Analytical Throughput for the Channel MAC Paradigm

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Abstract. It has been shown analytically [1],[2] that significant performance improvements as compared to existing technologies (e.g., IEEE 802.11) can be achieved in random access wireless networks. In [3] we proposed a fully distributed channel access paradigm based on the opportunistic communication principal called the Channel MAC paradigm suitable for distributed wireless networks such as ad hoc networks. In this paper, we analytically derive the throughput of the Channel MAC. It provides a throughput-limit on the channel-based MAC mechanism in shared multiple access environments without collisions or capturing effects. Both simulation and analytical results reveal possible performance improvement over existing techniques.

1 Introduction

The performance of ad hoc networks with Medium Access Control (MAC) protocols such as IEEE 802.11 falls well short of what is predicted by the theoretical models [1],[2]. This is mainly due to the inability of current MAC protocols to simultaneously take into account the dynamic channel conditions, decentralised channel access and unfairness in access to the common channel [4]. The concept of opportunistic communication has been shown to increase the performances of networks with centralised control [5],[6],[7]. Recently attempts has been made to apply the concept of opportunistic communication to networks with decentralised access mechanisms such as ad hoc and sensor networks [8],[9],[10],[11]. It has to be pointed out that these proposed schemes, although exploiting diversity as a way to determine who has priority for transmission, still use a slotted access system. Hence, in the absence of a central entity which would determine who will transmit based on the "best" channel, collisions will still occur because all nodes with good channel conditions will compete for resources at the beginning of the slot [12],[8],[11].

In [3], authors proposed a new MAC paradigm, called Channel MAC, which exploits the random nature of the fading channel to determine the channel access instances in a decentralised and distributed manner. Simulation results based on a Rayleigh fading channel showed that by using the new MAC paradigm, the network can achieve significantly higher throughput for all channel conditions as

H. Zhang et al. (Eds.): MSN 2007, LNCS 4864, pp. 375–386, 2007.

compared to 802.11 MAC scheme [3]. In this paper we model the Channel MAC protocol based on two-state channel model and show that the Channel MAC protocol always outperforms 802.11 MAC protocol.

The paper is structured as follows. The objectives and functionality of Channel MAC paradigm are briefly described in section II. This is followed by a description of the system model used in this study. Channel model and key definitions are illustrated here. Section IV describes the proposed analytical throughput model for the Channel MAC study. Simulation model and discussions on the results are given in the next section. Finally section VI concludes this work with our future concerns.

2 Channel MAC Scheme

The objective of Channel MAC paradigm is to use the concept of multiuser diversity to eliminate the scheduling-complexity of nodes in decentralised multihop networks. In Channel MAC, a station pre-arranges the instances at which it will send data-packets, based on the predicted channel gain between the node and the intended receiver and a channel gain threshold P_{th} for transmission. When the predicted channel gain goes above the P_{th} threshold the corresponding station potentially starts transmission. However, before sending data, a node will sense whether the channel is busy or not. If the medium is idle, i.e no other node is currently transmitting, the node starts transmission and continues it until the channel gain goes below the P_{th} threshold (i.e the channel goes into a fade). Number of packets transmitted during a good channel period depends on the packet size and the duration of the good channel period. If any other channel becomes good during transmission, the corresponding node will sense the channel being busy and will not transmit. It should be noted here that the carrier-sensing threshold of the nodes is set to a much lower value than the receiving threshold. Hence the transmitters should sense the medium busy even if the channel gain between a transmitter and an interfering node is low.

Given that each transmitter-receiver pair has an independent fading channel, the probability of two or more channels crossing the transmission threshold on a positive slope exactly at the same time is assumed to be negligible. However, due to finite propagation and sensing delay of the nodes, collision can occur, decreasing the throughput. We observe that the effects of collisions on the throughput is no worse than what is observed in other sensing based MAC protocols. It should be noted that the Channel MAC does not rely on random backoff mechanism to randomise the access to the shared medium. Instead, Channel MAC uses the random nature of channels between different node pairs to randomise the channel access. The decision to transmit is taken at each node without explicit knowledge of channel gain between other nodes in the neighbourhood, hence the fully distributed nature of the scheme.

