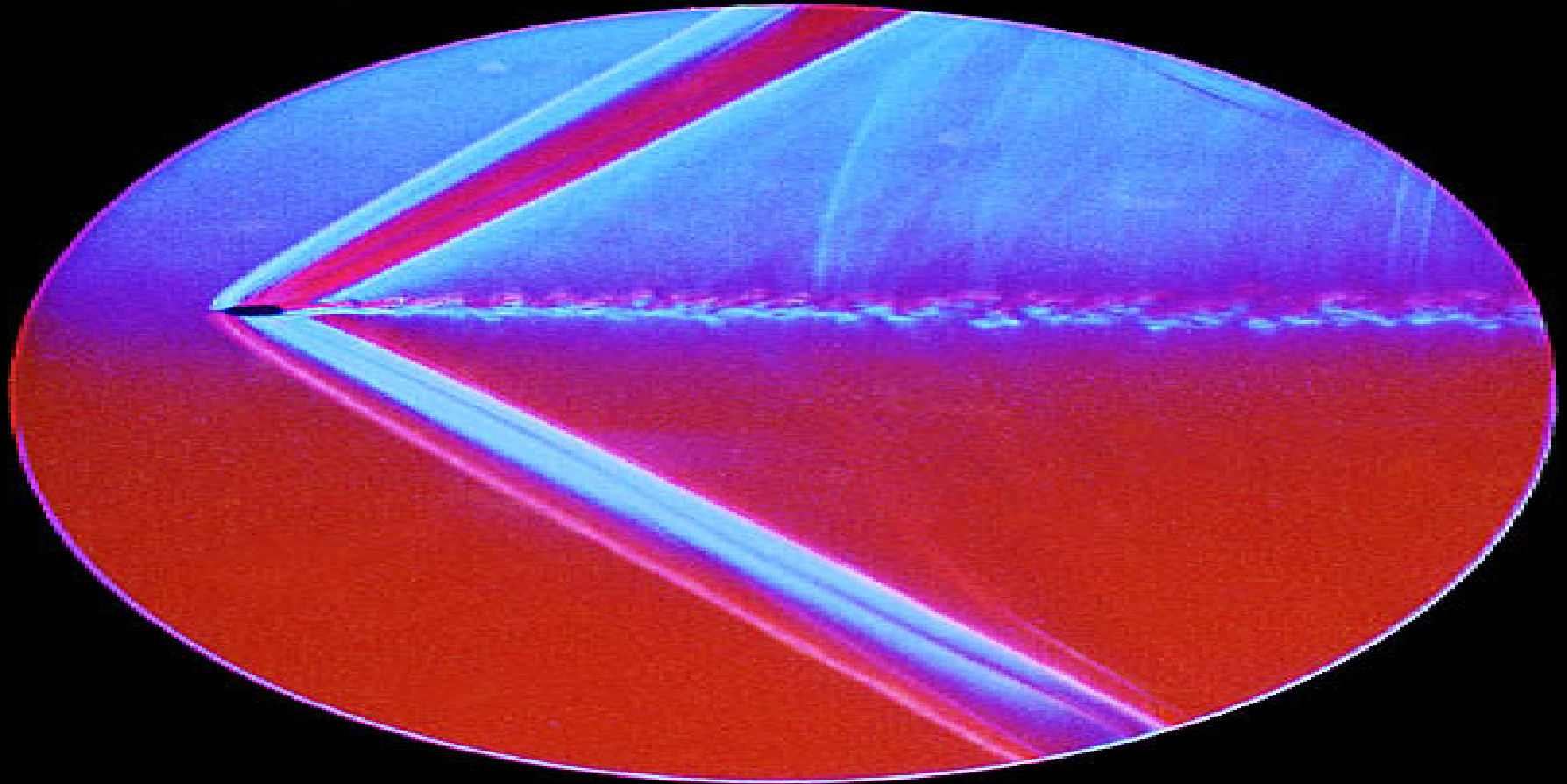


# THEORY OF MACH CONES IN DUSTY PLASMAS



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# OUTLINE

## □ Introduction:

- **What is a Mach cone?**
- **How do MCs look-like?**

## □ Theory of Mach Cones:

- **Space DPs**
- **Laboratory DPs**

## □ Summary

# □ Introduction

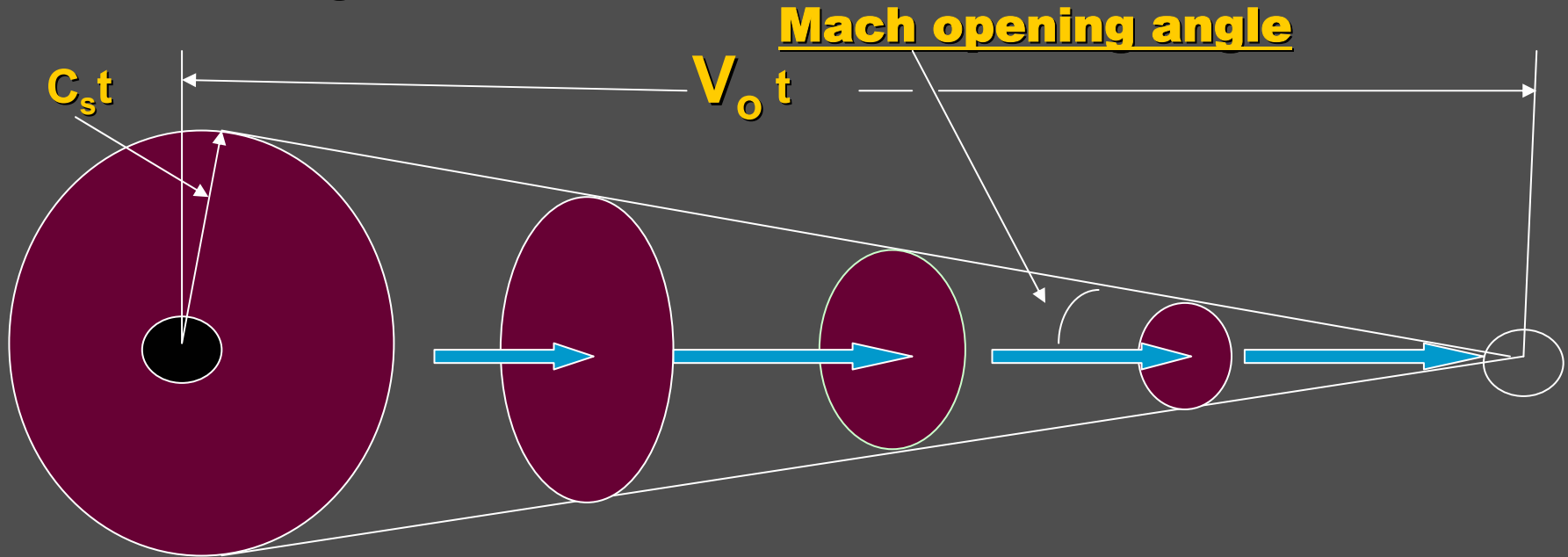
## ■ What is a Mach cone?

