A Generalized Model for Monitoring Accounts Receivable

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■ Credit managers and academics have a common interest in designing models that monitor the growth rate and level of accounts receivable. There is extensive literature [5, 8, 9, 12] showing that both days sales outstanding (DSO) and the aging schedule of accounts receivable depend on sales and, therefore, are not reliable measures for monitoring and controlling changes in accounts receivable. But while customer payment patterns have been identified as the key information source for monitoring and controlling accounts receivable [1, 3, 4, 5, 8, 9, 12], the literature has made only modest reference to the effect changes in sales patterns have on changes in accounts receivable.

In 1979 Carpenter and Miller (CM) [2] presented a framework that relates changes in receivables to sales pattern and/or collection experience effects. The CM analytical framework was based on a weighted DSO concept that was independent of both sales averaging and the pattern of sales. CM measured the change in receivables attributable to changes in collection experience and sales patterns and they also compared actual DSO and actual receivables to a standard. With the CM model credit managers could determine the relative contribution of a sales effect and/or a collection effect to changes in receivables. The CM model provided management a tool to improve the internal control of accounts receivables and the forecast of cash flows.

Although the CM model is a significant contribution to the financial management literature, we have discovered measurement errors that arise under specific conditions. We have also found a third effect, which we call a joint effect. We feel that our extension of the CM model provides a more insightful and intuitive explanation of the causes underlying changes in accounts receivable.

Our objectives are to present a set of seven conditions responsible for explaining changes in receiv-

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Exhibit 1. Sets of Conditions Responsible for Changes in Receivables

		Sales Patterns (S)		
		Up (↑)	No Change (NC)	Down (↓)
Collection Experience (CE)	Deteriorate (↑)	4	2	6
	No Change (NC)	3	1	3
	Improve (↓)	7	2	5

↑ - Receivables Increase

↓ - Receivables Decrease

ables; to develop a more general model for measuring sales and collection experience effects; to explain the presence of a joint interaction effect; and to discuss briefly the state-of-the-art of management information systems for measuring payment behavior and monitoring receivables and forecasting cash flows.

Receivable Behavior

In the early 1960s, Cyert, Davidson, and Thompson [3], Beranek [1], and Cyert and Thompson [4] found that a steady state transition matrix in a Markov chain model closely approximated the payment pattern process underlying accounts receivable behavior. They also observed that the transition matrix provided valuable insights for improving the management of accounts receivable. In the mid 1970s, Lewellen and Johnson [8], Lewellen and Edmister [9], Freitas [5], and Stone [12] observed that a firm's collection patterns are not in a continuous steady state condition because of seasonal, cyclical, or random events. Empirical support for unstable payment patterns was found by Kallberg and Saunders [7] when studying the payment behavior of individual customers. Building on Stone's [12] payment pattern model, Shim [11] developed a lagged regression approach to measure cash collection rates and bad debt expense rates by relating cash collections to credit sales of prior periods.

Carpenter and Miller [2] address the primary concern of financial and credit managers by focusing on the causes for accounts receivable changes. CM's model determines if a change in receivables is related to a change in sales growth and/or a change in collection experience. Expanding the CM framework, one finds that seven sets of conditions can be used to analyze changes in accounts receivable. Each of these seven conditions is appropriately identified and illustrated in Exhibit 1. The horizontal axis represents changes in receivables due to changes in sales patterns. Changes in sales are in turn related to changes in the demand for a firm's products. The vertical axis reflects changes in receivables related to collection experience. These changes in collection experience are in turn related to changes in a firm's credit policies.

Changes in sales patterns refer to changes in the level of sales occurring on a month-to-month basis. The pattern and trend of sales can change because of seasonal, cyclical, or random forces. The collection experience reflects the payment behavior of a firm's customers and is related to a firm's credit policy actions. Collection experience is characterized by the fraction of credit sales in a month that remain outstanding at the end of each subsequent month. For example, if the collection pattern for March is 90-60-20, it means 90% of March's sales are outstanding as receivables on March 31; 60% of February's sales are outstanding as receivables on March 31; and 20% of January's sales are outstanding on March 31.

An overview of the seven conditions shown in Exhibit 1 provides the logic underlying the revised algorithm. In Condition 1, receivables do not change because there is no change in sales patterns or collection experience. Under Condition 2, 100% of the change in receivables can be attributed to a change in collection experience. For example, receivables can increase because of lenient credit policies that result in a slowdown in customer payment patterns, *i.e.*, a deterioration in collection experience, which has no influence on sales. Alternatively, receivables can decrease because of tightened credit policies that have no impact on sales. That is, customers are paying earlier because of tightened credit policies and the tighter credit controls have not caused sales to decline.

Under Condition 3, the opposite extreme from Condition 2, 100% of the change in receivables stems from changes in demand for the firm's products. One case under Condition 3 occurs when an increase in receivables is caused solely by an increase in sales. The other case occurs when a decrease in receivables is totally related to a decrease in sales. In Condition 3 the increase or decrease in sales has not affected credit policy and/or collection behavior.

