Electromagnetics Theory

Electrostatic (Part 1)

Outline:

3.1 Maxwell equation

3.2 Charge & current distribution

3.3 Coulomb's Law

3.4 Gauss Law

3.5 Electric Scalar Potential

3.6 Conductors

3.7 Dielectrics

3.8 Electric Boundary conditions

3.9 Capacitance

Electrostatic

Electro – electric field

Static - (not moving)

- So, we can say that it's a time independent field.
- Depends only on position.
- Source – stationary charges

3.1 Maxwell equation

$$\nabla \bullet \vec{D} = \rho_v$$

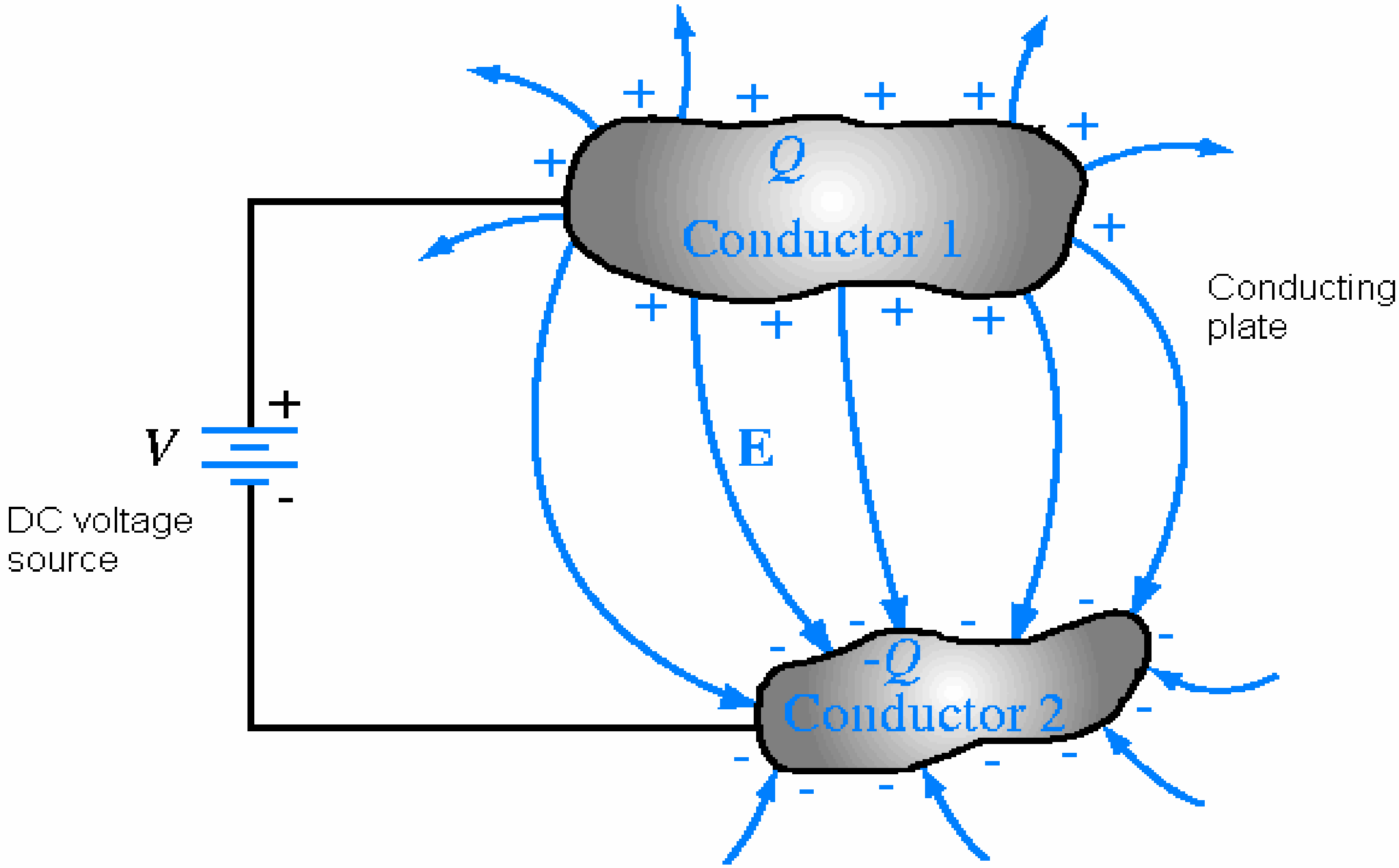
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Electric field

$$\nabla \bullet \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Magnetic field



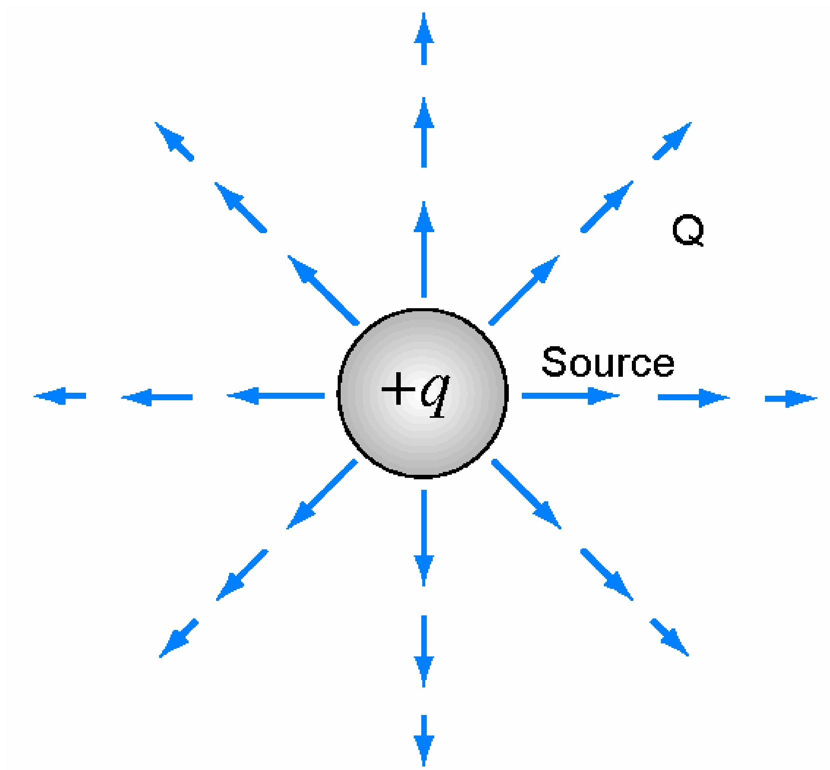
Q
Conductor 1
Conducting plate
V
DC voltage source
E
-Q
Conductor 2

