

Determining the Cash Discount in the Firm's Credit Policy

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Introduction

A key decision in the firm's credit policy is whether or not to offer a cash discount for the early payment of invoices; and, if a discount is offered, how much it should be. Credit terms involving cash discounts are common in many industries. A typical credit term is "2/10, net 30;" the customer may take a 2% cash discount from the stated invoice price if payment is made by the tenth day from the date of invoice. If the customer neglects (or chooses not) to pay by that date, the full invoice price is due on the thirtieth day.

Although there is at least one report that suggests discounts substantially affect profitability [2], the role cash discounts play in credit policy has received little attention in the finance literature. The purpose of this study is to structure the cash discount decision in terms of the tradeoff between costs and benefits. There are costs involved in giving up some percentage of the full invoice amount, but there are three possible

benefits. First, cash is received sooner, reducing the need to borrow or allowing more cash for investment. Second, sales volume may increase, since a discount is, in effect, a price reduction. Third, if customers can be induced to pay early, it may be possible to reduce bad debt losses.

Assume the firm now has some credit policy that results in a particular pattern of cash flows over time. We then examine how the pattern might change with the introduction of a cash discount. The decision criterion is the present values of the two cash flows. Our approach is similar to the discounted cash flow analysis of the accounts receivable problem described by Atkins and Kim [1], which we prefer to other techniques that do not explicitly consider the timing of the cash flows (see [3], for example). We find a break-even policy by computing the maximum discount the firm can offer given a set of assumptions about timing,

change in sales volume, fraction of credit sales expected to be paid with a discount, and any change in the bad debt loss rate. The firm would not want to offer a larger discount because the present value of cash flows with the discount policy would be less than the current policy. We also demonstrate how the decision model can be extended to the computation of an optimal discount rate, given relationships between the discount rate and 1) the fraction of customers taking the discount, and 2) the change in sales volume. The model is then applied to a variety of credit problems.

Computing the Maximum Discount

Case 1: Cash Discount Accompanied by a Change in Timing

We assume the firm's current credit policy offers no discount on a particular product line, resulting in cash payments, on average, on day N. "On average" refers to a simple arithmetic average. Appendix A shows that a time weighted average is more exact but that the arithmetic average is entirely sufficient for normal interest rates and periods under one year. Exhibit (1) illustrates the cash flows under the current credit policy. Initially, we assume the firm has no bad debt losses. This assumption is relaxed in Appendix B. The present value factor is $(1+i)^{-N}$, where i is the opportunity cost per day of the firm's funds. The appropriate determination of i is beyond the scope of this paper. The net present value of cash flows under the current policy is

$$NPV_0 = S(1+i)^{-N}, \quad (1)$$

where S represents total sales.

We now consider a proposed credit policy offering a cash discount of δ for early payment of invoices. We assume some fraction p of net sales will be paid on average on day M , while the remainder $(1-p)$ will be paid on average on day N' . Exhibit (2) represents the cash flow pattern under the proposed policy. Note that N' need not equal N — for example, if the percentage of customers taking the discount were higher among early payers than among late payers. The present value of cash flows under the proposed policy is the sum of the amount paid with a discount on day M plus the remainder paid in full on day N' :

$$NPV_1 = p(1-\delta)S(1+i)^{-M} + (1-p)S(1+i)^{-N'}. \quad (2)$$

In deciding on the discount rate, the firm should insure that the present value of cash flows under the proposed policy is at least as great as that under the current policy or,

Exhibit 1. Cash Flow Patterns Under Current Credit Policy

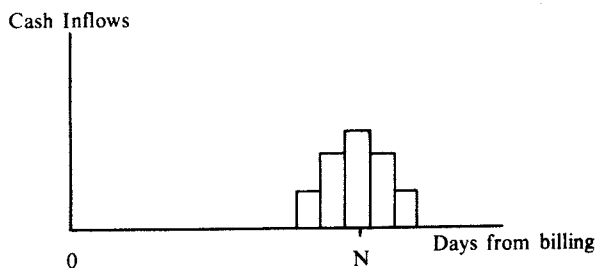
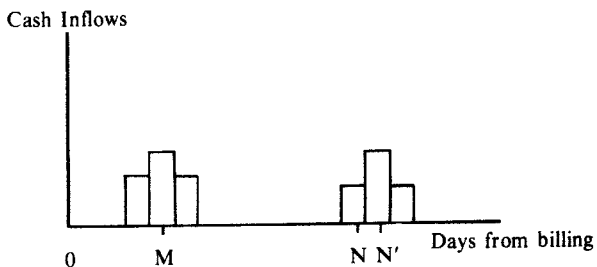