The story of Mach cone is very old [about 2 hundred years old]. To understand the formation of Mach cone, let us consider an object moving (with a speed  $V_o$ ) in a dispersive medium (of acoustic speed  $C_s$ ), and let us consider three situations:

1.  $V_o < C_s$ : The pressure pulses in front of the object are more closely spaced than those behind the object. This increases the frequency of the sound in front of the object, but decreases it behind the object. This is known as a **Doppler Shift**.

2.  $V_o = C_s$ : The pressure pulses in front of the object begin to pile up one on another. This causes a sudden pressure change. This leads to the formation of a **Shock Wave**.

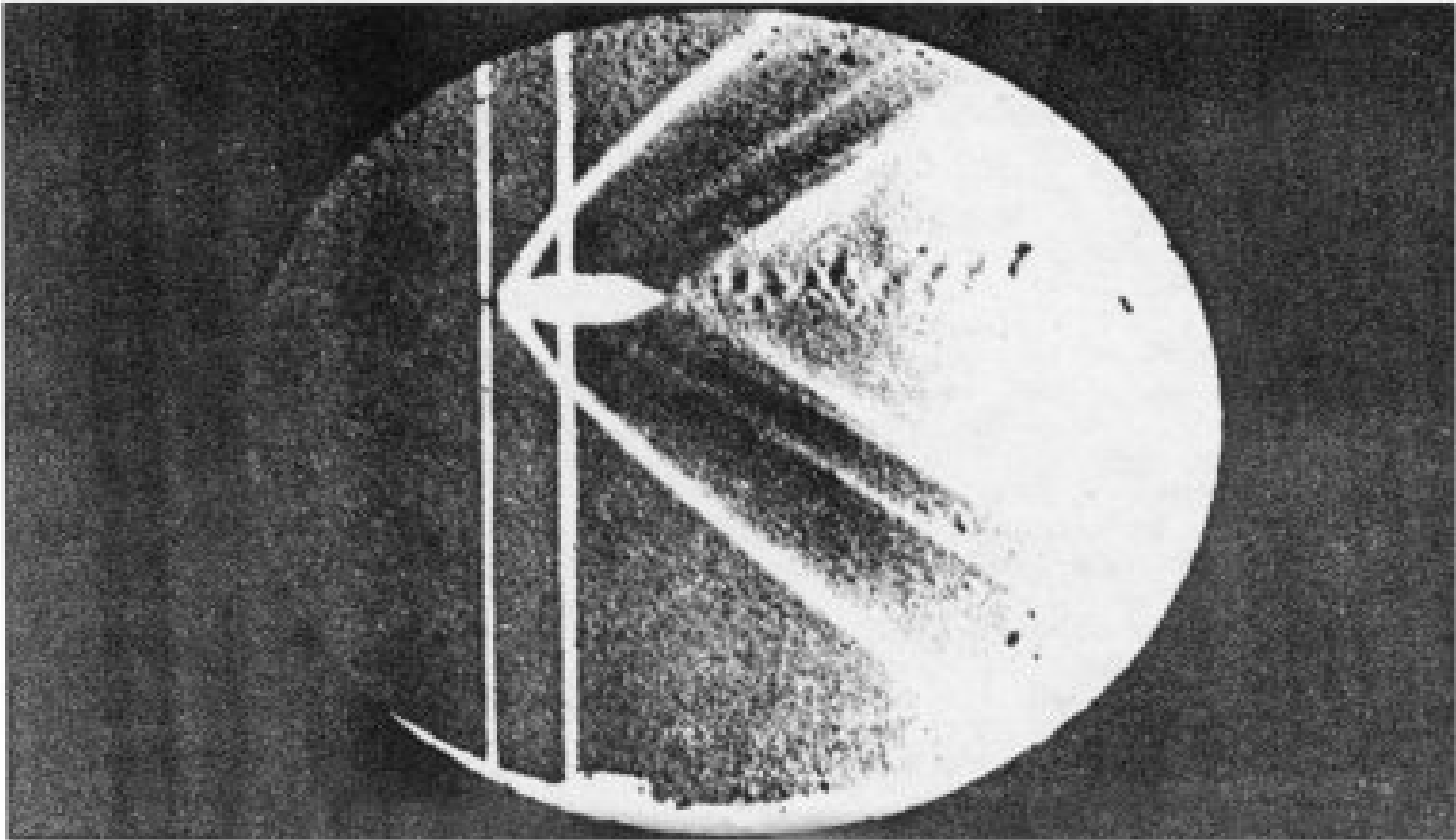
3.  $V_o > C_s$ : The object out-runs the generated pressure pulses. The loci of these out-run-generated pressure pulses form a cone: **Mach cone**. The region outside the cone: **zone of silence**.