3 System Model

Let us define a neighbourhood of $2n$ nodes, where $N_T \in (1, 2, \ldots, n)$ are the transmitters and $N_R \in (1, 2, \ldots, n)$ are the receivers. For symmetry let us assume that each transmitter $i \in N_T$ is communicating with receiver $i \in N_R$.

3.1 Channel Model

We consider a simple two-state channel model. It has either a non-fade state, "ON" with gain 1 or a fade state, "OFF", with gain 0. Suppose the distribution of each non-fade duration, $l_i (i \in \Re)$ is $f_n(x)$ with mean l. $\bar{l}$=min(l_i); for all i. Now, in light of the discussion of [13], the inter-arrival point distribution of one channel (instance at which the channel becomes good) can be approximated in general as a variant of the sum of Weibull distribution,

$$f(x) = \sum_z \lambda\beta_z(\lambda(x - \bar{l}))^{\beta_z - 1} exp((-\lambda(x - \bar{l}))^{\beta_z}) \tag{1}$$

where β_z is the shape parameter. The distribution depends on $l_i \in L$. Furthermore, $x > \bar{l}$ and $\bar{l} > 0$, i.e. any two arrival points of a channel are separated by a positive value.

For simplicity, we assume that the non-fade duration, termed Average Non-Fade Duration (ANFD), l, is constant, after which the channel goes into a fade with exponentially distributed fade duration as shown in figure 1. The instantaneous (i-th) idle time of $\bar{n}$ channel, denoted as $\Theta_{\bar{n}i}$, is an exponentially distributed random variable with the mean Θ; where $\bar{n} \in n, i \in \Re$. Hence probability of good channel, p, can be calculated as follows:

$$p = \frac{l}{l + \Theta} \tag{2}$$

We assume that all the channels in the network have the same p value.

3.2 Propositions

When the number of users in the network is 1 (this system is termed 1-user Channel MAC), the resulting transmission pattern of the network is identical to the channel model.

Proposition 1: In 1-user Channel MAC, the arrival point (start of transmission) process is approximated by a Renewal process.

Proof: The inter-arrival distribution can be considered as a shifted exponential of the form $f(x) = \lambda e^{-\lambda(x-l)}; (x \geq l)$ where $1/\lambda$ is the mean of the inter-arrival time. It follows from [14] that the arrival point process is approximated as a renewal process. □

Using $\beta = 1$ and $z = 1$ (a single Weibull distribution) in Equation 1, we get the shifted exponential distribution of the inter-arrival process of 1-user Channel

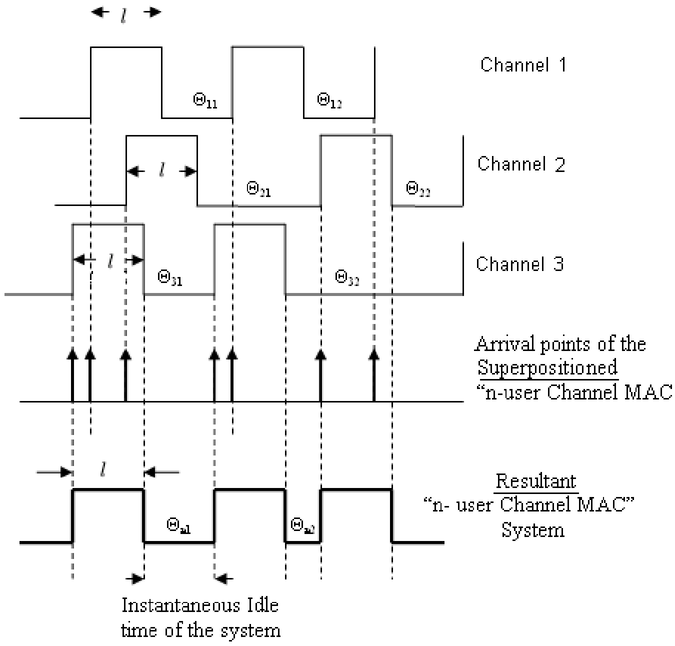

Fig. 1. Two-state channel model

MAC. We define "Expected period of 1-user Channel MAC", T_p as the expected renewal period [14] of the process. T_p can be expressed in terms of the number of arrival points per unit time period (i.e. Level Crossing Rate, r) as follows:

$$T_p = 1/r = 1/\lambda + l = t + l \tag{3}$$

where $t = 1/r - l$ is the expected idle time for 1-user Channel MAC as shown in Figure 2.