Exhibit 2. Cash Flow Patterns Under Proposed Credit Policy



$$NPV_1 \geq NPV_0. \quad (3)$$

Substituting Equations (1) and (2) into Equation (3), and solving for δ gives

$$\delta \leq 1 - (1+i)^{M-N'} \left[1 - \frac{1}{p} + \frac{(1+i)^{N'-N}}{p} \right]. \quad (4)$$

Equation (4) specifies a range of possible discounts that would make this condition hold. We define δ_{max} to be the maximum justifiable discount rate to offer or that discount rate which equates NPV_1 and NPV_0 ,

$$\delta_{max} = 1 - (1+i)^{M-N'} \left[1 - \frac{1}{p} + \frac{(1+i)^{N'-N}}{p} \right]. \quad (5)$$

Equation (5) specifies the maximum cash discount the firm should consider. A higher rate cannot be justified on a present value basis because the costs of reduced revenues would exceed the benefits of receiving cash sooner.

We note that δ_{max} depends on p , the fraction of sales being paid with a discount. This value could be estimated using a variety of techniques such as past experience with individual accounts, market surveys, or observations of similar terms for other product lines.

If the average payment date before the credit policy

change equals the average payment date after ($N=N'$), we can simplify δ_{\max} to

$$\delta_{\max} = 1 - (1+i)^{M-N}. \quad (6)$$

Now the maximum feasible discount is independent of p . N would equal N' if 1) all customers were taking the same amount of time to pay under the old credit terms, or 2) the same dollar fraction of early and late paying accounts were moved forward.

Numerical Example

Under the firm's current credit terms, 50% of credit sales are collected on day 60 and 50% on day 120. The average collection day is therefore 90. The firm wants to know what discount it can offer for payment by day 10. It is assumed that all the customers now paying on day 60 will take the discount while the late customers will not. Financing costs are 10%. In terms of Equation (5), $N=90$, $N'=120$, $M=10$, $p=.50$, and $i = .10/365 = .00027$. The maximum feasible discount rate is

$$\delta_{\max} = 1 - (1.00027)^{10-120} \left[1 - \frac{1 - (1.00027)^{120-90}}{.5} \right] \\ = .0137 = 1.37\%.$$

This means the firm could not benefit from offering a discount of greater than 1.37%.

While this example uses a very simple payment pattern, the model can easily handle complex ones.

Case 2. Cash Discount Accompanied by Changes in Timing of Payments and Sales Volume

Changing the cash discount could result in a change in sales volume. When sales volume changes we must also consider the change in cash outflow reflecting the change in variable costs. Let g represent the fractional change in sales volume. If v represents the variable costs per sales dollar, then a change of sales volume gS will result in the change of cash outflow vgS . The present value of this cash outflow depends on the firm's pattern of cash payments for variable costs. Let Q be the average payment date of variable costs. The present value of cash flows under this credit policy, assuming fraction p of credit sales will be paid on day M , will be

$$NPV_2 = p(1-\delta)S(1+g)(1+i)^{-M} + \\ (1-p)S(1+g)(1+i)^{-N} - vgS(1+i)^{-Q}. \quad (7)$$

Setting $NPV_0 = NPV_2$ and solving for δ gives

$$\delta_{\max} = 1 - (1+i)^{M-N'} \\ \left[1 - \frac{1}{p} + \frac{(1+i)^{N'-N} + vg(1+i)^{N'-Q}}{p(1+g)} \right]. \quad (8)$$

Numerical Example

A gas station currently accepts only cash. The owner is considering the possibility of honoring a national bank card. Experience of other stations shows this would likely result in a 10% increase in sales. Payments from the bank card company are discounted 5%. The owner expects 50% of his customers will take advantage of the bank card. If variable costs are 80% payable on average on day zero, would the gas station owner be better off accepting the bank card or sticking to a cash only basis? Assume cost of funds to be 10% or .027% per day.

In this case, $N=N'=0$, $M=0$, $Q=0$, $p=.5$, $g=.1$ and $v=.8$. Using Equation (8),

$$\delta_{\max} = 1 - \left[1 - \frac{1}{.5} + \frac{1 + .8(.1)(1.0)}{.5(1.10)} \right] = \\ .0364 = 3.64\%.$$

The maximum justifiable discount rate in this case is 3.64% compared to the bank card 5% discount. Hence, the firm should not accept the bank card.

It is easy to see how sensitivity analysis could be employed in this problem to examine the impact of various changes in sales, or in different fractions of customers taking the discount. For example, we could use Equation (8) to determine the minimum volume increase that would be necessary to justify acceptance of the bank card. Setting $\delta_{\max} = .05$ and solving for g , we find g must equal 14.3% before the station would break even.

Determination of the Optimal Discount Rate

We now consider the obvious interdependence of the discount rate with both the fraction of sales to be paid with a discount and the change in sales volume. The higher the discount, for example, the higher the number of customers taking the discount. If these dependencies can be specified, an optimal discount can be computed instead of simply the maximum feasible discount.