3.2 Charge & current distribution

Stationary Charge

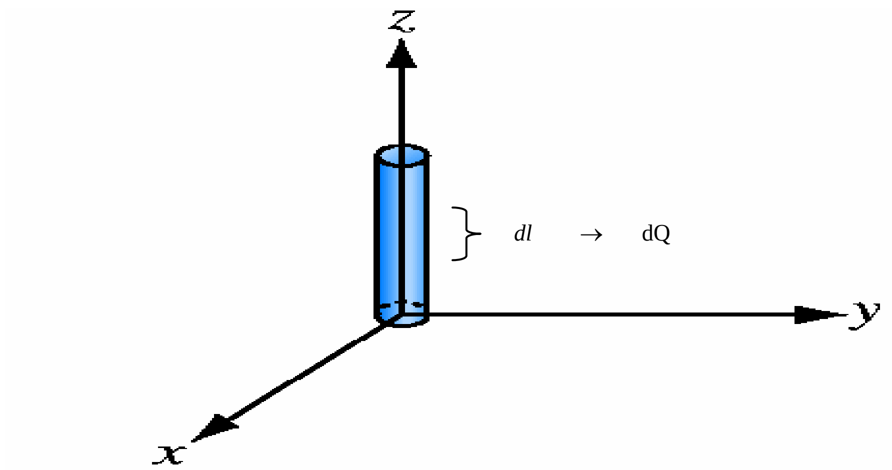
- Point Charge (titik)
- Line Charge
- Surface Charge
- Volume Charge

1. Point Charge



+ve – arrow out
-ve – arrow in

2. Line Charge



$dQ \propto dl$ -For a bigger sample more charge is carried

$$dQ = \rho_l dl$$

(ρ_l is line charge density [c/m])

$\int$

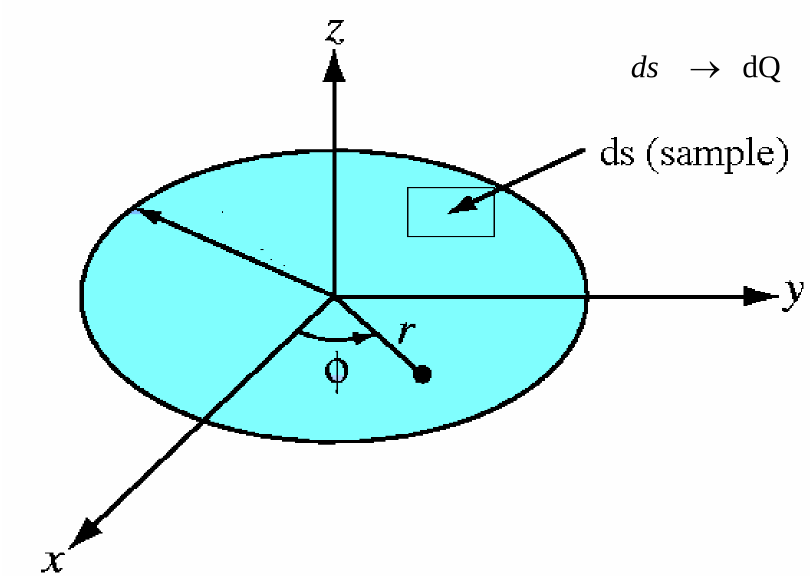
$$Q = \int dQ = \int_l \rho_l dl$$

dl → Cartesian (dx, dy, dz)

dl → Cylindrical $(dr, rd\varphi, dz)$

dl → Spherical $(dr, rd\theta, r\sin\theta d\varphi)$

3. Surface Charge



ds (area)
dQ (charge)

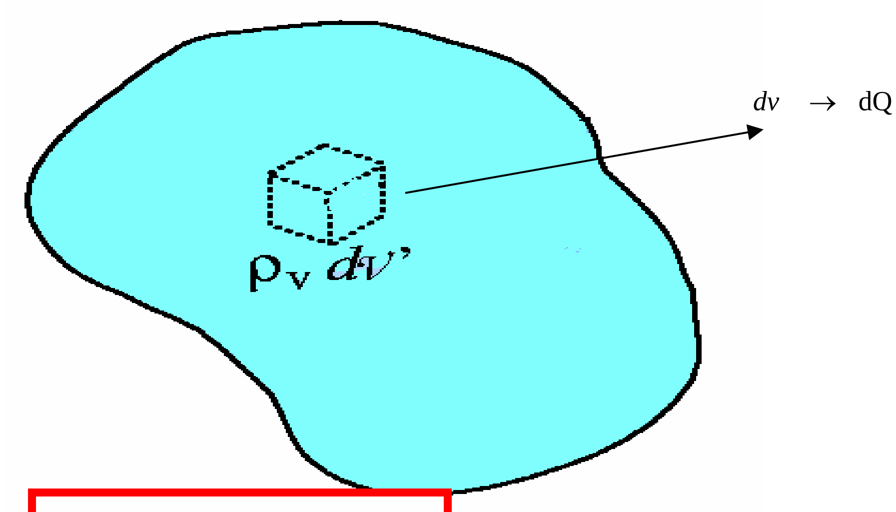
$$dQ \propto ds$$

$$dQ = \rho_s ds$$

, ρ_s is surface charge density [c/m^2]

$$Q = \int dQ = \int \rho_s ds$$

4. Volume Charge



$$dQ \propto dv$$

$$dQ = \rho_v dv$$

$$Q = \int dQ$$

$$= \int_v \rho_v dv$$

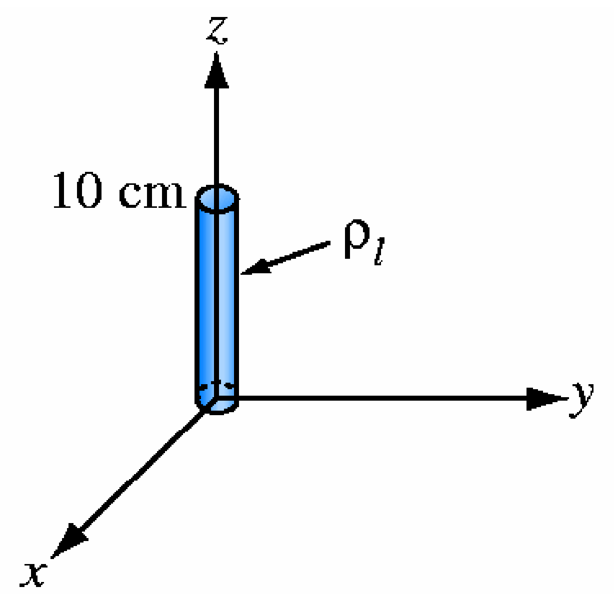
$$\iiint$$

dv → Spherical $(r^2 \sin\theta dr d\theta d\phi)$

dv → Volume charge density [c/m^3]