The region inside the cone: **zone of action**. The angle that the cone makes with the horizontal: **Mach opening angle**. The Mach opening angle is, therefore, defined as  $\theta = \sin^{-1}(C_s/V_o)$ , since  $\sin\theta = C_s t/V_o t$ . In our investigation, we replace  $C_s$  by  $\omega/k = V_p$  for different waves in different media of our interest.

## ▪ **How do MCs look-like?**

- **The shape of Mac cones observed in a solid: a bullet experiment performed by Mach himself:**



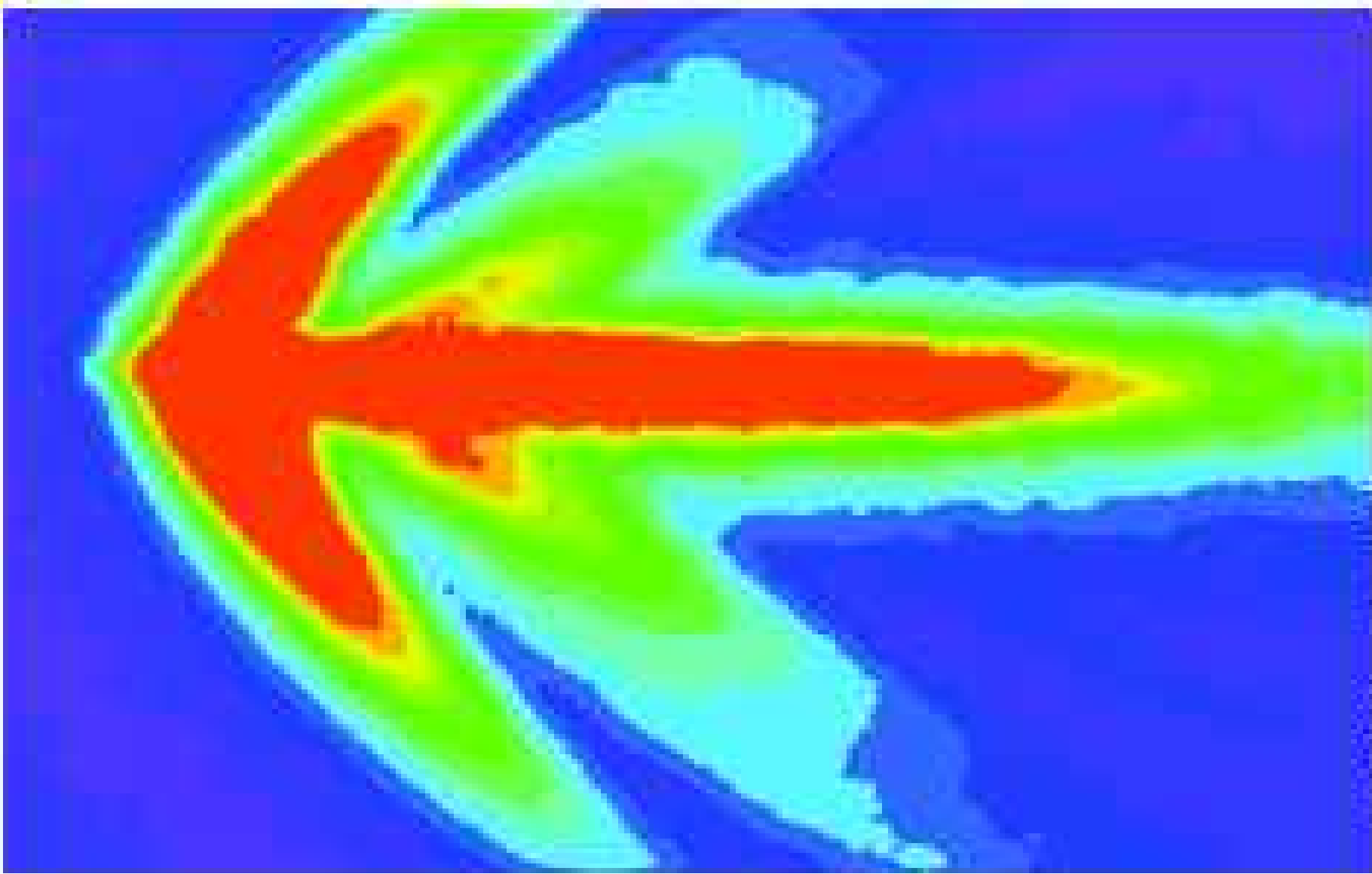
- **The patterns similar to Mach cones formed by a duck in a pond:**



- **The formation of Mach cones by a supersonic aircraft in (space) gas:**



- Experimentally observed (multiple) Mach cones in a dusty Plasma (Samsonov et al 2000):

