_v}{\varepsilon} \quad \Rightarrow \quad \nabla \bullet (\nabla V) = -\frac{\rho_v}{\varepsilon}$$

Or

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$$

← Poisson's equation

- If medium has no free changes thus eq (3) reduce to zero

$$\nabla^2 V = 0$$

← Laplace equation

….. To be continued …..

Thank You