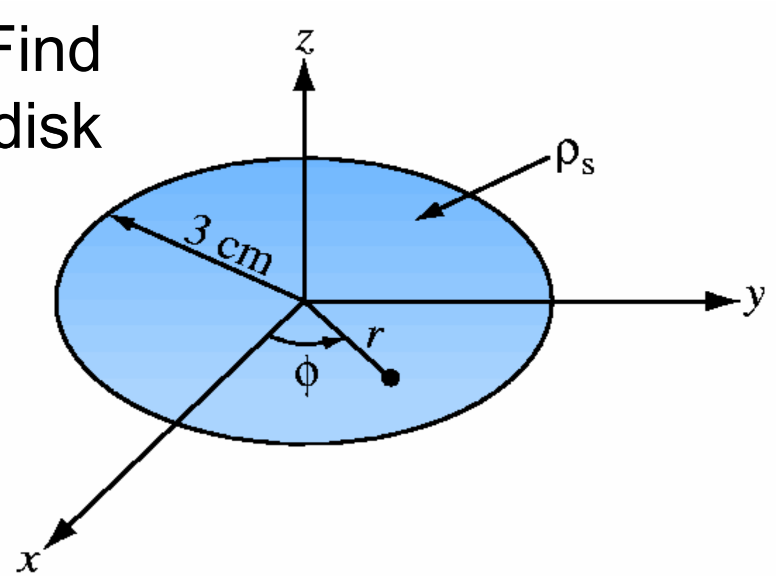
Example 3.1 – Line Charge Distribution

Calculate the total charge Q contained in a cylindrical tube of charge oriented along the z-axis as shown below. The line charge density is $\rho l = 2z$, where z is the distance in meters from the bottom end of the tube. The tube length is 10cm.



Example 3.2 – Surface Charge Distribution

The circular disk of electric charge shown below is characterized by an azimuthally symmetric surface charge density that increases linearly with r from zero at the center to 9 C/m^2 at r = 3 cm. Find the total charge present on the disk surface.



Charge Density

- Line Charge Density , ρ_l
- Surface Charge Density , ρ_s
- Volume Charge density , ρ_v

1. Line Charge Density (ρ_l)

➡ When charge is distributed along a line

We know that $dQ = \rho_l dl$

Thus

$$\rho_l = \frac{dQ}{dl} = \lim_{\Delta l \to 0} \frac{\Delta Q}{\Delta l} \quad \text{[c/m]}$$

Where

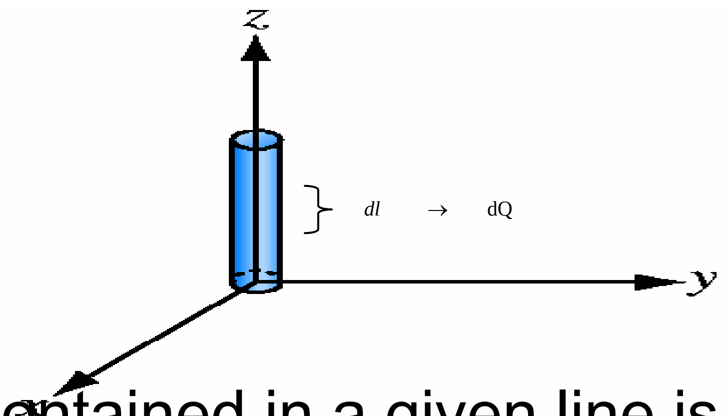
1) Δq is the charge contained in Δl

2) ρ_l is defined at a given point in space, specified by at a given time

1. Line charge density (Cont'd)

3) ρ_l represent the average charge per unit line for a line Δl

4) We can say that " the charge is distributed along a line"



5) Thus total charge contained in a given line is

$$Q = \int dQ = \int_l \rho_l \, dl$$

2. Surface Charge Density (ρ_s)

ρ_s → When dealing with conductor
↘ Electric charge is distributed across the surface material

$$dQ = \rho_s ds$$

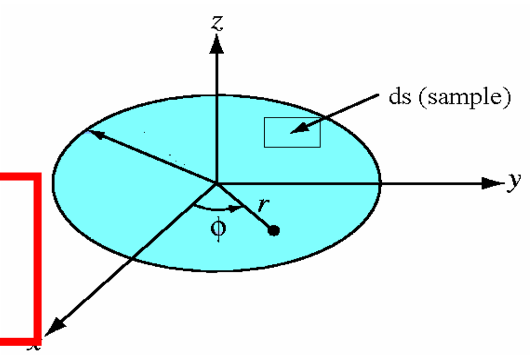
$$\rho_s = \frac{dQ}{ds} = \lim_{\Delta s \to 0} \frac{\Delta Q}{\Delta s} \quad [c/m^2]$$

Where ΔQ - charge present across an elemental surface area Δs

Thus

Total charge;

$$Q = \int dQ = \int \rho_s ds$$



3. Volume Charge Density (ρ_v)

ρ_v → Similarly is charge is distributed in a volume/space

We know

$$dQ = \rho_v dv$$

thus

$$\rho_v = \frac{dQ}{dv} = \lim_{\Delta v \to 0} \frac{\Delta Q}{\Delta v} \quad [c/m^3]$$

Where ΔQ - charge contained in Δv

Example 3.3 - Charge Density

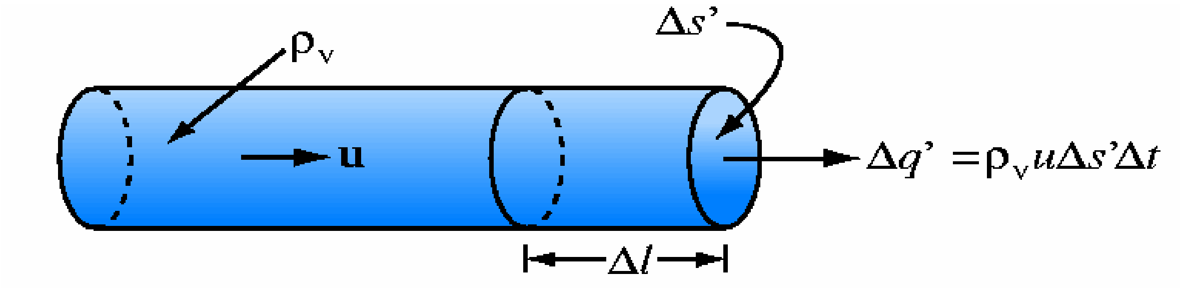
A uniform, spherical volume charge distribution contains a total charge of 10-8 C. If the radius of this spherical volume is 2 x 10-2 m, find ρ_v.

