

Microeconomics 1 - Part 1

Solutions for assignment 1

Marco Petracco-Giudici

1 Problem 1.B.3

We need to demonstrate that $v(x) \geq v(y) \Leftrightarrow x \geq y$. This is very easy since $v(x)$ is a strictly increasing transform of $u(x)$, which means that $u(x) \geq u(y) \Leftrightarrow v(x) \geq v(y)$. The result follows immediately.

2 Problem 2.D.2

Call H the total endowment of hours available to the consumer. Calling s the hourly wage expressed in Euros, this amount of time is worth a maximum of $H * s$ Euros to the consumer.

He can use this money for two uses: buying units of goods x at price p or "buy" leisure h at the price s (this is an **opportunity cost**). The resulting consumption set has the form: $X = \{(x, h) : px + hs \leq Hs\}$

3 Problem 3.B.1

(a) The preference ordering \succsim on X is strongly monotone if $y \geq x$ and $y \neq x$ imply $y \succ x$. We need to show that if \succsim is strongly monotone then it is also monotone: if we have $y \gg x$ this implies $y \geq x$ and $y \neq x$, coupled with strong monotonicity this implies $y \succ x$ which is all we need to prove that strong monotonicity implies monotonicity.

(b) Starting from an **interior** point x of X we can always find a point y such that $\|y - x\| \leq \varepsilon$ and $y \gg x$. By monotonicity this point is such that $y \succ x$, which is enough to prove local nonsatiation of \succsim .

4 Problem 3.C.5

(a) A preference ordering is homothetic if $x \sim y \Rightarrow \alpha x \sim \alpha y$.

Suppose first that \succsim can be represented by an Homogeneous function of Deg. 1. Consider two bundles $x, y \in X$ such that $x \sim y$. Since $u(\cdot)$ represents \succsim we can write $u(x) = u(y)$ and as well that $\alpha u(x) = \alpha u(y)$. But

by the homogeneity of $u(\cdot)$ it descends that $\alpha u(x) = u(\alpha x)$ which means that we can write the last equality as $u(\alpha x) = u(\alpha y)$ which is equivalent to saying that $\alpha x \sim \alpha y$ as we wanted to demonstrate.

To demonstrate the reverse implication suppose that \succsim is homotetic. Consider a bundle $x \in X$ and a scalar $\alpha > 0$. As we are supposing that the ordering is continuous, following the construction of Figure 3.C.1 in MWG we can say that there must be a function $u(x)$ such that for any $x \in X$ we can write $u(x)e \sim x$ and as well that $u(\alpha x)e \sim \alpha x$. But since $u(\cdot)$ is homotetic we know that the first inequality implies that $\alpha u(x)e \sim \alpha x$. By transitivity then we can conclude that $u(\alpha x)e \sim \alpha u(x)e$ which implies that $u(\alpha x) = \alpha u(x)$ for any x , meaning that $u(\cdot)$ is homogeneous, as we wanted to demonstrate.

(b) Suppose that \succsim can be represented by $u(x) = x_1 + \phi(x_2 \dots x_L)$. Consider two bundles $x, y \in X$ such that $x \sim y$. By the definition of utility function $u(x) = u(y)$ and we can write $u(x) + \alpha = u(y) + \alpha$ for any positive scalar α . By the functional form of $u(\cdot)$ we can write

$$u(x) + \alpha = \alpha + x_1 + \phi(x_2 \dots x_L) = u(x + \alpha * e_1)$$

$$u(y) + \alpha = \alpha + y_1 + \phi(y_2 \dots y_L) = u(y + \alpha * e_1)$$

where $e_1 = \{1, 0, \dots, 0\}$. Hence $u(x + \alpha e_1) = u(y + \alpha e_1)$ and by the definition of utility function $x + \alpha e_1 \sim y + \alpha e_1$ which is what we wanted to demonstrate.

5 Problem 3.D.1

The Walrasian demand function for a Cobb-Douglas utility function is $x(p, w) = \left\{ \frac{\alpha w}{p_1}, \frac{(1-\alpha)w}{p_2} \right\}$

(a) Both terms of the demand vector are multiplied and divided by a constant, so demand remains constant

$$(b) p \cdot x = \frac{\alpha w}{p_1} p_1 + \frac{(1-\alpha)w}{p_2} p_2 = \alpha w + (1-\alpha)w = w$$

(c) We know that the Cobb-Douglas is *strictly* quasi-concave, and it is clear that the demand correspondence is single-valued (i.e. it is a demand *function*)