Assume that p , the fraction of credit sales paid with a discount, depends on the size of the cash discount offered, or $p=f(\delta)$. We define the optimal discount to be that discount which maximizes the present value of

the cash flows, NPV_1 . This will occur when $dNPV_1/d\delta=0$. Using Equation (2), and letting $f'(\delta)$ represent the first derivation with respect to δ ,

$$\frac{dNPV_1}{d\delta} = -f(\delta)S(1+i)^{-M} + (1-\delta)S(1+i)^{-M}f'(\delta) - S(1+i)^{-N'} f'(\delta) = 0. \tag{9}$$

Cancelling the redundant S we obtain

$$-f(\delta) (1+i)^{-M} + [(1-\delta) (1+i)^{-M} - (1+i)^{-N'}] f'(\delta) = 0. \tag{10}$$

The solution to Equation (10) depends on the specific relationship between p and δ . The relationship should be such that $p = 0$ when no discount is offered, and $p = 1$ when the discount approaches 100%. An example of a very simple function satisfying this relationship is $p = \delta$. By substituting $f(\delta) = \delta$ and $f'(\delta) = 1$ into Equation (10), we obtain

$$\delta^* = \frac{[1 - (1+i)^{M-N'}]}{2}. \tag{11}$$

Optimal Discount Example

Under the firm's current credit terms, payments are collected on average on day 90, with no discount offered. The firm discounts cash flows at 10%. It is assumed that, if some customers take a discount, the average payment date for the remaining customers will not differ much from 90 (or $N' = N$). Assume that the relationship between δ and p is approximated by $p = 20\delta$ (or with a 2.5% discount 50% of the customers would pay early). What is the optimal discount rate, if any, the firm should offer?

Using Equation (11), the optimal discount rate is computed:

$$\delta^* = \frac{[1 - (1 + \frac{.10}{365})^{10-90}]}{2} = .0108 = 1.08\%$$

Hence, the firm can afford to offer a discount of 1.08% (or in more realistic terms, 1%).

Optimal Discount When Sales Volume Changes

To determine an optimal discount rate in the case of both timing and sales volume changes, it is necessary to specify how both p and g depend on δ . Let $p = f(\delta)$ and $g = h(\delta)$. NPV_2 from Equation (7) then becomes

$$NPV_2 = f(\delta) (1-\delta)S [1+h(\delta)] (1+i)^{-M} + [1-f(\delta)]S [1+h(\delta)] (1+i)^{-N'} - vh(\delta)S (1+i)^{-Q}. \tag{12}$$

For almost any realistic functions for $f(\delta)$ and $h(\delta)$, determining δ^* from the derivative of Equation (12) is most efficiently handled using computer approximation methods rather than finding an explicit solution.

Summary and Conclusions

This paper shows how the cash discount decision in a firm's credit policy can be structured in terms of timing of payments, change in sales, variable costs, the firm's cost of funds, the proportion of sales expected to be paid with a discount, and the bad debt loss rate. The model focuses on the maximum feasible discount rate for a given set of circumstances. The firm cannot afford to offer a discount above this specified rate. Second, by taking into account the dependencies between the size of the discount and both the fraction of sales paid with a discount and the change in sales volume, a model can be developed to specify the optimal discount rate.

With the discount problem structured in this manner, we can make several observations pertinent to credit policy decisions.

1. Products with different variable costs should have different credit terms. In general, the lower the variable costs, the higher the feasible discount.
2. Since the cash discount offered depends on the firm's own cost of funds, managers should consider changing credit policy as the firm's opportunity costs change.
3. The timing of cash flows is critical to the discount decision model. The manager must consider not only how the discount shifts some payments forward but also how it affects the timing of cash flows from non-discount taking customers.
4. Since the cash discount depends on how sensitive demand is to price changes, knowledge of price elasticity is important.
5. As demonstrated in Appendix B, bad debt losses affect the maximum justifiable discount. Firms with higher bad debt loss rates can afford to offer higher cash discounts, provided the discount helps reduce the loss.

References

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Appendix A

When using present value methods, care must be taken to define the average time of two or more cash flows occurring at different times. For example, assume a cash flow of A dollars on day N_1 and B dollars on day N_2 . The present value of these cash flows is

$$PV_1 = A(1+i)^{-N_1} + B(1+i)^{-N_2}$$

We wish to compute an average time, N^* , such that the present value of A+B discounted at rate i for N^* periods will have the same present value as PV_1 . In symbols:

$$(A+B)(1+i)^{-N^*} = A(1+i)^{-N_1} + B(1+i)^{-N_2}$$

$$(1+i)^{-N^*} = \frac{A}{A+B}(1+i)^{-N_1} + \frac{B}{A+B}(1+i)^{-N_2}$$

$$(1+i)^{-N^*} = f_A(1+i)^{-N_1} + f_B(1+i)^{-N_2}$$

where $f_A = A/(A+B)$ and $f_B = B/(A+B)$. By taking logs and solving for N^* , we get

$$N^* = \frac{-\ln[f_A(1+i)^{-N_1} + f_B(1+i)^{-N_2}]}{\ln(1+i)}$$

The same procedure works for any number of cash flows (M), so we may write:

$$N^* =$$

$$\frac{\ln[f_1(1+i)^{-N_1} + f_2(1+i)^{-N_2} + \dots + f_M(1+i)^{-N_M}]}{\ln(1+i)}$$