Proposition 2: The shifted exponential distribution function (defined in Proposition 1) results in a non-Poisson renewal arrival process.

Proof: The hazard rate of the distribution, $r(x)$, is λ when $x \geq l$. But if $x < l$, $r(x) = 0$. It follows that the arrival process is a non-Poisson renewal process. □

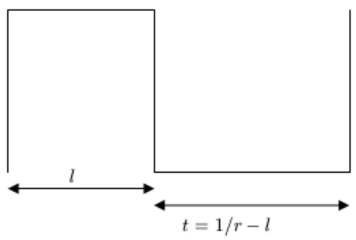

Fig. 2. Expected period of 1-user Channel MAC

Arrival points of n-user Channel MAC. We define the "Superpositioned n-user Channel MAC" as the superposition ([14], pp 101-104) of arrival points of n independent channels. We assume that, at each instance, exactly one channel becomes good (i.e transition from OFF to ON). Corresponding station then can transmit data given that no one else is transmitting at that instance. Following the operation of the Channel MAC scheme, we can identify the transmission periods and idle periods of the network with n users, which we term as "Resultant n-user Channel MAC" system.

Note the difference between *Resultant* and *Superpositioned* n-user Channel MAC. In Resultant n-user Channel MAC, the number of arrival points (i.e transition from OFF to ON) cannot be greater than the number of arrival points in the Superpositioned n-user Channel MAC. It is due to the fact that some of the arrival points of the Superpositioned n-user system may not contribute to throughput in Channel MAC operation as they may occur while another node is transmitting.

We further assume that in Superpositioned n-user Channel MAC, arrival points of individual channels are "sparse". i.e., in any particular $\bar{A}$ set of arrival points occurring in a random time-interval, there will be with high probability, at most one point from each process. In addition, no arrival points from one channel dominates over others. Hence equal number of arrival points from different channels should be present in any random interval.

4 Modelling Channel MAC Throughput

4.1 Superposition of Point Processes

It is known that the superposition of two independent Renewal processes is itself a Renewal process iif all three processes are Poisson [15]. Since the arrival points of 1-user Channel MAC does not constitute a Poisson process (by Proposition 2), the Superpositioned n-user Channel MAC is not a Renewal process either. To simplify the analysis, practical applications such as the superposition of arrival processes in a "Single server queuing model" consider approximation based approaches where the superimposed point process is approximated as a renewal process [16].

It is also well-known that the superposition of independent and uniformly sparse processes converges to a Poisson process as the number of processes and the sparseness increase. Such convergence results were first examined by Palm, in 1943 and Khinchin in 1955 under rigid assumptions [17]. A general Poisson limit theorem for independent superpositions was obtained by Grigelionis in 1963 [18]. This theorem states that if points of each individual processes are (a) suitably sparse and (b) no one process dominates the rest, then the distribution of the point process is close to Poisson. Corresponding results for dependent (mixing) Point processes with Poisson and compound Poisson process in the limit can be found in [19]. All these works conclude that a Poisson process is often a good approximation for a superposition process if many processes are being superposed.

Based on the discussions we state the following proposition:

Proposition 3: The arrival points of the Superpositioned n-user Channel MAC converges asymptotically to a Poisson Point process as per our assumptions.