Condition 4 in Exhibit 1 highlights the case where lenient credit policies are responsible for an increase in receivables. These policies have a spillover effect on sales, which contributes to the increase in receivables. At the same time an increase in demand is also fundamentally responsible for an increase in sales. The demand generated by sales results in a subsequent increase in receivables, which may spillover and cause collection experience to deteriorate. Thus, an increase in receivables may be partially related to a pure sales effect, a pure collection effect, and/or a joint interaction effect between sales and collection. An example of Condition 4 is a credit card promotion that results in increased sales, but also brings in new, but slow paying customers. In summary, when credit policies are lenient and sales are increasing, we observe there are three effects responsible for a change in receivables — sales, collection, and joint.

In contrast to scenario 4, we observe stringent credit policies in Condition 5. Tightened credit policies and procedures represent a two-edged sword that might result in lower receivables and also a reduction in sales. Simultaneously, a decline in demand is reflected in declining sales and subsequently lower receivables. The reduction in sales may also manifest itself in a tightening of credit policies. For example, if sales for a fad type product declined rapidly, the company could immediately stop extending credit, and sell only for cash. Because a portion of the sales decrease is related to restrictive credit policies, we find a part of the decrease in receivables is caused by a joint interaction between sales and collection. The interaction of improved collection patterns and declining sales creates a joint effect. Thus, under conditions of tightened credit policies, we are likely to observe receivables declining because of an improvement in collection experience, a reduction in the sales, and joint effects.

Under Conditions 6 and 7, opposing forces moderate the change in receivables. Under Condition 6, for example, credit policies are lenient causing receivables to increase, but a decline in demand causes sales to decrease and receivables to decline. The size and direction of the change in receivables depends on whether the decline in demand has a greater effect than the lenient credit policy. Under Condition 7, tightened credit policies result in improved collection experience and declining receivables, while increased demand causes sales and also receivables to increase. The size and direction of the change in receivables depend on whether the increase in demand is more prominent than tightened credit policy. In summary, from a sales and credit manager's perspective, Condition 7 is the most preferred outcome of the seven scenarios in Exhibit 1 and Condition 6 is the least attractive outcome of the scenarios in Exhibit 1.

Generalized Model

In measuring the change in accounts receivable, CM presented a framework that includes a sales pattern effect (SPE) and a collection experience effect (CEE). These two effects can be summarized as follows:

Exhibit 2. Algorithms by Carpenter and Miller (CM) and by Gentry and De La Garza (GD) for Measuring the Pattern Effects that Cause a Change in Receivables

Condition	Description	Pattern Effects	Algorithms: CM	Algorithms: GD
1	NC in CE or S	None		
2	↑ or ↓ in CE and NC in S ($S_j = S_i$)	CEE	$\Delta CE \times S_j$	$\Delta CE \times S_i$
3	↑ or ↓ in S and NC in CE	SPE	$\Delta S \times CE_i$	$\Delta S \times CE_i$
4	↑ in CE and ↑ in S	SPE	$\Delta S \times CE_i$	$\Delta S \times CE_i$
		CEE	$\Delta CE \times S_j$	$\Delta CE \times S_i$
		JE		$\Delta S \times \Delta CE$
5	↓ CE and ↓ in S	SPE	$\Delta S \times CE_i$	$\Delta S \times CE_j$
		CEE	$\Delta CE \times S_j$	$\Delta CE \times S_j$
		JE		$-\Delta S \times \Delta CE$
6	↑ in CE and ↓ in S	SPE	$\Delta S \times CE_i$	$\Delta S \times CE_i$
		CEE	$\Delta CE \times S_j$	$\Delta CE \times S_j$
7	↓ in CE and ↑ in S	SPE	$\Delta S \times CE_i$	$\Delta S \times CE_j$
		CEE	$\Delta CE \times S_j$	$\Delta CE \times S_i$

Legend
S = sale patterns
CE = collection experience pattern
NC = no change
↑ or ↓ = see Exhibit 1
i = oldest month
j = current month
SPE = sales pattern effect
CEE = collection experience effect
JE = joint effect

$$SPE_j = \Delta Sales \times Collection\ Experience(t_i), \quad (1)$$

$$CEE_j = \Delta Collection\ Experience \times Sales(t_j). \quad (2)$$

The symbol Δ stands for the difference between a value in t_j, the current month, and a standard benchmark in time t_i. The relationships between SPE and CEE and the seven conditions that cause receivables to change are presented in Exhibit 2. In Exhibit 2 one can observe that CM use either Equation (1) and/or (2) for measuring SPE and CEE for each of the seven conditions. This static assumption of a dynamic process causes measurement errors in the CM model.

The CM algorithm produces a correct measure of SPE or CEE under four sets of conditions: That is, when the cause of the change is either totally related to collection experience (Condition 2) or totally related to sales experience (Condition 3); when demand and sales are declining in conjunction with loosened credit control that results in deteriorating collection experience (Condition 6), and when there is no change in both sales and collection experience (Condition 1). However, the CM algorithm does not measure SPE and CEE correctly when both sales and collection experience cause receivables to increase (Condition 4) or to decrease (Condition 5) or in the mixed case (Condition 7) when sales are increasing and collection experience is improving.

Examples will best show how the measurement error can occur. To illustrate Condition 7, we assume that sales increase from \$60 in March to \$90 in June and the collection experience improves from 90% of March's sales outstanding on March 31 to 80% of June's sales outstanding on June 30. Receivables have increased from \$54 in March to \$72 in June for a net increase of \$18. This example is based on an exhibit developed by Carpenter and Miller [2, p. 39].