$$\rho_v = 2.984 \times 10^{-3} cm^{-3}$$

Current Density, $\vec{J}$

1. Let us consider fig 3.2(a)

   A tube of charge with volume density ρ_v



- $\vec{u}$ = charge velocity (along axis of the tube)
- over period of Δt , the charge move a distance

$$\Delta l = u\Delta t$$

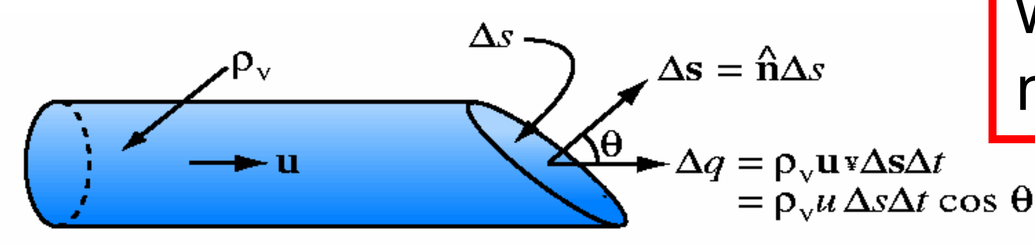
Current Density (cont'd)

2. The amount of charge that crosses the tube's cross-sectional surface $\Delta s'$ in time Δt is therefore:-

$$\Delta q' = \rho_v u \Delta s' \Delta t$$

3. More general case →

Where the charge are $\hat{n}$ flowing through a surface Δs whose surface normal not necessarily parallel to $\vec{u}$



$$\Delta \vec{s} = \hat{n} \Delta s$$

$$\Delta q = \rho_v \vec{u} \bullet \Delta \vec{s} \Delta t = \rho_v u \Delta s \Delta t \cos \theta$$

$\Delta s' =$ Cross section

Current Density (cont'd)

4. Then the current is given by :-

$$\Delta I = \frac{\Delta q}{\Delta t} = \rho_v \vec{u} \bullet \Delta \vec{s}$$

$$\boxed{\Delta I = \vec{J} \bullet \Delta \vec{s}}$$

where

$$\boxed{\vec{J} = \rho_v \vec{u}} \quad [\text{A/m}^2]$$

$\vec{J}$ is current density

Current density (cont'd)

5. Therefore, total current flow

$$I = \int_s \vec{J} \bullet d\vec{s} \quad [A]$$

When current (I) is generated by actual movements of charge matter. ← Convection Current

J ← Called Convection Current Density

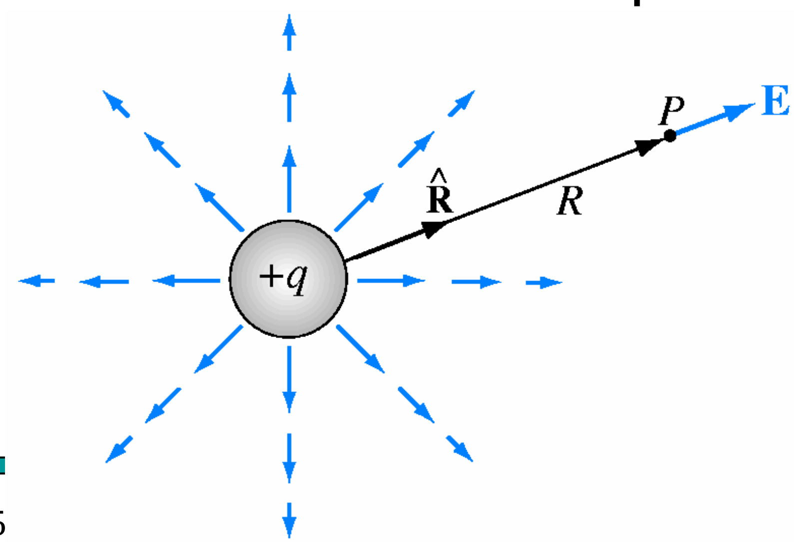
Conduction Current → Atoms of the conducting medium do not move
- obey Ohm's law

3.3 Coulomb's Law

States that an isolated charge q induces an electric field $\vec{E}$ at every point in space at every specific point P,

$$\vec{E} = \frac{q}{4\pi\varepsilon R^2}\hat{R} \quad \text{[V/m]}$$

Where $\hat{R}$ = unit vector pointing from q to p



ε = electrical permittivity

R = distance between q and p

Coulomb's Law (cont'd)

The force acting on test charge q' in the presence of electric field, $\vec{E}$ is

$$\vec{F} = q'\vec{E} \quad [N]$$

Where unit

$\vec{F}$ → N (Newtons)

q' → C (Coulombs)

$\vec{E}$ → [N/C] = V/m

Coulomb's Law (cont'd)

$\vec{D}$ and $\vec{E}$ are related by

$$\vec{D} = \varepsilon \vec{E}$$

With $\varepsilon = \varepsilon_r \varepsilon_0$

where

$\varepsilon_0 = 8.85 * 10^{-12}$ = **electrical permittivity of free space** $(\varepsilon_r = 1)$

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

→ **Relative permittivity** or **Dielectric constant**

Coulomb's Law (cont'd)

Conclude that :

$$\vec{E} = \sum_{i=1}^{n} \frac{Qi}{4\pi\varepsilon Ri^{2}} \hat{R}i$$

Where $\hat{R} = \dfrac{vector}{magnitud}$

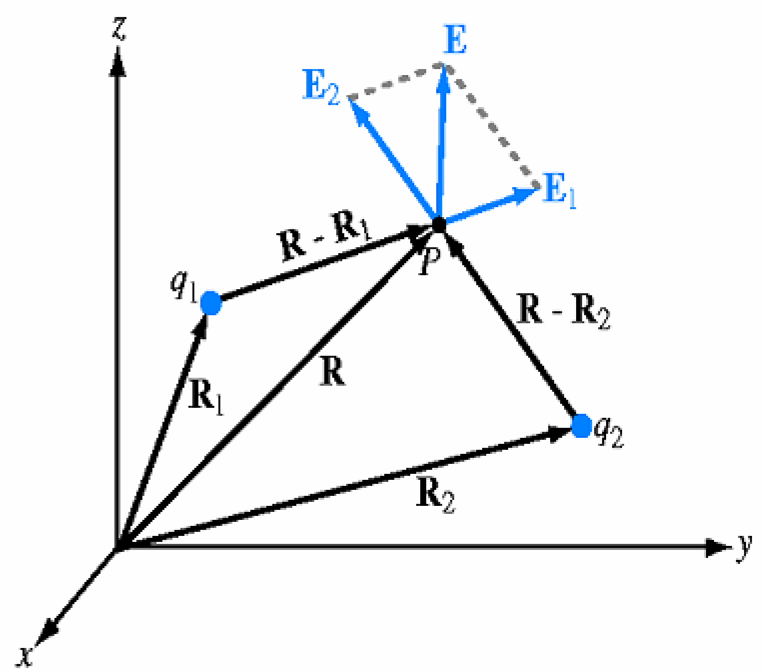
Coulomb's Law (cont'd)

