Effective Field theory and Nuclear Interactions

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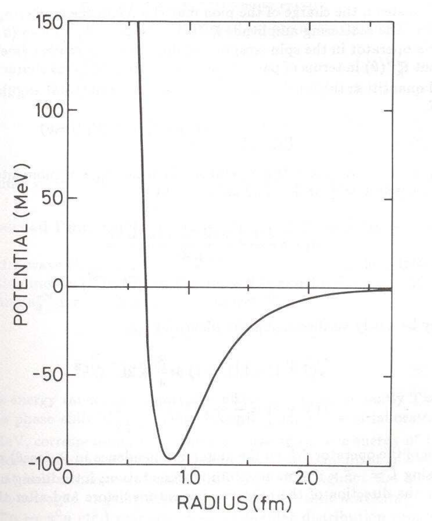
Outline

- Conventional NN interaction potential
- QCD and Nuclear Physics - Connection
- Effective Field theories
- Renormalization Group Approach to NN interactions
- Conclusions

Conventional NN interaction

- Potential as a function of inter-particle distance r

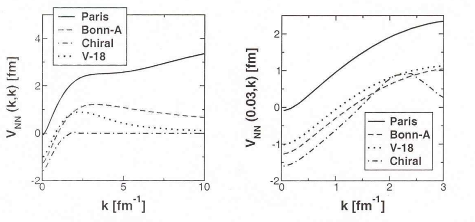


Conventional NN interaction

- In the big picture: Build up a microscopic theory for finite nuclei and nuclear matter.(Many-Body Systems)
- Main Features of conventional NN interactions:
  - short-range repulsion
  - intermediate and long-range attraction
  - long-range attraction and of “finite" range
- What do we know about the physics of NN potential?
  - Long-range part of the potential: One-Pion exchange
  - Intermediate and short range: scalar and vector meson exchanges

High-Precision Potentials

- High-Precision Potentials in momentum space



- Ingredients: One-pion exchange, 2-Pion exchange, short-distance: Different models use different ingredients.

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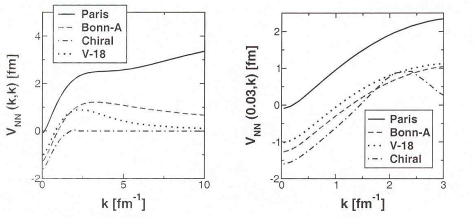

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- Reproduce two-nucleon data to $(\frac{\chi^2}{d.o.f.} \approx 1.)$

Nuclear Physics and QCD

- Theory of strong interactions: Quantum Chromodynamics (QCD)
- Interaction between light hadrons: Nuclear Physics (momentum scale $\approx 300\,\mathrm{MeV}$)
- different degrees of freedom (Quarks and gluons - QCD; Hadrons and Mesons - Nuclear Interaction)

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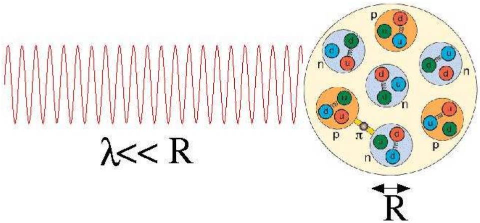
What is the connection? - Enter Effective Field Theories - EFT



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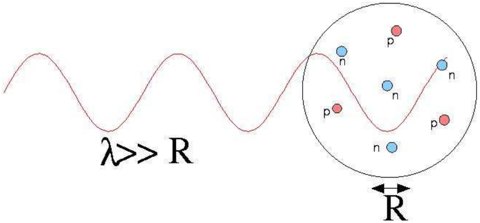

Effective Theories - ET

Phenomena at low-momenta cannot probe the details of the high-momentum structure.

- Example: Particle (mass m) at a height h from surface of earth: Gravitational Potential Energy difference

$$dU = mgh$$

  - Newton's Gravitation theory - ET - limit of small space-time curvature:

$$dU = \frac{GMm}{r_i} - \frac{GMm}{r_f}$$

  - Setting: $r_i = R; r_f = R + h$,

$$dU = \frac{GMm}{R^2} \frac{R}{R+h} h$$

$$dU = mgh(1 - \frac{h}{R} + \frac{h^2}{R^2} + \cdots)$$

Nuclear Effective Field Theory - EFT

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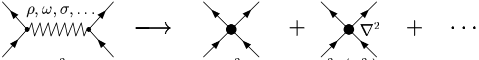

$$\frac{g^2}{\mathbf{q}^2 + m^2} \longrightarrow \frac{g^2}{m^2} - \frac{g^2}{m^2}\left(\frac{q^2}{m^2}\right) + \cdots$$

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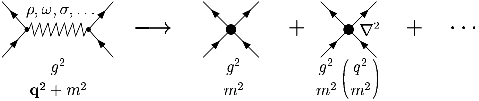