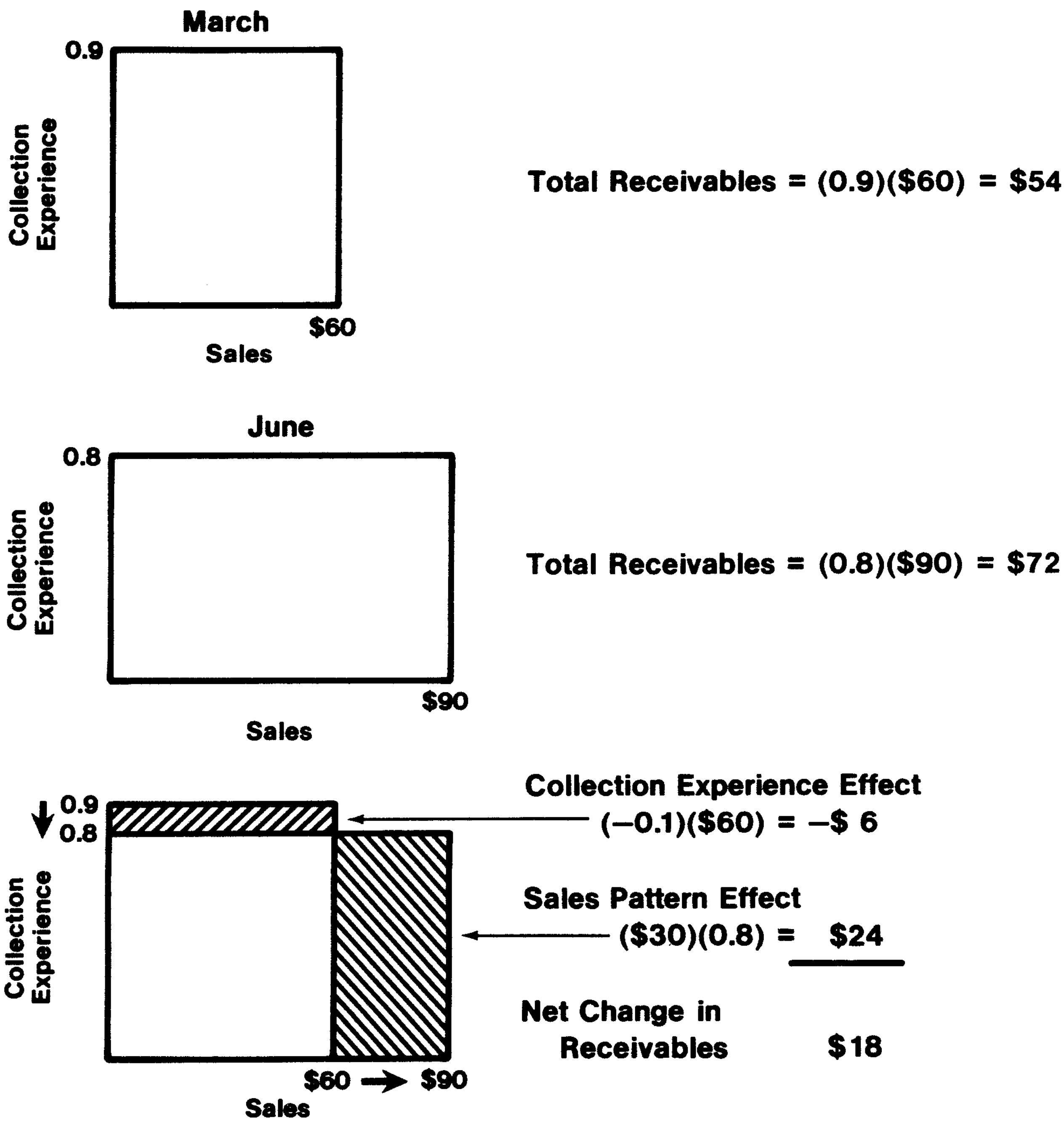
	March	June	Net Changes in Receivables
Sales	\$60	\$90	
% of sales outstanding at end of period	0.90	0.80	
Accounts receivable	\$54	\$72	+\$18

From Equation (1), we find CM's sales pattern effect is

$$(\$90 - \$60)(0.90) = +\$27,$$

and, from Equation (2), we determine CM's collection

Exhibit 3. Collection Experience Effect and Sales Pattern Effect



experience effect is

$$(0.80-0.90)(\$90) = -\$9.$$

Thus, CM attribute the +$18 net change in accounts receivable [$27 + (–$9) = $18] to a $27 incremental increase in sales and to a $9 improvement in collection experience.