3.3-1 Electric field due to multiple point charge

Where **R** = lead – tail

Thus for q_1 :

$$\vec{E}_1 = \frac{q_1}{4\pi\varepsilon R^2}\hat{R}$$



Where

$$R = \left|\vec{R} - \vec{R}_1\right|$$

$$\hat{R} = \frac{\left(\vec{R} - \vec{R}_1\right)}{\left|\vec{R} - \vec{R}_1\right|}$$

Electric field due to multiple point charge

Thus $$\vec{E}_1 = \frac{q_1\left(\vec{R}-\vec{R}_1\right)}{4\pi\varepsilon\left|\vec{R}-\vec{R}_1\right|^3}$$

Same for $\vec{E}_2$: $$\vec{E}_2 = \frac{q_2\left(\vec{R}-\vec{R}_2\right)}{4\pi\varepsilon\left|\vec{R}-\vec{R}_2\right|^3}$$

$\therefore$ at **P** due to charge q1 & q2 is equal to the vector sum of $\vec{E}_1$ & $\vec{E}_2$.

General $\vec{E}$ ⇨

$$\vec{E} = \frac{1}{4\pi\varepsilon}\sum_{i=1}^{N}\frac{q_i\left(\vec{R}-\vec{R}_i\right)}{\left|\vec{R}-\vec{R}_i\right|^3}$$

Example 3.4 – Electric field due to 2 point charge (pg 73)

Two point charge with $q_1 = 2 \times 10^{-5}$ C and $q_2 = -4 \times 10^{-5}$ C are located in a free space at (1,3,-1) and (-3,1,-2), respectively, in a Cartesian coordinate system. Find $\vec{E}$

a) The electric field at (3,1,-2)
b) The force on a 8×10^{-5} C charge located at that point.

All distance are in meters.

$$(a) \quad \vec{E} = \frac{\hat{x} - 4\hat{y} - 2\hat{z}}{108\pi\varepsilon_0} \times 10^{-5} \quad (V/m)$$

$$(b) \quad \vec{F} = \frac{2\hat{x} - 8\hat{y} - 4\hat{z}}{27\pi\varepsilon_0} \times 10^{-10} \quad (N)$$

3.3-2 Electric field due to a charge distribution

Electric field can be divided into **3 types of charge distribution**:

a) Line charge

b) Surface charge

c) Volume charge

a) Line charge

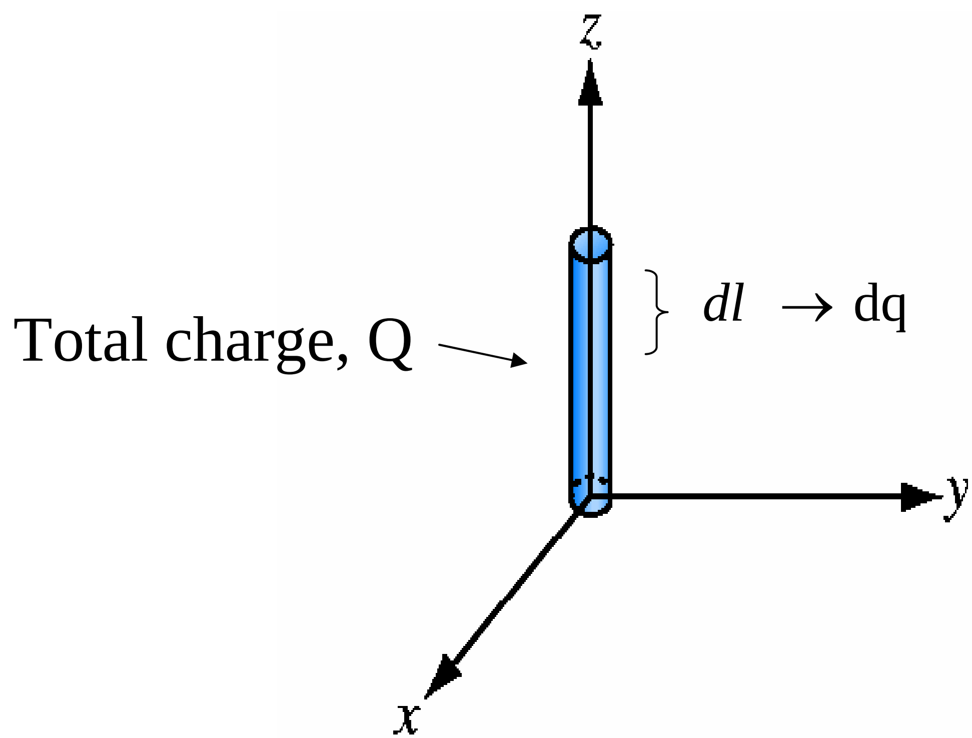
We know $\vec{E} = \dfrac{q}{4\pi\varepsilon R^2}\hat{R}$