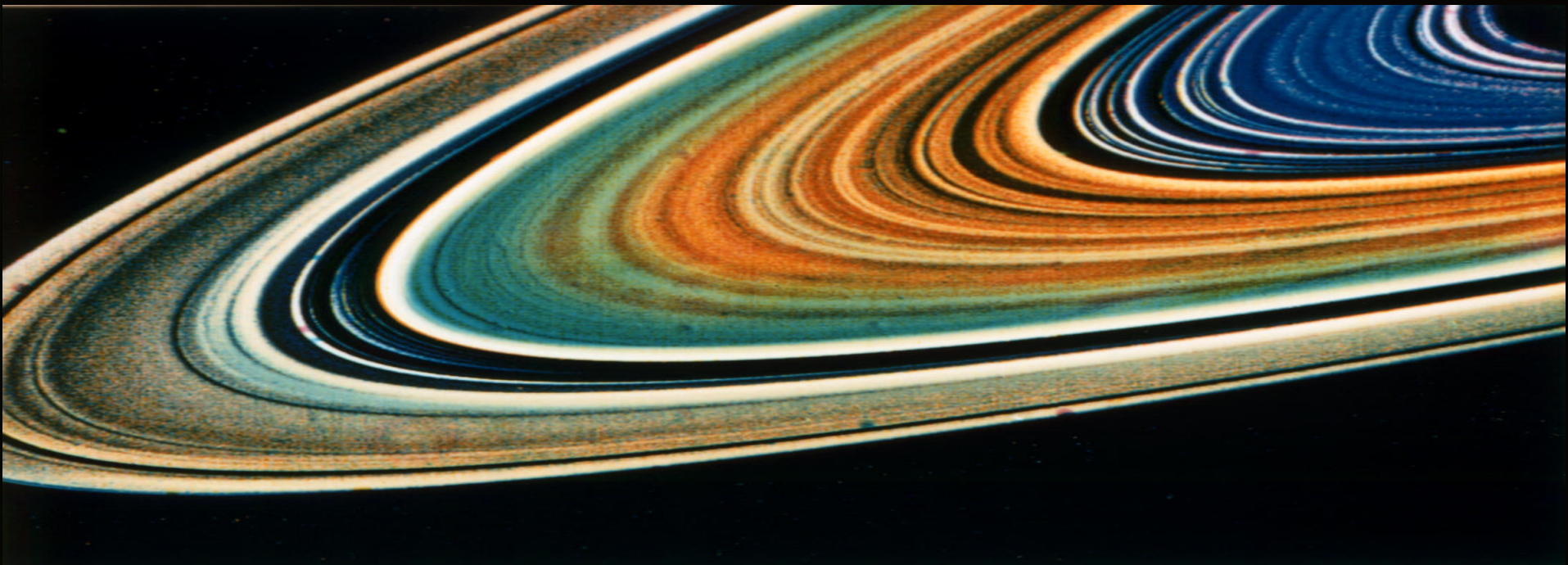


credit: University of Iowa

# □ Theory of Mach Cones

## ▪ Space DPs: Saturn's Dusty Rings

We first consider the possibility for the formation of Mach cones in Saturn's dusty rings. The figure below: mysterious spokes of Saturn's B-ring: The trails of dust grains: due to the es repulsion between charged dust particles and boulders.



# Dust Dynamics: Gravito-electrodynamics of dust in Saturn's rings:

The angular speed  $\omega_d$  of negatively charged dust particles in Saturn rings [Mendis et al. 1982]:

$$\omega_d = \frac{1}{2r^3} \left[ -\omega_{cd} \pm \sqrt{\omega_{cd}^2 + 4r^3(\Omega_k^2 + \omega_{cd}\Omega_p)} \right]$$

$r$  = dust particle position normalized by planet radius  $R_p$ ;  
 $\omega_{cd}$  = dust gyro-frequency;  $\Omega_k$  = Kepler frequency;  $\Omega_p$  = planetary spin rate;  $\pm$  = prograde (reprograde) motion of dust particles.

This means that a large boulder and a small dust particle will move at different speeds. The difference in speeds  $V_d$  is [Havnes et al. 1995] :

$$V_d = rR_p(\omega_d - r^{-3/2}\Omega_k)$$

## • DA Mach cones

The dispersion relation for DA waves propagating in DMP of Saturn's rings is [Shukla & Mamun 2003]:

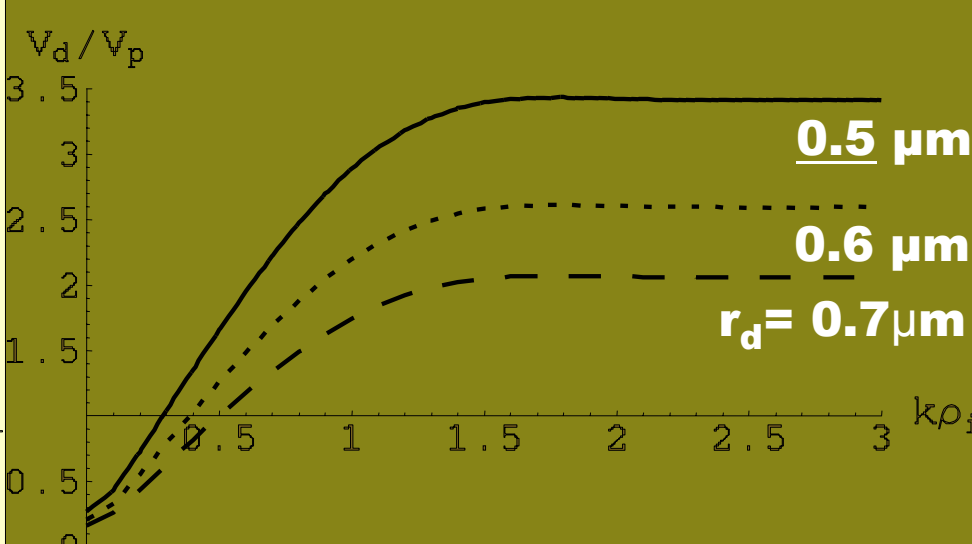
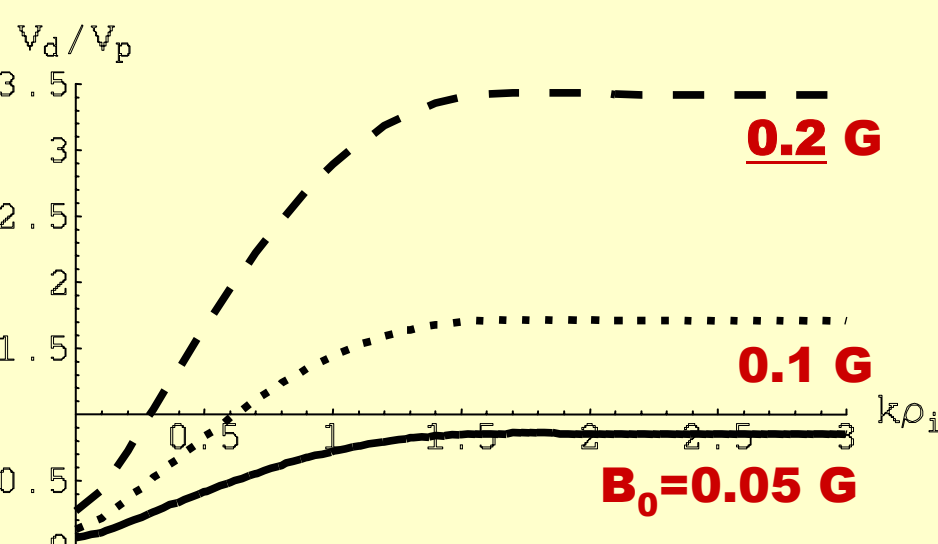
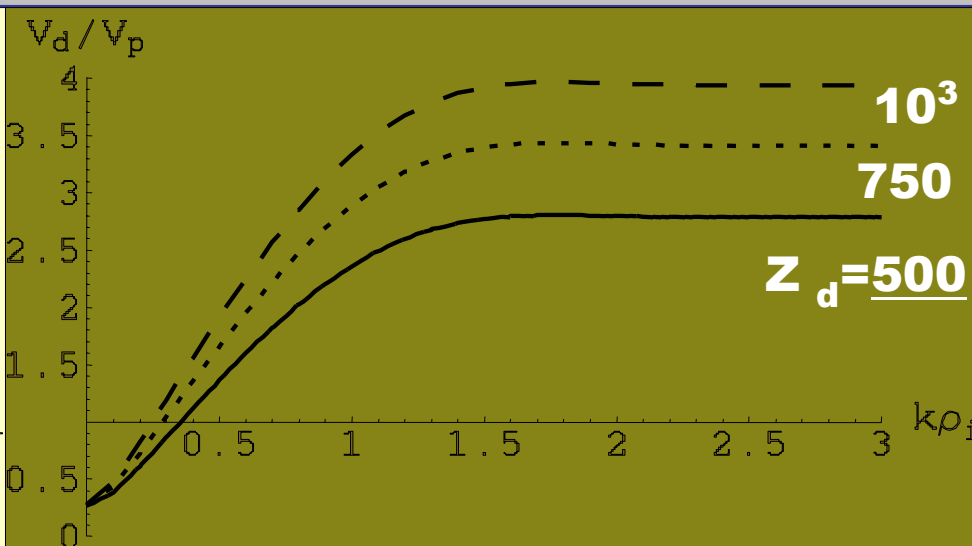
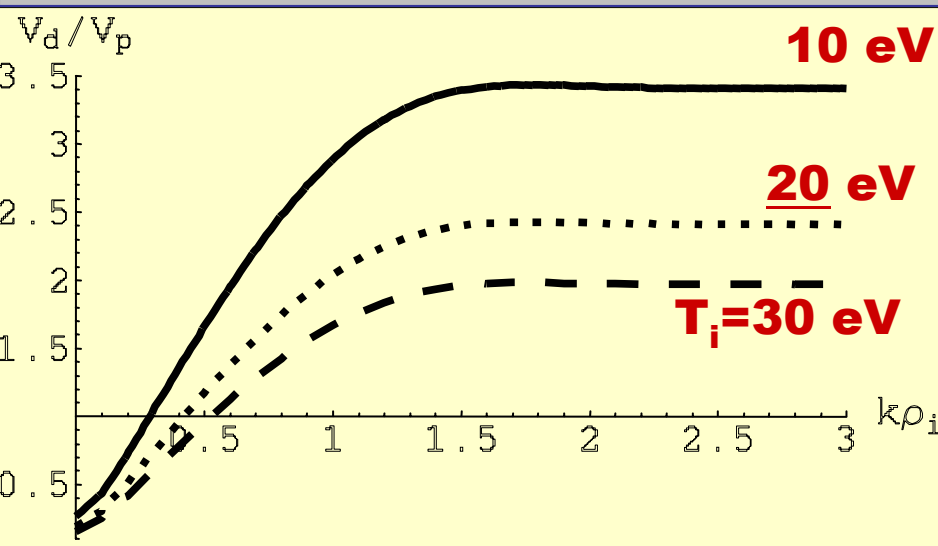
$$\left[ 1 - \frac{1}{\alpha} \Gamma_0(b_i) \right] \omega^2 + \frac{2}{\alpha} \Gamma_1(b_i) \frac{\omega^4}{\omega_{ci}^2} = \frac{k^2 C_D^2}{\alpha}$$

where  $\Gamma_{0,1} = I_{0,1} \exp(b_i)$ ,  $I_0$  ( $I_1$ ) is the modified Bessel function of 1st (2nd) order,  $b_i = (k_{\perp} \rho_i)^2$ ,  $C_D = \lambda_{Di} \omega_{pd}$ , and  $\alpha = 1 + (k_{\perp} \lambda_{Di})^2 + n_{e0} T_i / n_{i0} T_e$ .

For short wavelength limit (viz.  $b_i \gg 1$ ):  $\omega = k C_D / \alpha$  which is the dispersion relation of Rao et al. [1990].