Our (GD) generalized algorithm extends CM's model and offers a different interpretation of the above

Exhibit 4. Information for Examples

	Collection Patterns in percent for periods t_0, $t+1$ and $t+2$											
	Condition 4				Condition 5				Condition 7			
Month	Sales	0	+1	+2	Sales	0	+1	+2	Sales	0	+1	+2
1	$100	80	40	10	100	80	41	10	100	80	41	10
2	102	82	40	10	98	78	40	10	102	78	40	10
3	104	85	41	10	97	77	39	10	104	77	39	10
4	108	87	42	11	95	75	38	10	108	75	38	10
5	110	89	43	12	92	72	37	10	110	72	37	10
6	112	90	44	13	90	70	36	9	112	70	36	9
7	114	92	45	13	89	68	35	9	114	68	35	9
8	115	94	45	13	86	67	34	9	115	67	34	9
9	118	95	45	13	85	65	33	9	118	65	33	9
10	120	97	46	13	83	64	32	9	120	64	32	9
11	122	99	47	14	81	62	32	8	122	62	32	8
12	125	100	49	14	80	60	31	8	125	60	31	8

example. The GD algorithm for Condition 7 is

$$SPE_j = \Delta S \times CE_j, \quad (3)$$

$$CEE_j = \Delta CE \times S_i. \quad (4)$$

Equations (3) and (4) are also found in Exhibit 2 as the GD algorithm for Condition 7. Exhibit 3, based on Equations (3) and (4), geometrically depicts what actually happened to accounts receivable between March and June. It can be observed that whether CM's model or GD's model is used, the net increase in accounts receivable is $18. However, in the lower one-third of Exhibit 3 the GD algorithm produces a sales pattern effect of $24 compared to $27 in the CM model; and the collection experience effect is −$6 compared to −$9 in the CM model. This divergence from CM's model may be explained by splitting their Equations (1) and (2) into two components as follows:

Sales pattern effect
$$= (\$30)(0.8) \quad + (\$30)(0.1) = \quad \$27$$
Collection experience effect
$$= (-0.1)(\$60) \quad - (0.1)(\$30) = \quad \underline{-\$\,9}$$
Net change in accounts receivables $= +\$18$

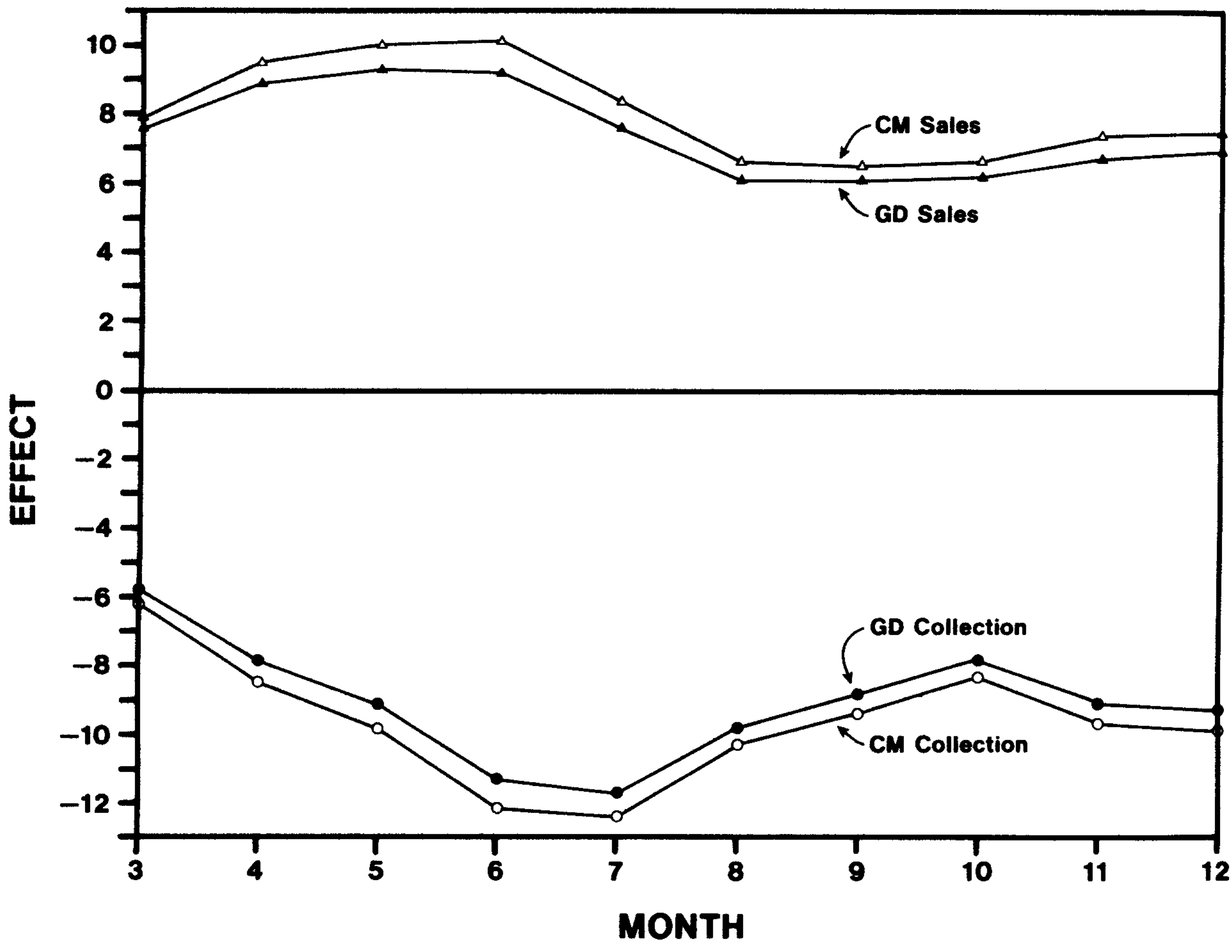
Comparing the information in the lower one-third of Exhibit 3 to the preceding equations, it can be seen that the left term in both equations matches the shaded areas, while the right term matches the area of a corner that does not exist. CM's Equation (1) assigns the $3 account receivable corner to the sales pattern effect and then, their Equation (2) subtracts the same account receivable corner from the collection experience effect, thereby cancelling each other and resulting in a net change in accounts receivable of $18. The preceding example shows the original CM model overstates the sales pattern effect and the collection experience effect compared to the GD generalized model.