To see how much N^* differs from a simple arithmetic average, we will recompute the average for the numerical example of Case 1:

$$\begin{aligned} \text{There } f_1 = .50, f_2 = .50, N_1 = 60, N_2 = 120, \\ \text{so } N^* = \\ \frac{-\ln[.50(1 + \frac{.10}{365})^{-60} + .50(1 + \frac{.10}{365})^{-120}]}{\ln(1 + \frac{.10}{365})} = 89.8 \end{aligned}$$

The arithmetic average is given by $.5(60) +$

$.5(120) = 90$. A difference of 0.2 days is hardly worth the effort required to use the more exact formulation.

Appendix B. Cash Discount Accompanied by Changes in Timing, Sales Volume, and Bad Debt Loss Rate.

When a discount is offered, it is possible that in some cases the bad debt loss rate is reduced. This might happen, for example, when a firm goes from a policy of accepting checks to accepting a credit card. It would also be the case where "cash only" is instituted as the alternative policy.

We define net sales to be $S(1-b)$, where b is the fraction of sales never collected, or the bad debt loss rate. The present value of cash flows under the current policy is

$$NPV_0' = (1-b)S(1+i)^{-N} \quad (B-1)$$

We assume that under the alternative credit policy some customers responsible for the bad debt loss will take advantage of the discount, thereby reducing the overall loss. We define the fraction of total sales thus restored by k . Net sales under the current policy would then be $S(1-b+k)$. If we let p represent the fraction of net sales (including restored sales) paid with a discount on day M , the present value of cash flows becomes:

$$NPV_1' = \frac{p(1-\delta)S(1-b+k)(1+i)^{-M} + (1-p)S(1-b+k)(1+i)^{-N'}}{(1+i)^{-N'}} \quad (B-2)$$

Setting $NPV_0' = NPV_1'$ and solving for δ gives

$$\delta_{\max} = 1 - (1+i)^{M-N'} \left[1 - \frac{1}{p} + \frac{(1-b)(1+i)^{N'-N}}{p(1-b+k)} \right] \quad (B-3)$$

From this equation we see that as k increases, other things being equal, δ_{\max} increases. Hence, if a firm has reason to believe that offering a discount would reduce bad debt losses, it can offer a higher discount than that justified by simple timing effects. The relationship between discounts and bad debt loss reduction is not well understood and deserves empirical investigation.

An equation analogous to Equation (8) can be developed to include bad debt losses as well. If we let NPV_2' be the present value of cash flows under the proposed policy:

$$NPV_2' = \frac{p(1-\delta)S(1-b+k)(1+g)(1+i)^{-M} + (1-p)S(1-b+k)(1+g)(1+i)^{-N'} - vgS(1+i)^{-Q}}{(1+i)^{-N'}} \quad (B-4)$$

Setting $NPV'_2 = NPV'_0$ and solving for δ gives

$$\delta_{\max} = 1 - (1+i)^{M-N'}$$

$$\left[1 - \frac{1}{p} + \frac{(1-b)(1+i)^{N'-N} + vg(1+i)^{N'-Q}}{p(1+g)(1-b+k)} \right]. \quad (B-5)$$

Equation (B-5) is the most general equation for computing δ_{\max} with previous expressions being special cases.

Example

The firm currently accepts cash only and is considering meeting its competition by offering terms of net 30. It is expected that some payments will be stretched beyond 30 days with the average cash flow

on day 45. Extending credit will increase bad debt losses to an estimated 1%, but sales should increase 3% with no price change. Cost of funds is .05% per day, and variable costs are 80% of sales paid on average on day 0. It is likely that all customers will take advantage of the more liberal policy. Should the firm extend credit for 30 days?

In terms of Equation (B-5), $i=.0005$, $p=1.0$, $b=0$, $k=-.01$, $g=.03$, $m=45$, $N'=N=0$, $v=.80$, and $Q=0$. Solving for δ_{\max} gives -2.7% . The negative "discount" implies the firm should not accept the proposed change since it would have to raise prices by 2.7% just to break even. This example shows how well the model can handle a common type of credit policy problem in which timing, bad debt loss, and sales volume change simultaneously.

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