We have numerically analyzed  $V_d$  and  $V_p = \omega/k$ . The numerical results can be interpreted as follows:

**Wavelength regimes of DAWs for which Mach cones are formed in DMPs of Saturn's rings:  $r=2$ ,  $n_{i0}=10\text{ cm}^{-3}$ ,  $n_{e0}=50\text{ cm}^{-3}$ ,  $T_e=30\text{ eV}$ ,  $\delta=85$  and underlined values for other parameters [Shukla & Mamun: Phys. Lett. A 315, 258 (2003)].**



# • DM Mach Cones

The dispersion relation for three branches [viz. shear dust Alfvén waves, fast (+) and slow (-) magnetosonic waves] of DM waves propagating in DMP of Saturn's rings are [Mamun et al: JTP Lett. (2003)]:

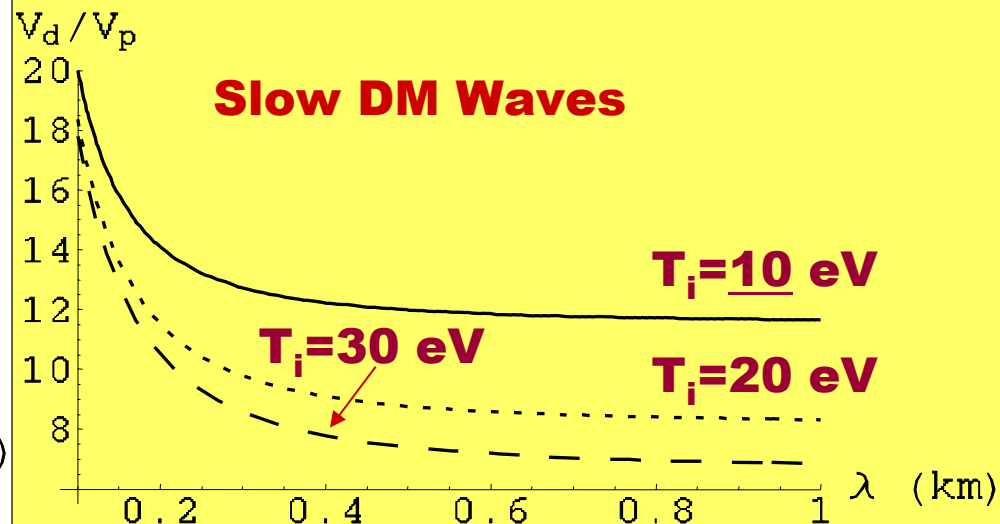
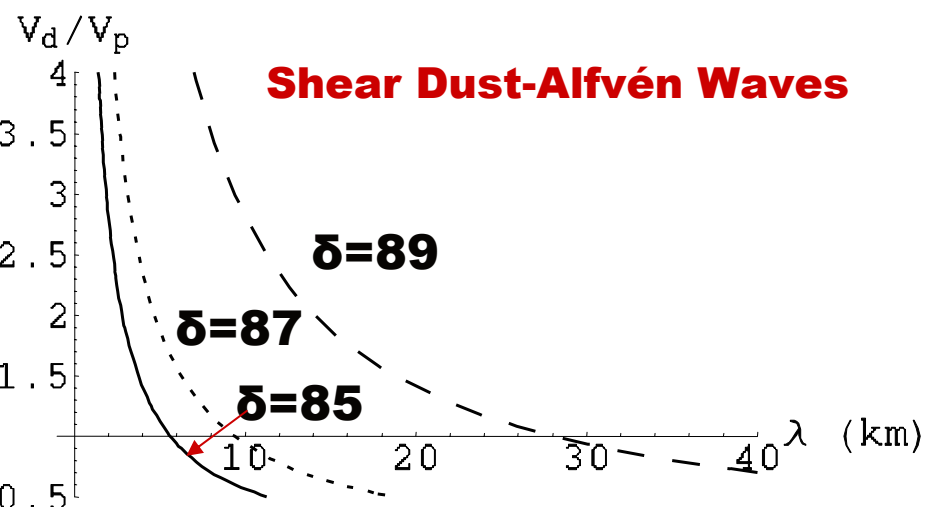
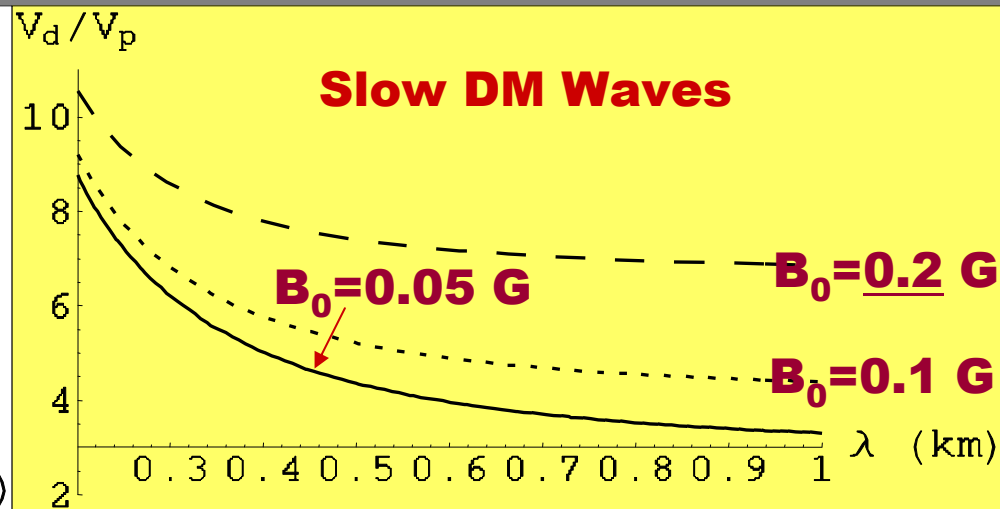
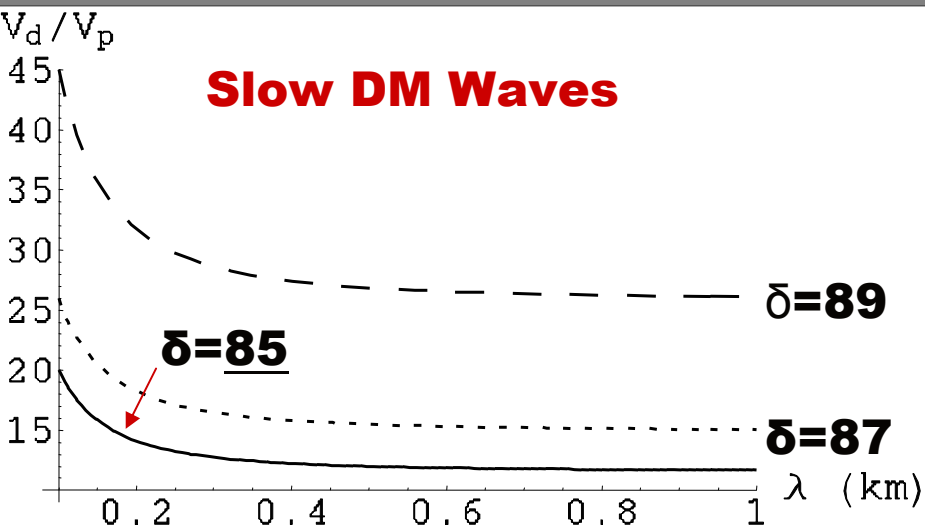
$$\frac{\omega}{k} = \frac{V_A \cos \delta}{\sqrt{1 + k^2 \lambda_i^2}}$$