We developed an example, both to create an intuitive feeling for the results generated by the generalized algorithm under Condition 7 and to highlight the differences between CM's and GD's algorithms. The example assumes annual sales have increased 25% and annual collection patterns improved approximately 25%. The example data are given in Exhibit 4, while Exhibit 5 illustrates graphically the sales pattern (SPE) and collection experience effects (CEE) based on the data in Exhibit 4. Exhibit 5 shows that CEE ranges from −$5.8 in month 3 to −$11.7 in month 7 for the GD model and −$6.2 in month 3 to −$12.4 in month 7 for CM's model; also the SPE for GD's model fluctuates from $6.1 in months 8 and 9 to $9.3 in month 5 and the CM model generates SPE from $6.5 in month 9 to $10.1 in month 6. A primary purpose of these algorithms is to provide management with performance measures of its sales and credit staff. We observe from Exhibit 5 that CM's model consistently overstates the contribution of the sales pattern effect and the collection experience effect and thereby provides management an inflated perspective of sales and collection experience.

An Extension: The Joint Effect

Under Conditions 2, 3, 6, and 7 in Exhibit 1, changes in receivables are totally attributable to a collection experience effect, a sales pattern effect, or a combination of the two when the effects act in opposite

Exhibit 5. Condition 7

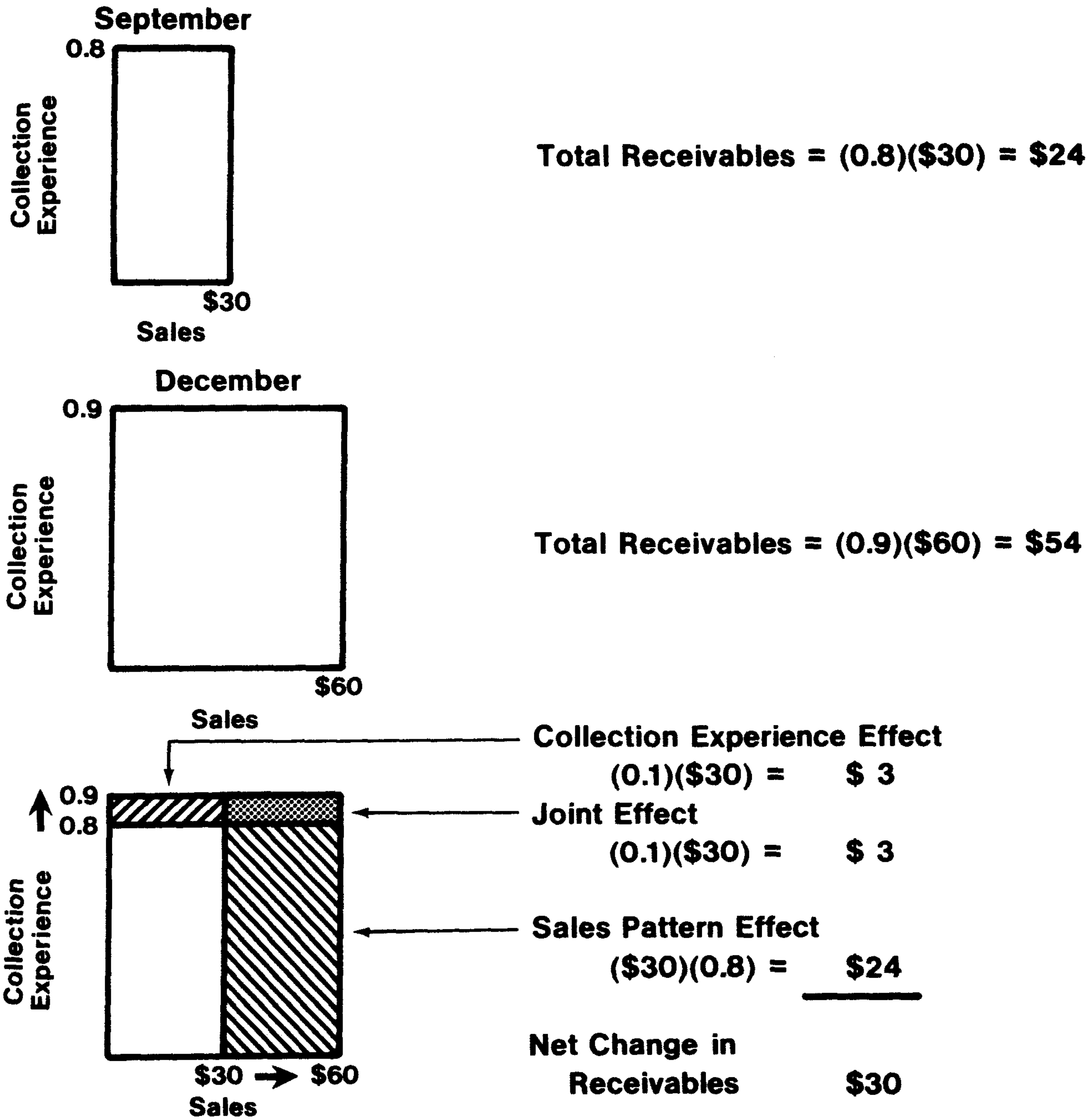


directions. However, when both effects cause receivables to increase (Condition 4) or to decrease (Condition 5), a joint effect is introduced. A joint effect occurs under either Condition 4 or Condition 5 and the equations for determining the three effects are presented in Exhibit 2.

An example from CM's Exhibit [2, p. 39] is developed to illustrate how the measurement error under Condition 4 occurs. The example assumes sales are $30 in September and increase to $60 in December. The collection experience deteriorates from 0.8 in September to 0.9 in December. Both sales and collection effects contribute to an increase in receivables.

Using CM's Equation (1), one finds the $30 increase in sales contributes $24 ($30 × 0.8) to receivables and CM's Equation (2) determines a $6 (0.1 × $60) increase resulting from deteriorating collection experience. The net increase in receivables according to CM's algorithm is $30 ($24 + $6). The GD model in Exhibit 6 shows that the sales effect is $24, which is identical to CM's calculation. However, GD's model in Exhibit 6 uses $\Delta CE \times S_t$ to calculate collection experience and the joint effect (JE) is ΔSales × ΔCollection Experience. Using the generalized model, Exhibit 6 shows the collection experience effect is $3 (0.1 × $30) and the joint effect is $3 (0.1 × $30). In comparing the two collection effects, it can be observed that CM's equations assign the $3 joint effect to the collection experience effect. It also can be deduced from Equations 1 and 2 that CM's model allocates the joint effect to the sales pattern effect when there is a reduction in sales and an improvement in the collection experience, Condition 5. Exhibit 6 illustrates that the joint effect is a combination of the sales and collection