4.2 Expected Idle Time of Resultant n-User Channel MAC

Proposition 4: In Poisson Point process, if n number of arrival points occur in an interval T, the expected delay of the first arrival point in T is $\frac{1}{n+1} \times T$.
Proof: According to ([14], pp. 46; [20], pp. 125) the arrival points of a homogeneous Poisson process with constant rate of arrival are independently and uniformly distributed over the interval. In other words, if n i.i.d. uniform random variables on $[0, \hat{t}] \equiv T$ are arranged in increasing order, they represent n successive occurrence times of a Poisson process. The average spacing of them is $\frac{\hat{t}}{n}$. □

It can be observed that the the expected idle time, $E[I]$ of the system decreases with the increasing number of channels. As per our assumptions the Superpositioned n-user Channel MAC is approximated as Poisson Point process (see Proposition 3). Then we derived the expected delay of first arrival point for a Poisson arrival process (see Proposition 4). Based on Proposition 3 and 4, we can derive $E[I]$ for the Resultant n-user Channel MAC as follows:

After a successful transmission by any channel, independent 0,1,...,∞ arrivals(s) may occur during immediate next fade duration, t, of that channel. Hence the expected idle time is the the weighted average of idle times for all possible number of arrival points.

$$E[I] = \sum_{i=0}^{\infty} P_i I_i \tag{4}$$

where P_i is the probability that i arrival occurs in t and I_i is expected delay of first occurrence of arrival point in this case. P_i is Poisson distributed as per our assumptions.

$$\begin{aligned} E[I] &= \sum_{i=0}^{\infty} \frac{(nrt)^i}{i!} e^{-nrt} \frac{1}{i+1} t \\ &= \frac{1}{nr} \left(1 - e^{-nrt}\right) \end{aligned}$$

where $t = 1/r - l$.

4.3 Analytical Throughput Measurement

The expected period of arrival point process for the Resultant n-user Channel MAC, $\hat{T_p}$ is the summation of the expected duration of successful transmission, l and expected idle time, $E[I]$. The average channel utilization or throughput, S

of Channel MAC is given by the ratio of l to the expected period of the Resultant n-user Channel MAC [21].

$$S = \frac{l}{\hat{T}_p} = \frac{l}{l + E[I]} = \frac{l}{l + \frac{1}{nr}\left(1 - e^{-nrt}\right)} \quad (5)$$

where $t = 1/r - l$.

5 Simulation

5.1 Simulation 1: Fixed l and Exponential Fade Duration

In this section we simulate the performance of Channel MAC based on the channel model (fixed l and exponentially distributed fade duration with mean $1/r - l$) discussed in section 3.1. The simulation approach we used is to generate n independent channels with same l and average fade duration $1/r - l$. When one or more channel "ON" periods overlap only the first channel to go to "ON" state after a non-zero idle period contributes to the throughput.

We have not considered the collision in analytical form. But in the simulation, we perceive the collisions as follows: if more than one start points of the overlapped non-fade durations of channels are same, we ignore all of them as it indicates a collision. The next immediate non-fade duration of the channel gets the opportunity to transmit for simplicity in our simulation. Practically, parameters such as packet-length, control packets, data-rate, etc, determine the next arrival point which will contribute to the throughput after a collision. The full impact of packet collisions will be observed by the discrete event simulation of the Channel MAC. By a Monte Carlo simulation we derive the throughput of the system over a number of simulation runs. Furthermore, we assume same p for all the stations in multiple node simulations.

5.2 Simulation 2: Rayleigh Fading Model

In the second simulation approach, we generate a set of "ON" and "OFF" intervals based on a Rayleigh distribution. The channel gain characteristics (i.e the times where channel gain is greater than the threshold) are Rayleigh distributed. p, which is equivalent to the probability that the channel gain H_i, is above a certain threshold, $H_{\hat{T}}$ is given by

$$p = \exp^{(-\frac{H_{\hat{T}}^2}{h_0^2})} \quad (6)$$

where h_0 is the average value of fading.