But due to the charge dq

Thus, $\vec{E} = \int_l \dfrac{dq}{4\pi\varepsilon R^2}\hat{R}$

$$\vec{E} = \int_l d\vec{E}$$

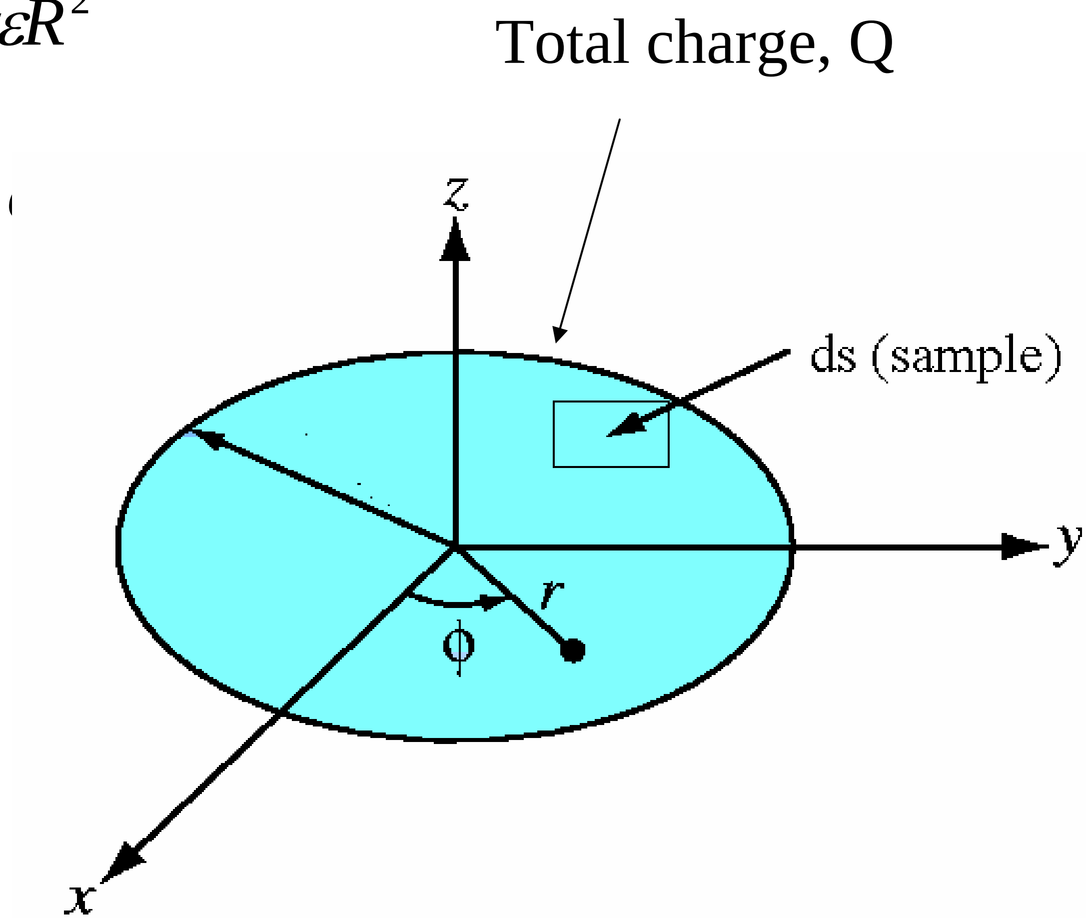
Where $dq = \rho_l dl$



b) Surface charge

We know $\vec{E} = \dfrac{q}{4\pi\varepsilon R^2}\hat{R}$

But due to the charge



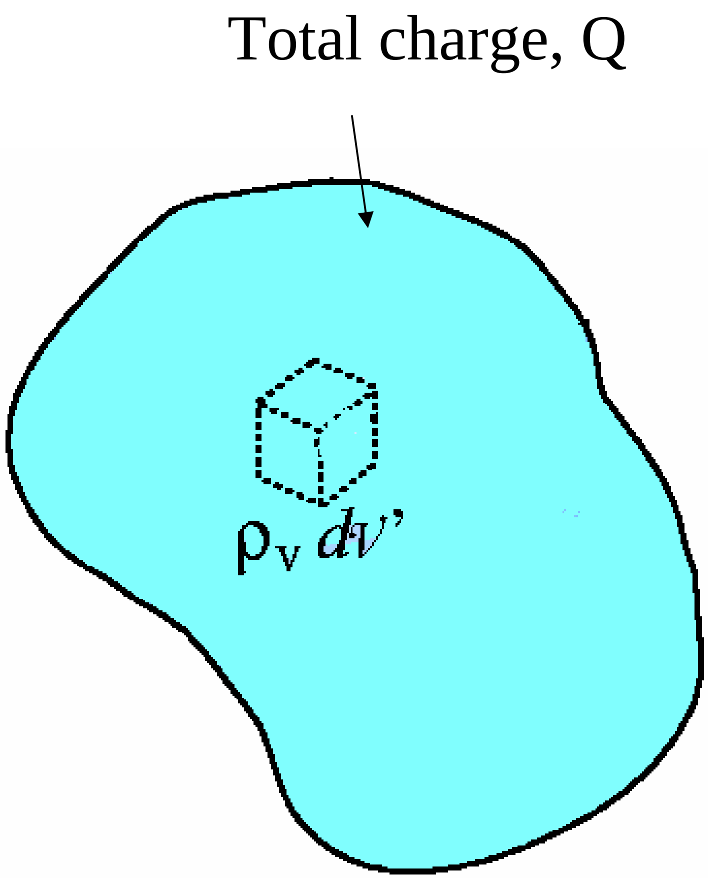
Thus, $\vec{E} = \int_l \dfrac{dq}{4\pi\varepsilon R^2}\hat{R}$

$$\vec{E} = \int_l d\vec{E}$$

Where $dq = \rho_s ds$

c) Volume charge

We know $$\vec{E} = \frac{q}{4\pi\varepsilon R^2}\hat{R}$$



But due to the charge dq

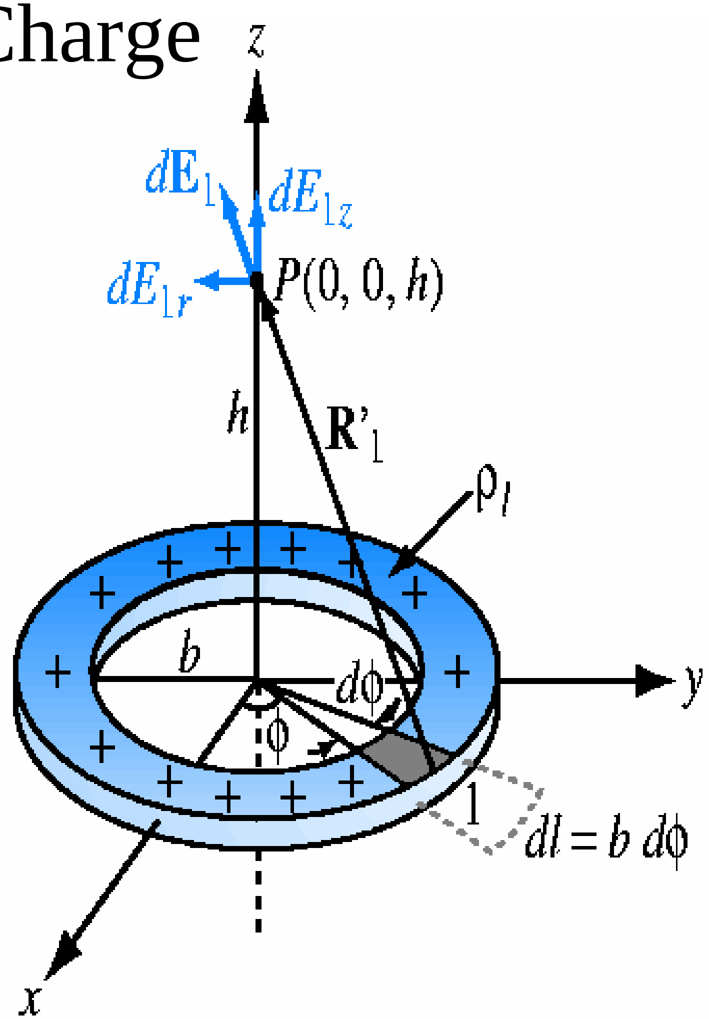
Thus, $$\vec{E} = \int_l \frac{dq}{4\pi\varepsilon R^2}\hat{R}$$

