ELECTROMAGNETICS THEORY
BEKP3553
1-2006/2007

Tutorial 3: (**Assignment 1**; 5-8 students in a group; Due: 25th August 2006)

Electrostatics

A. Charge and Current Distributions

1. A cube 2m on a side is located in the first octant in a Cartesian coordinate system, with one of its corners at the origin. Find the total charge contained in the cube if the charge density is given by $\rho_V = xy^2e^{-2z}$ (mC/m^3)

2. Find the total charge contained in a cylindrical volume defined by $r \leq 2$ m and $0 \leq z \leq 3$ m if $\rho_V = 10\, rz$ (mC/m^3)

3. Find the total charge contained in a cone defined by $R \leq 2$ m and $0 \leq \theta \leq \pi/4$, given that $\rho_v = 20R^2\cos^2\theta$ (mC/m^3).

4. If the line charge density is given by $\rho_l = 12\, y^2$ (mC/m), find the total charge distributed on the y-axis from $y = -5$ to $y = 5$.

5. If $\mathbf{J} = 2xz\, \mathbf{a_y}$ (A/m^2), find the current I flowing through a square with corners at (0,0,0), (2,0,0), (2,0,2), and (0,0,2)

B. Coulomb's Law

6. A square with sides of 2 m has a charge of 20 μC at each of its four corners. Determine the electric field at a point 5 m above the center of the square.

7. Three point charges, each with $q = 3$ nC, are located at the corners of a triangle in the x-y plane, with one corners at the origin, another at (2cm, 0, 0), and the third at (0, 2cm, 0). Find the force acting on the charge located at the origin.

8. Charge $q_1 = 4\ \mu$C is located at (1cm, 1cm, 0) and charge q_2 is located at (0, 0, 4cm). What should q_2 be so that $\mathbf{E}$ at (0, 2cm, 0) has no y-component?

9. Point charges Q_1 and Q_2 are, respectively, located at (4, 0, –3) and (2, 0, 1). If Q_2=4nC, find Q_1 such that
   (a) The **E** at (5, 0, 6) has no *z*-component.
   (b) The force on a test charge at (5, 0, 6) has no *x*-component.

C. Gauss's Law

10. Let $\mathbf{E} = xy\,\mathbf{a}_x + x^2\,\mathbf{a}_y$, find
   (a) Electric flux density **D.**
   (b) The volume charge density ρ_v

11. Three concentric spherical shells r = 1, r = 2, and r = 3 m, respectively, have surface charge distributions 2, –4, and 5 μC/m^2.
   (a) Calculate the electric flux through r = 1.5 m and r = 2.5 m.
   (b) Find **D** at r = 0.5, r = 2.5, and r = 3.5 m.

12. Charge Q_1 is uniformly distributed over a thin spherical shell of radius a, and charge Q_2 is uniformly distributed over a second spherical shell of radius b, with $b > a$. Apply Gauss's law to find **E** everywhere.

13. An infinitely long cylindrical shell extending between r = 1 m and r = 3 m contains a uniform charge density ρ_{vo}. Apply Gauss's law to find **D** in all regions.

D. Electric Scalar Potential

14. In free space, $V = x^2y(z + 3)$ V. Find
   i. **E** at (3, 4, –6)
   ii. The charge within the cube $0 < x,y,z < 1$

15. The circular disk of radius a shown in Fig. 3.7 (Ulaby pp.75) has uniform charge density ρ_s across its surface.
   (a) Obtain an expression for the electric potential V at a point P (0, 0 , z) on the z-axis.
   (b) Use the result to find **E** and then evaluate it for $z = h$. Compare your final expression with Eq. (3.24), which was obtained on the basis of Coulomb's law.

16. A circular ring of charge of radius a lie in the x-y plane and is centered at the origin. Assume also that the ring is in air and carries a uniform density ρ_l .
   (a) Show that the electrical potential at (0, 0, z) is given by $V = \dfrac{\rho_l a}{2\varepsilon_0 (a^2 + z^2)^{1/2}}$
   (b) Find the corresponding electric field **E**.

E. Boundary Conditions

17. A Cartesian coordinate system is divided into two regions. Region 1 has $x < 0$ and is occupied by a dielectric having relative permittivity $\varepsilon_{r1} = 2$, whereas region has $x > 0$ and is occupied by a dielectric having relative permittivity $\varepsilon_{r2} = 9$. If the electric field in region 1 at $x = 0$ given by $\mathbf{E}_1 = \hat{\mathbf{x}}2 + \hat{\mathbf{y}}3 - \hat{\mathbf{z}}4$, determine $\mathbf{E_2}$ and $\mathbf{D_2}$ in region 2 at the interface. Assume charge free boundary.

18. Two extensive homogeneous isotropic dielectric meet on plane $z = 0$. For $z \geq 0$, $\varepsilon_{r1} = 4$ and for $z \leq 0$, $\varepsilon_{r2} = 3$. A uniform electric field $\mathbf{E}_1 = \hat{\mathbf{x}}5 - \hat{\mathbf{y}}2 + \hat{\mathbf{z}}3$ kV/m exists for $z \geq 0$. Find
    (a) $\mathbf{E}_2$ for $z \leq 0$
    (b) The angles E_1 and E_2 make with the interface

19. Region $y \leq 0$ consists of a perfect conductor while region $y \geq 0$ is a dielectric medium ($\varepsilon_{r1} = 2$). If there is a surface charge of 2 nC/m^2 on the conductor, determine **E** and **D** at
    (a) $A(3, -2, 2)$
    (c) $B(-4, 1, 5)$