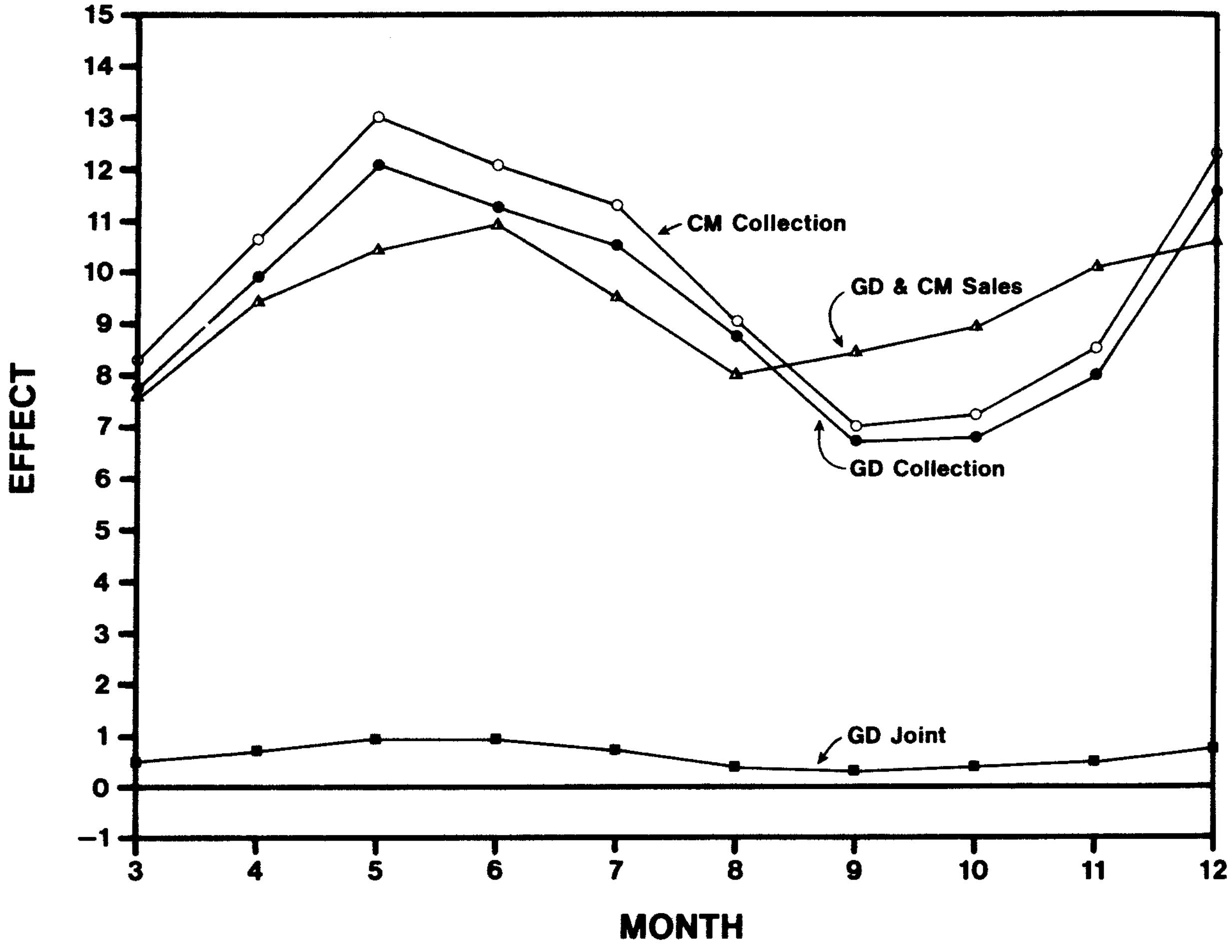
Exhibit 6. The Three Effects



effects. Thus, CM's model for the aforementioned set of conditions overstates the collection experience effect and does not include the joint effect.

To better understand and interpret the GD algorithms that introduce the joint effect and to highlight the differences between the generalized algorithms and those of CM, we developed more extensive examples for Conditions 4 and 5, based on the data in Exhibit 4. The Condition 4 example assumes that sales have increased 25% for the year (from $100 to $125) and that collection experience has deteriorated approximately 25%. Both sales and collection experience have con-

Exhibit 7. Condition 4