In the simulation, for a given p value we derive the channel gain threshold, $H_{\hat{T}}$. Then we generate a channel model, covering a time period $\hat{T}$, in the form of

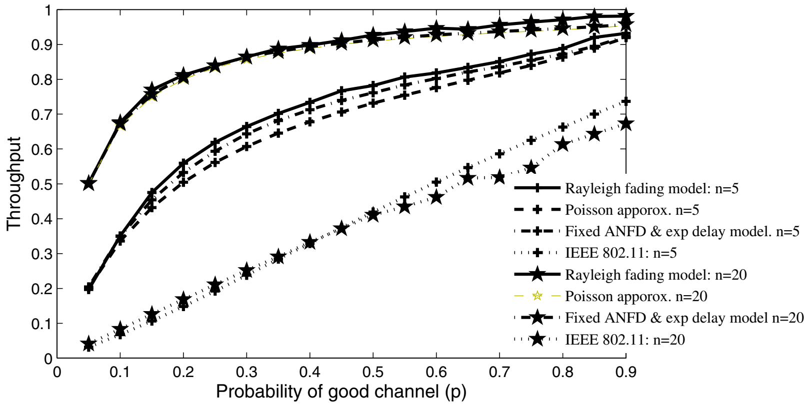

Fig. 3. Throughput vs. p for different number of stations

a set of time intervals, $\mathbf{\Lambda} = \{\lambda_1, \lambda_2, \ldots, \lambda_i, \ldots\}$, where the channel gain is above the threshold $H_{\hat{T}}$. These $\mathbf{\Lambda}$ time periods are the transmission intervals of a node when the probability of good channel is p. For n nodes, n sets of independent $\mathbf{\Lambda}$ time intervals, were generated. In case of overlapping transmission intervals from different nodes, only the first transmission interval in the overlapping group contributes to the throughput. We assume same p for all the stations in multiple node simulations.

Network throughput of Channel MAC for different probabilities of good channel is presented in Figure 3. Performance of IEEE 802.11 under fading channel conditions based on [22] are also shown in this figure.

5.3 Results Comparison

In spite of the slight variations in results (particularly for lower number of nodes), it can be noted that Channel MAC outperforms 802.11 for all p values in all cases. It can be noted that for higher number of nodes Channel MAC achieves higher throughput at lower p values, increasing the potential operating range. Furthermore the total throughput of the network will continue to increase with increasing number of nodes due to multiuser diversity, contrary to the performance of most other medium access control protocols (i.e., IEEE 802.11). For example for $n = 5$, throughput of the system is greater than 0.8 for $p > 0.6$, while for $n = 20$, same level of throughput can be achieved for $p > 0.2$. Comparing to a typical operating setting of IEEE 802.11 MAC where p=0.9 and $n = 5$, Channel MAC outperforms the corresponding throughput by 26% with the same number of nodes. It grows to 43% when $n = 20$ for both systems.

In Figure 4, The throughput vs number of stations in the Channel MAC and the IEEE 802.11 is shown at $p = 0.1$ and 0.85. It can be noted that the discrepancy between simulation and analytical model results decreases at lower p.

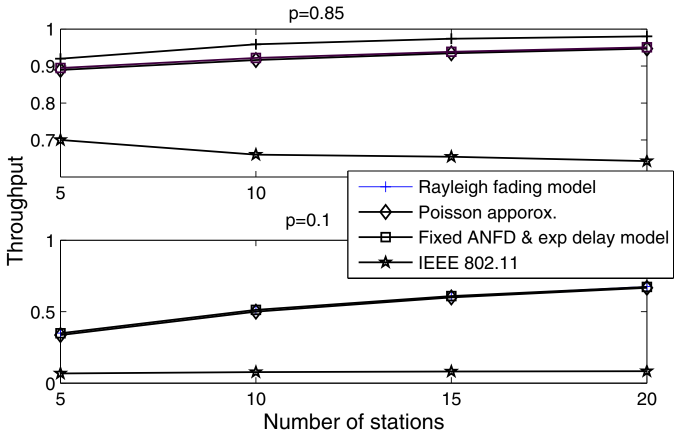

Fig. 4. Throughput vs. number of stations for $p = 0.1$ and $p = 0.85$

Furthermore, the simulation results should approach to the Poisson approximation with the increasing number of nodes. A detailed analysis of this discrepancy is given in Appendix A.