$$\frac{\omega}{k} = \frac{1}{\sqrt{2}} \left[ \frac{V_A^2}{1 + k^2 \lambda_i^2} + C_d^2 \pm \sqrt{\left( \frac{V_A^2}{1 + k^2 \lambda_i^2} + C_d^2 \right)^2 - \frac{4V_A^2 C_d^2 \cos^2 \delta}{1 + k^2 \lambda_i^2}} \right]^{1/2}$$

where  $V_A$  = dust-Alfvén speed,  $C_D$  = dust-acoustic speed,  $\lambda_i$  = ion-skin depth, and  $\delta$  = angle between  $\underline{k}$  and  $\underline{B}_0$ .

We have numerically analyzed  $V_d$  and  $V_p = \omega/k$ . The numerical results can be interpreted as follows:

**Wavelength regimes of DMWs for which Mach cones are formed in Saturn's dusty rings:  $r=2$ ,  $n_{i0}=10\text{ cm}^{-3}$ ,  $n_{e0}=50\text{ cm}^{-3}$ ,  $Z_d=500$ ,  $r_d=0.5$  and underlined values for other parameters [Mamun et al.: JEPT Lett. 77, 541 (2003)].**



## ■ Laboratory DPs

We consider Lab. DMP containing weakly coupled electrons and ions, and strongly correlated dust. We employ both QLC approximation (Rosenberg & Kalman 1997) and GH model (Kaw & Sen 1998). The dispersion relation involving  $\Psi=V_p/V_d$  is [Mamun et al.: Phys. Rev. Lett. 92, 095005 (2004)]:

$$\alpha + \frac{2V_d^2 \Lambda_1}{\lambda_{Di}^2 \omega_{ci}^2} \Psi^2 - \frac{\omega_{pd}^2}{k^2 V_d^2 \Psi^2 - \omega_{pd}^2 \mathcal{D}(k)} = 0$$

where  $\alpha=1+(\omega_{pe}/\omega_{ce})^2+(1-\Lambda_0)/(k\lambda_{Di})^2$ ,  $V_d$  is used here as free parameter,  $\Lambda_{0,1}=I_{0,1}\exp(b_i)$ ,  $\omega_{pd}$ ,  $\omega_{pe}$ ,  $\omega_{ce}$  and  $\omega_{ci}$  are dp, ep, ec and ic frequencies, respectively.