tributed to an increase in receivables. The resulting sales pattern and collection experience effects are graphed in Exhibit 7. The sales pattern effect shown in Exhibit 7 is identical for both the CM and the GD model. However, the CM model consistently overstates the collection effect by the amount of the joint effect which ranges from \$0.3 in month 9 to \$0.9 in months 5 and 6.

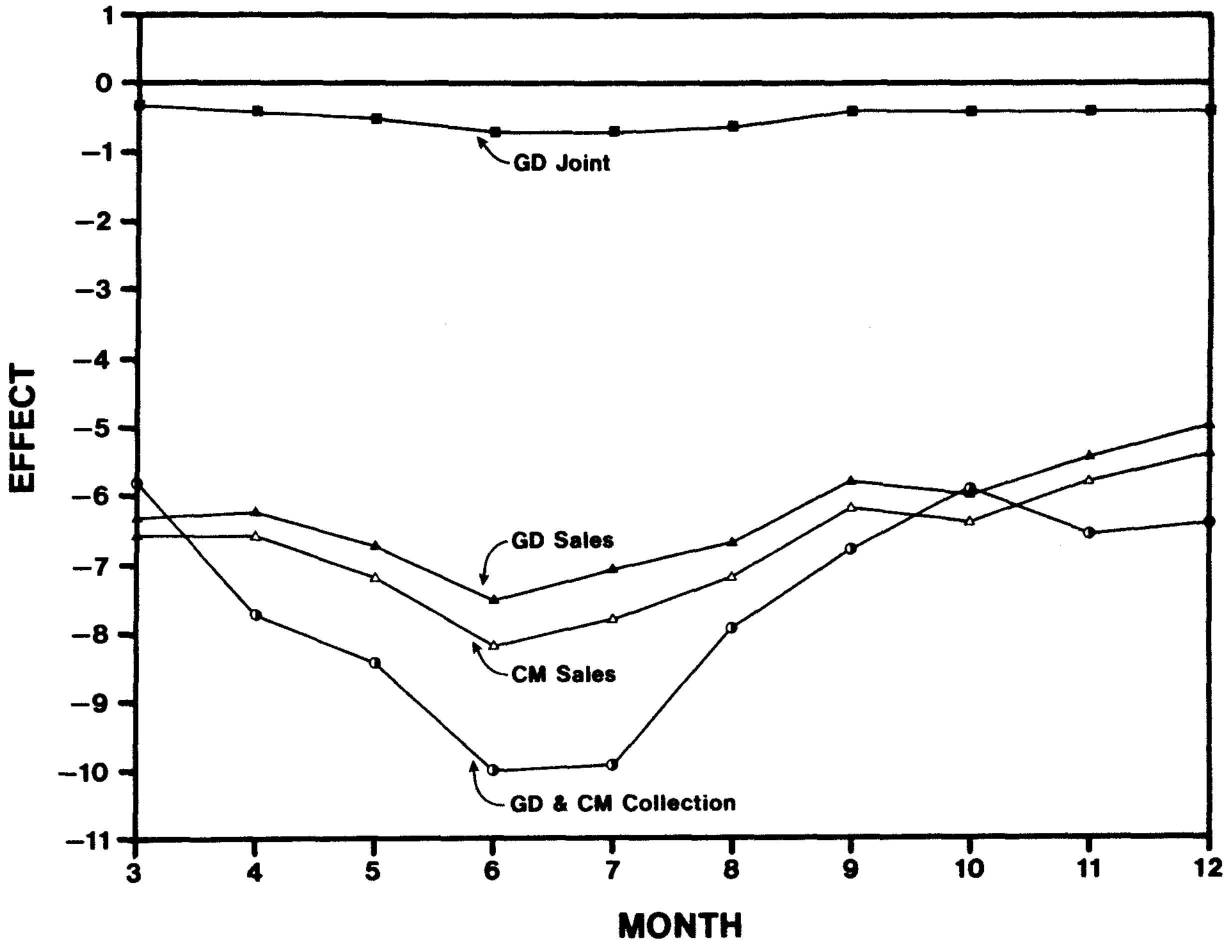
The Condition 5 example assumes a 25% annual decrease in sales and approximately a 25% improvement in collection experience. The relative contribution of the collection experience dominates the example and Exhibit 8 illustrates that it is identical for both models. Exhibit 8 shows that the CM model overstates the sales effect in the revised model by the amount of the joint effect. In summary, the CM algorithm overstates the collection effect in Condition 4 and overstates the sales effect in Condition 5.