6 Conclusion

In this paper we evaluate analytically the Channel MAC paradigm. The analytical and simulation results presented show that Channel MAC can achieve higher throughput than IEEE 802.11 in distributed wireless networks. Moreover, the throughput in channel MAC scheme increases with increasing number of nodes, due to the multi user diversity of the system. Drawbacks of this scheme are the bandwidth required to exchange channel information between transmitter-receiver pairs and the added processing power required to predict channel conditions.

The analytical model accurately captures the behaviour of Channel MAC protocols for the two-state channel model presented in the paper. Furthermore through the comparison of analytical results to a channel model based on Rayleigh fading we have shown for large values of n the analytical model presented here, closely matches the performance of Channel MAC protocol in Rayleigh fading channels. In future work we aim to develop an analytical model to capture more general channel models.

Acknowledgements

The authors are grateful to Professor Lars Rasmussen, University of South Australia for useful feedback on the paper and general discussions.

References

1. Gupta, P., Kumar, R.: The capacity of wireless networks. IEEE Transactions on Information Theory 46(2), 388–404 (2000)
2. Grossglauser, M., Tse, D.: Mobility increases the capacity of ad-hoc wireless networks. IEEE/ACM Transactions on Networking 44(4), 477–486 (2002)
3. Ashraf, M., Jayasuriya, A., Perreau, S., Rasmussen, L.: Channel mac: A novel medium access control paradigm for wireless ad hoc networks. In: Australian Telecommunication Networks and Applications Conference, pp. 404–408 (December 2006)
4. Sagduyu, Y., Ephremides, A.: The problem of medium access control in wireless sensor networks. IEEE Wireless communications 11(6), 44–53 (2004)
5. Goldsmith, A., Varaiya, P.: Capacity of fading channels with channel side information. IEEE Transactions on Information Theory 43(6), 1986–1992 (1997)
6. Tse, D., Hanly, S.: Multi-access fading channels: Part i: Polymatroid structure, optimal resource allocation and throughput capacities. IEEE Transactions on Information Theory 44(7), 2796–2815 (1998)
7. Hanly, S., Tse, D.: Multi-access fading channels: Part ii: Delay-limited capacities. IEEE Transactions on Information Theory 44(7), 2816–2831 (1998)
8. Qin, X., Berry, R.: Exploiting multiuser diversity for medium access control in wireless networks. In: Proceedings of INFOCOM, vol. 2, pp. 1084–1094 (2003)
9. Srihari, A., Lang, T.: Exploiting decentralized channel state information for random access. IEEE Transactions on Information Theory 51(2), 537–561 (2005)
10. Venitasubramaniam, P., Adireddy, S., Tong, L.: Opportunistic aloha and corss layer design for sensor networks. In: Proceedings of MILCOM, pp. 705–711 (October 2003)
11. Zhao, Q., Tong, L.: Distributed opportunistic transmission for wireless sensor networks. In: Acoustics, Speech, and Signal Processing (ICASSP 2004), vol. 3, pp. 833–836 (May 2004)
12. Knopp, R., Humblet, P.: Information capacity and power control in single-cell multi-user communications. In: Proceedings of IEEE International Conference on Communications, pp. 331–335 (June 1995)
13. Feldmann, A., Whitt, W.: Fitting mixtures of exponentials to long-tail distributions to analyze network performance models. In: Proceedings of INFOCOM, pp. 1096–1104 (1997)
14. Cox, D., Isham, V.: Point Processes. Chapman and Hall, Australia (1980)
15. Cinlar, E.: Superposition of point processes. Stochastic point processes: statistical analysis, theory and applications, pp. 549–606 (1972)
16. Whitt, W.: Approximating a point process by a renewal process, i: Two basic methods. Operations Research 30(1), 125–147 (1982)
17. Schuhmacher, D.: Estimation of distances between point process distributions, PhD thesis, University of Zurich (2005)
18. Grigelionis, B.: On the convergence of sums of random step processes to a poisson process. Theory of Probability Applications 2(8), 177–182 (1963)
19. Banys, R.: On superpositions of random measures and point processes, NY. Mathematical statistics and probability theory, Lecture notes in Statistics, vol. 2, pp. 26–37 (1978)
20. Mieghem, P.: Performance analysis of communications networks and systems. Cambridge University Press, Cambridge (2006)