$$\vec{E} = \int_l d\vec{E}$$

Where $dq = \rho_v dv$

Example 3.5 - $\vec{E}$ of a Ring Charge

A ring of charge of radius *b* is characterized by a uniform line charge ρ_l density of positive polarity. With the ring in free space and positioned in the x-y plane as shown, determine the electric field intensity $\vec{E}$ at a point *P*(0,0,h) along the axis of the ring at a distance h from its center.



$$\vec{E} = \frac{\rho_l b h}{2\varepsilon_0 \left(b^2 + h^2\right)^{3/2}} \hat{z} (V/m)$$

Exercise

An infinite sheet of charge with uniform surface charge density ρ_s is located at z = 0 (x-y plane), and another infinite sheet with density $-\rho_s$ is located at z = 2m, both in free space. Determine $\vec{E}$ in all regions.

$$z < 0 \quad : \vec{E} = 0$$

$$0 < z < 2m \quad : \vec{E} = \frac{\rho_s \hat{z}}{\varepsilon_0}$$

$$z > 2m \quad : \vec{E} = 0$$

3.4 Gauss's Law

•From the 1st Maxwell's equation:

Differential form of Gauss's Law ➡

$$\nabla \bullet \vec{D} = \rho_v \qquad \text{............} \qquad (1)$$

•Equation in EMT often convert back & forth between differential & integral.

•To convert eq (1) into integral form, multiply both side by d_v & volume integral.

$$\int_v \nabla \bullet \vec{D}\, dv = \int_v \rho_v dv = Q \quad \text{..........}(2)$$

•Where Q is the total charge enclosed in Volume.

Gauss's Law (cont.)

Knowing that

$$\int_v \nabla \bullet \vec{E}\, dv = \oint_s \vec{E} \bullet d\vec{s} \text{ ....divergence theorem}$$

Thus we get

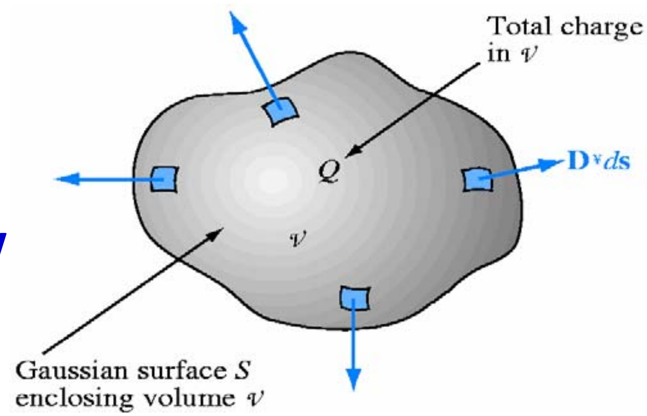
$$\int_v \nabla \bullet \vec{D}\, dv = \oint_s \vec{D} \bullet d\vec{s} \text{ ....}(3)$$

Gauss's Law (cont.)

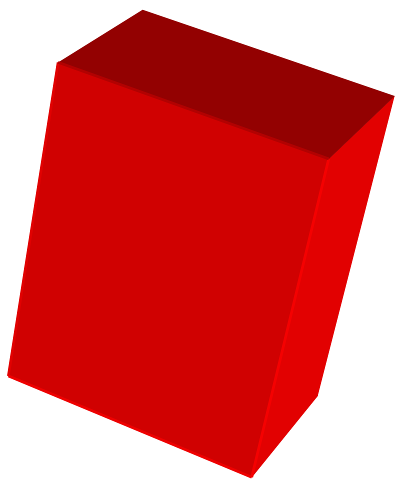
By comparing eq (2) & (3) we get

$$\oint_s \vec{D} \bullet d\vec{s} = Q_{en}$$

Gauss's law



Where ***S*** is called a Gaussian surface.



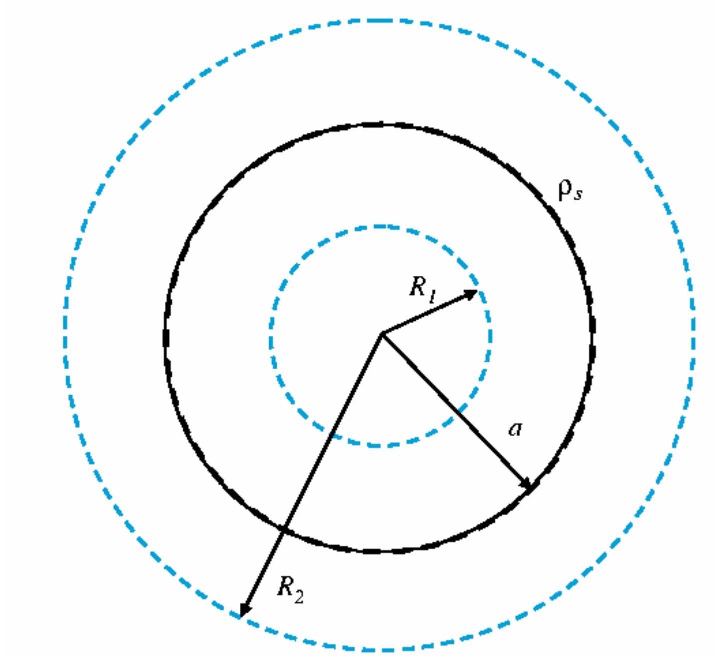
$$\oint_s \vec{D} \bullet d\vec{s} = \int_{S_1} \vec{D} \bullet d\vec{s}_1 + \ldots + \int_{S_6} \vec{D} \bullet d\vec{s}_6$$