The joint effect is based on the $\Delta CE \times \Delta S$. By monitoring sales, collection, and joint effects described in the generalized model, management can have more accurate measures of why receivables have changed and can be in a position to reward sales or collection effects appropriately.

The size of the joint effect depends on the size of the changes in sales or collection experiences. For example, under Condition 4 in Exhibit 2, if sales increased from \$30 to \$60 and collection experience deteriorated from 0.1 to 0.9, a very large relative change, the three effects would be:

Sales effect	=	\$30 × 0.1 =	\$3
Collection effect	=	0.8 × \$30 =	24
Joint effect	=	\$30 × 0.8 =	24
Total change in receivables			\$51

Exhibit 8. Condition 5



The total change in receivables is \$51 (\$3 + \$24 + \$24) and the joint effect comprises 47% (24/51) of the total change.[1] Thus, a large relative change in either sales or collection with a small change in the other effect can, at the limit, result in almost 50% of the change in receivables being contributed by the joint effect.

If sales and collection experience each made relatively large moves of equal size, all three effects would contribute equally to the change in receivables. For example, suppose sales decreased from \$60 to \$30 and the collection experience improved from 0.8 to 0.4. Both sales and collection experience were 50% smaller, which caused receivables to be smaller. The three effects resulting from this Condition 5 example are as follows:

Sales effect	=	−\$30 × 0.4	=	−\$12
Collection effect	=	−0.4 × \$30	=	−\$12
Joint effect	=	−(−\$30) × −0.4	=	−\$12
Total change in receivables				−\$36

The total reduction in receivables was \$36 and one-third (−12/−36) of the relative decline was contributed by each effect.

State-of-the-Art

Stone [12] and Shim [11] have developed methodologies for determining collection patterns. Stone rec-

[1] A weighted average method could be used to reallocate the joint effect in the appropriate proportions to the sales and collection effects. For example, the sales effect contributes 1/9 (3/27) of the \$24 joint effect or \$2.66 (1/9 × \$24). The collection effect contributes 8/9 (24/27) of the \$24 joint effect or \$21.33. By reallocating the joint effect, the revised sales effect would be \$5.66 (\$3 + \$2.66) and the revised collection effect would be \$45.33 (\$24 + \$21.33). Algorithms for calculating the weighted average reallocation of the joint effect are in the Appendix.

ommends measuring the patterns within a division or a business unit that have common credit terms, because medium or large-sized corporations seldom have a common collection pattern for all business units or divisions.

Recently Gillespie [6] discussed the state-of-the-art in management information systems that provide the technology to determine on-line collection behavior from customers. He stated that only a few companies currently have an integrated treasury management and accounting system capable of measuring collection patterns for their major product lines. With integrated treasury and accounting systems, a firm can acquire on-line information on specific items clearing and run it directly into the receivables ledger. One of the greatest benefits of such a system is that a company can do accurate payment cycle forecasting. Previously there was little information available to managers about payment cycle cash flows [7, p. 26].

Conclusions

The generalized model provides algorithms for measuring the collection, sales and joint effects that underlie changes in accounts receivable. Although the relationships are complex, the generalized model supplies financial managers and credit analysts an internal information tool for determining why a change in receivables has occurred and what policy or operational changes to make in order to control accounts receivable. The technology is now available to design on-line monitoring systems that will provide powerful insight to management concerning payment behavior and allow substantive improvement in cash-flow forecasting. This model also defines the foundations for developing a similar contribution analysis that would include accounts payable and inventories, *i.e.*, the remaining components of the cash conversion cycle described by Richards and Laughlin [10].

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Appendix. Weighted Average Algorithms for Allocating the Joint Effect to the Sales and Collection Effects for Conditions 4 and 5

Condition 4

$$SPE = \Delta S \times CE_i + \frac{\Delta S \times CE_i}{(\Delta S \times CE_i) + (\Delta CE \times S_i)} \Delta S \times \Delta CE$$

$$CEE = \Delta CE \times S_i + \frac{\Delta CE \times S_i}{(\Delta S \times CE_i) + (\Delta CE \times S_i)} \Delta S \times \Delta CE$$

Condition 5

$$SPE = \Delta S \times CE_j - \frac{\Delta S \times CE_j}{(\Delta S \times CE_j) + (\Delta CE_j \times S_j)} \Delta S \times \Delta CE$$

$$CEE = \Delta CE \times S_j - \frac{\Delta CE \times S_j}{(\Delta S \times CE_j) + (\Delta CE \times S_j)} \Delta S \times \Delta CE$$