21. Kleinrock, L., Tobagi, F.: Packet switching in radio channels: Part 1- carrier sense multiple access modes and their throughput-delay characteristics. IEEE Transactions of Communications 23(12), 1400–1416 (1975)
22. Pham, P., Perreau, S., Jayasuriya, A.: New cross layer design approach to ad hoc networks under rayleigh fading. IEEE Journal on selected areas in communications 23(1), 28–39 (2005)

Appendix A: Difference in Poisson Approximation and Simulation Results of Channel MAC Throughout

The expected throughput between the Poisson approximation and that of the simulation 2 differs as observed in figure 3 and 4. It is related to the discrepancies between the independence measure of Poisson approximation and that of the shifted exponential assumption of the channel model.

A shifted exponential distribution was assumed for the inter-arrival process of 1-user Channel MAC. Its density is

$$f(x) = \begin{cases} \hat{L}e^{-\hat{L}(x-d)} & x \geq d \\ 0 & otherwise \end{cases} \tag{7}$$

where $\hat{L}^{-1}$ is the mean of the exponential variable and d is constant. The two parameters $\hat{L}$ and d are related to the mean μ and variance σ^2 of the shifted exponential density by

$$\mu = \hat{L}^{-1} + d; \sigma^2 = \hat{L}^{-2} \tag{8}$$

Since $d = l$ and $\mu = 1/r$ in 1-user Channel MAC, it follows that the Co-efficient of Variation (CV) of the inter-arrival process, c_a is

$$c_a = \frac{\sigma}{\mu} = \frac{1/\hat{L}}{1/\hat{L} + l} = 1 - p \tag{9}$$

In case of the superpositioned n-user Channel MAC, the mean of the exponential variable L and CV, c_{sup} are related to the component counterparts by following equations [16]:

$$L = \sum_{i=1}^{n} L_i = nL_i; c_{sup} = \sum_{i=1}^{n} (\frac{L_i}{L})c_i \tag{10}$$

where L_i and c_i are the mean of the exponential variable and CV of the ith component process, $i \in n$. As we consider same L_i values for all the channels, it follows that

$$c_{sup} = c_i = 1 - p \tag{11}$$

In simulation 2, due to approximation error in the Rayleigh fading model of the n-user Channel MAC, we get the non-linearity pattern in figure 5. It differs from the linear function of equation 9. Therefore, the simulation result approaches to the Poisson approximation at lower p values.

Again the mean arrival rate of the superpositioned n-user Channel MAC, $\mu_s = 1/nr = 1/L + d_s$.

$$\therefore d_s = \frac{1}{n}\left(\frac{1}{r} - \frac{1}{L_i}\right) = \frac{l}{n} \tag{12}$$

$d_s = 0$ results in a Poisson arrival point process. Hence the absolute value of d_s indicates a measure of independence. In simulation 2, the distribution of the arrival point process approaches to Poisson as $d_s \to 0$. For a fixed value of n, l increases with p. d_s is also increased evidently. Hence, for lower values of n (=5,10, etc e.g.,), the simulated results deviate more from the Poisson approximation as p increases. Consecutively, as n increases, $d_s \to 0$, indicating the distribution of the arrival process approaches to Poisson.

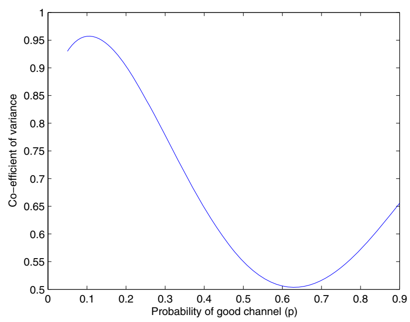

Fig. 5. approximate c-p curve pattern (smoothed) for 1-user Channel MAC by Simulation 1