Example 3.6 - Gauss's Law (pg 79)

A thin spherical shell of radius a, carries a uniform surfaces charge density ρ_s . Use Gauss's Law to determine $\vec{E}$.

a) $R \leq a$

b) $R \geq a$



$$(a)\ \vec{E}_1 = \vec{D}_1 = 0$$

$$(b)\ \vec{E}_2 = \frac{\rho_s a^2}{\varepsilon R^2}\hat{R} \quad V/m$$

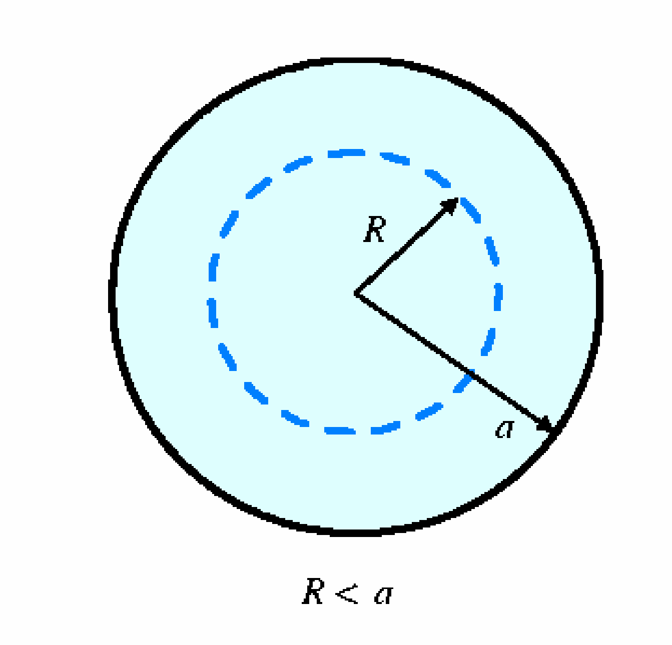
$$\vec{D}_2 = \frac{\rho_s a^2}{R^2}\hat{R} \quad c/m^2$$

Exercise - Gauss's Law (pg 79)

A spherical volume of radius a, carries a uniform volume charge density ρ_v. Use Gauss's Law to determine $\vec{D}$.

a) $R \le a$

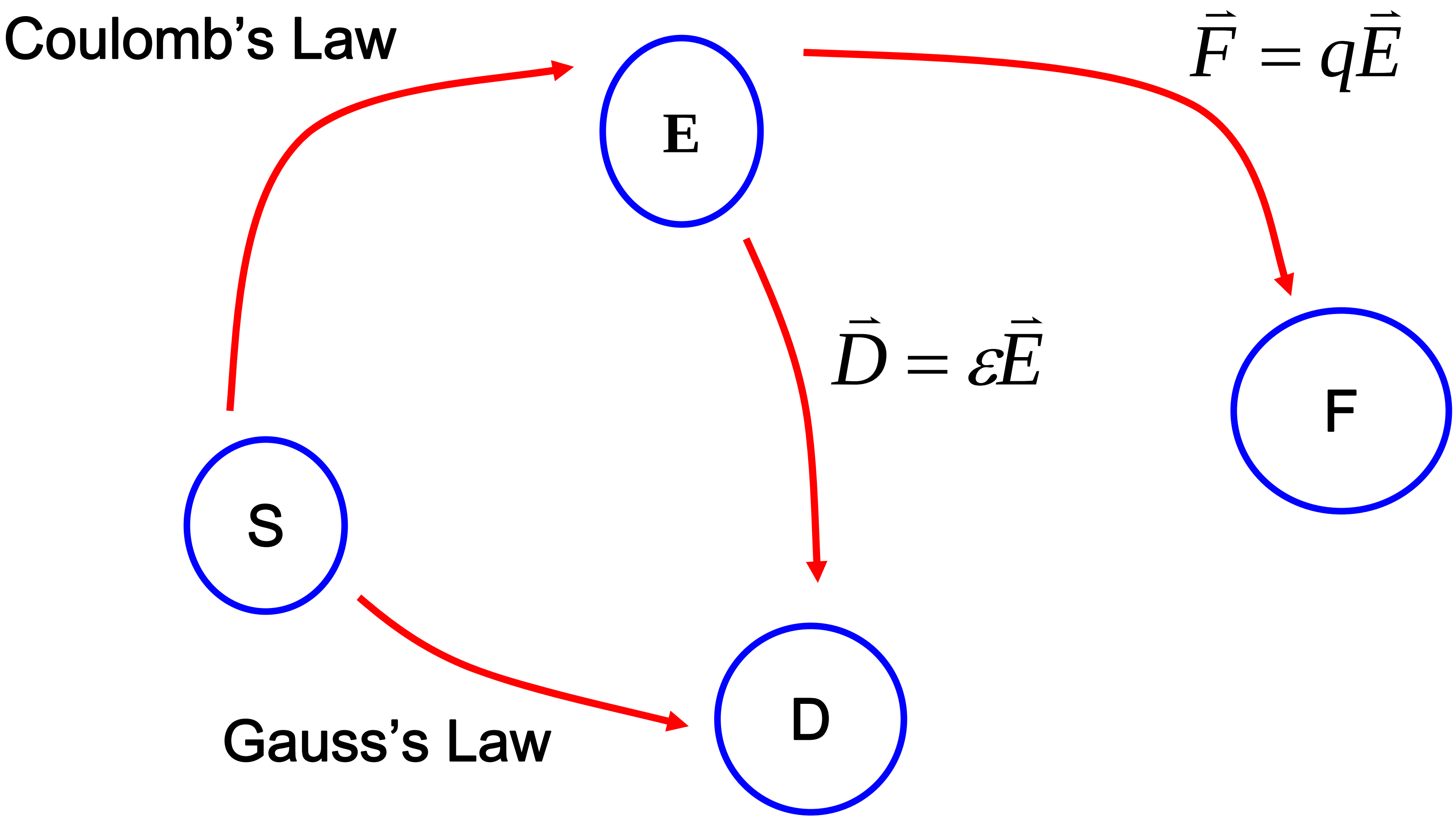
b) $R \ge a$



$$(a)\ \vec{D}_2 = \frac{\rho_v R}{3}\hat{R} \quad c/m^2$$

$$(b)\ \vec{D}_2 = \frac{\rho_v a^3}{3R_1^{\ 2}}\hat{R} \quad c/m^2$$

Conclusion


Coulomb's Law
E
$\vec{F} = q\vec{E}$
$\vec{D} = \varepsilon\vec{E}$
F
S
Gauss's Law
D

….. To be continued …..

Thank You