For QLCA and GHM  $D(k)$  can be approximated as

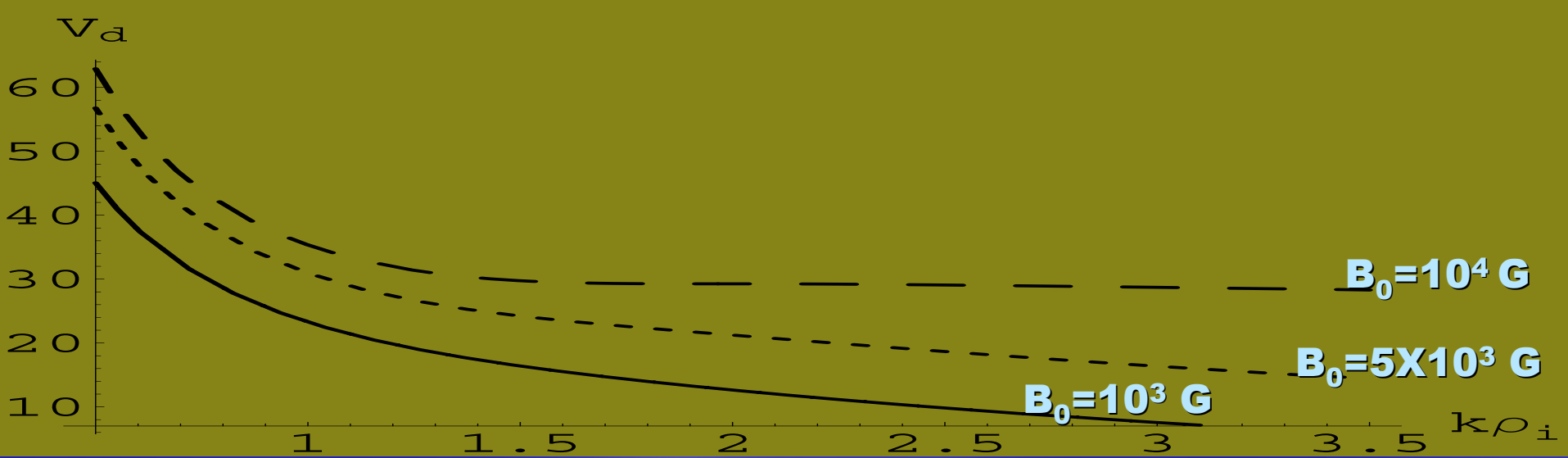
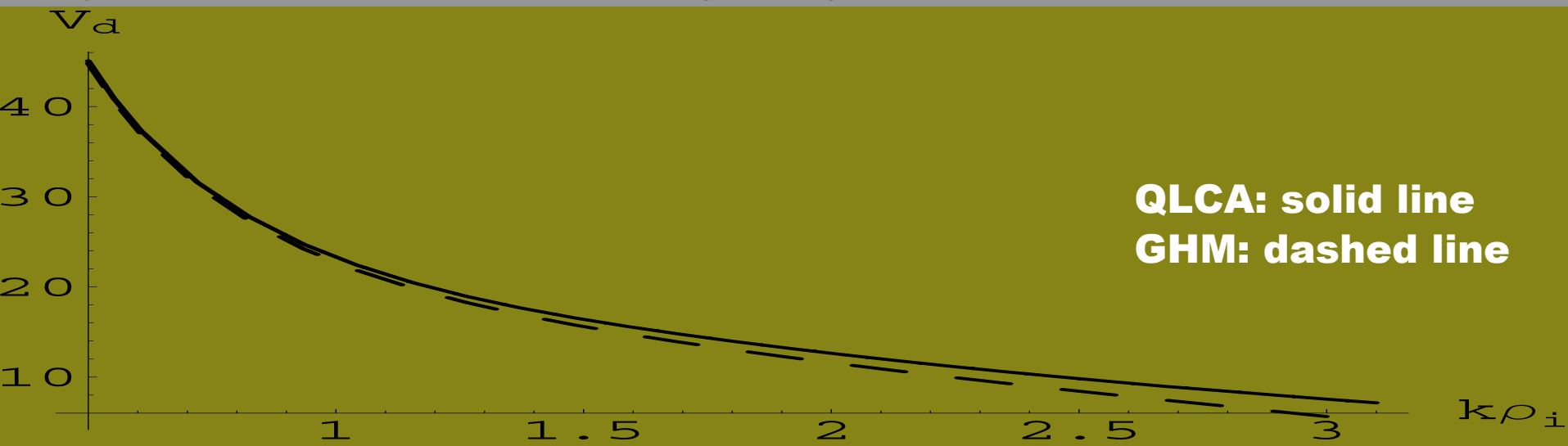
$$\mathcal{D}_{RK}(k) \simeq f k^2 a_d^2$$

$$\mathcal{D}_{KS}(k) \simeq \gamma_d \mu_d k^2 \lambda_{Dd}^2$$

where  $f = -[4(0.9 + 0.05(a_d/\lambda_D)^2)]/45$  for  $\Gamma_d \gg 1$  and  $d \ll \lambda_D$ ;  $a_d$  = dust grain radius,  $d$  = inter grain radius,  $\gamma_d$  = adiabatic index,  $\lambda_D$  = effective Debye radius;  $\lambda_{Dd}$  = dust Debye radius;  $\mu_d$  = compressibility.

We numerically analyzed, and found  $\psi=1$  curves in  $(V_d, k)$  space for typical laboratory dusty plasma parameters for both QLC approximation and GH model. The numerical results can be interpreted as follows:

$V_p/V_d=1$  curves in  $(V_d, k\rho_i)$  space for modified DAWs in a SCDP:  $n_{d0}=2\times 10^5 \text{ cm}^{-3}$ ,  $Z_d=10^3 \text{ cm}^{-3}$ ,  $n_{i0}=3\times 10^9 \text{ cm}^{-3}$ ,  $T_e=2 \text{ eV}$ ,  $T_i=1 \text{ eV}$ ,  $T_d=0.07 \text{ eV}$ ,  $B_0=10^4 \text{ G}$ , and  $r_d=0.3 \text{ }\mu\text{m}$  [Mamun et al: Phys. Rev. Lett. 92, 095005 (2004)].



# □ Summary

- **DA Mach cones in Space and Laboratory DPs:** The condition for the formation of DA Mach cones and prediction of their basic features, which are relevant to both space and laboratory DPs [Shukla & Mamun: Phys. Lett A. (2003); Mamun & Shukla: Geophys. Res. Lett. (2004); Mamun and Shukla: Phys. Scripta (2005)].
- **DM Mach cones in Space DPs:** The condition for the formation of DM Mach cones and prediction of their basic features, which are relevant to Saturn's dusty rings [Mamun, Shukla & Bingham: JETP Lett. (2003); Shukla, Mamun & Bingham: Phys. Scripta (2004); Mamun and Shukla: Phys. Scripta (2005)].

- **DA Mach cones in strongly coupled laboratory DPs:** The condition for existence of DA Mach cones in strongly coupled dusty plasma, and proposal for new lab. exp. [Mamun et al: Phys. Rev. Lett. (2004)].
- We finally hope that the theoretical predictions that we have presented here will be verified by new space and laboratory experiments.

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## **ACKNOWLEDGEMENT**

**Prof. P K Shukla**

**[for his guidance to work in DPP and in MCT]**

**Prof. R Bingham & Prof. G E Morfill**

**[for their suggestions/ideas to develop our MCT]**

*Last, but, of course, not the least, I would like to*



**THANK YOU ALL**