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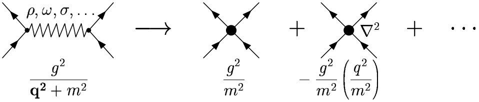

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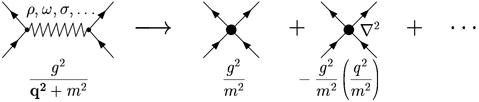

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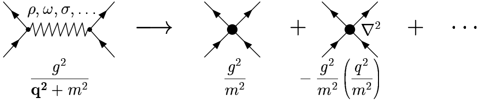

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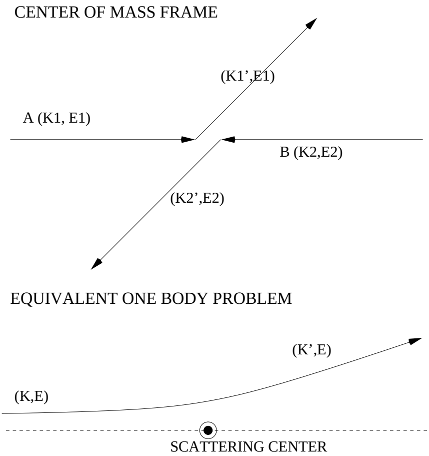
- Chiral Effective Theory Lagrangian:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^2_{\pi\pi} + \mathcal{L}^1_{\pi \mathrm{N}} + \mathcal{L}^2_{\pi \mathrm{N}} + \mathcal{L}^3_{\pi \mathrm{N}} + \cdots$$

- Connection to QCD: Pions - Pseudo-Goldstone Bosons

Renormalization Group Approach

- Two-Body Scattering



Renormalization Group Approach

- Lippmann Schwinger Equation:

$$|\Psi\rangle = |\phi\rangle + \frac{1}{E - \widehat{H}_0 \pm i\epsilon} \widehat{V} |\Psi\rangle$$

- Multiply by $\widehat{V}$:

$$\widehat{V}|\Psi\rangle = \widehat{V}|\phi\rangle + \widehat{V} \frac{1}{E - \widehat{H}_0 \pm i\epsilon} \widehat{V} |\Psi\rangle$$

- Introduce $\widehat{T}^{\pm}(E)$:

$$\widehat{T}^{\pm}(E)|\phi\rangle = \widehat{V}|\Psi\rangle,$$

$$\widehat{T}^{\pm}(E)|\phi\rangle = \widehat{V}|\phi\rangle + \widehat{V} \frac{1}{E - \widehat{H}_0 \pm i\epsilon} \widehat{T}^{\pm}(E)|\phi\rangle,$$

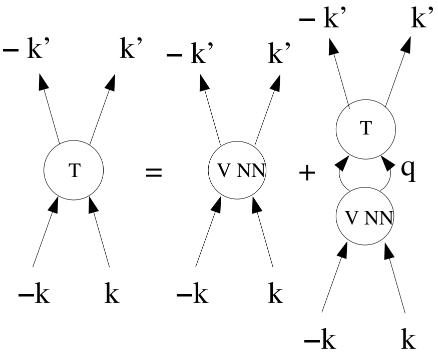
$$\widehat{T}^{\pm}(E)|\phi\rangle = \widehat{V}|\phi\rangle + \widehat{V} G_0(E \pm i\epsilon) \widehat{T}^{\pm}(E)|\phi\rangle$$

Renormalization Group Approach

- Lippmann-Schwinger equation for the l^{th} partial wave:

$$\langle k'|T_l(k^2)|k\rangle = \langle k'|V_l|k\rangle + \frac{2}{\pi}\mathcal{P}\int_0^\infty q^2 dq \frac{\langle k'|V_l|q\rangle\langle q|T_l(k^2)|k\rangle}{k^2 - q^2}.$$

- Schematically $\cdots$



Renormalization Group Approach

- Lippmann-Schwinger equation for the l^{th} partial wave:

$$T_l(k',k;k^2) = V_{\rm eff}(k',k) + \frac{2}{\pi}\mathcal{P}\int_0^{\Lambda} q^2 dq \frac{V_{\rm eff}(k',q)T_l(q,k;k^2)}{k^2-q^2}.$$

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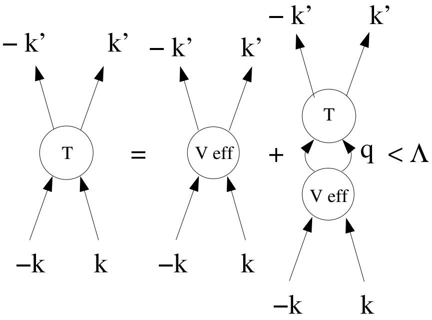


Renormalization Group Approach

- Physical Observables are independent of the cut-off, therefore:

$$\frac{dT_l(k',k;k^2)}{d\Lambda} = 0; \qquad T_l(k,k;k^2) \propto \sigma_l$$

$\rightarrow$ **R.G. EQUATION**

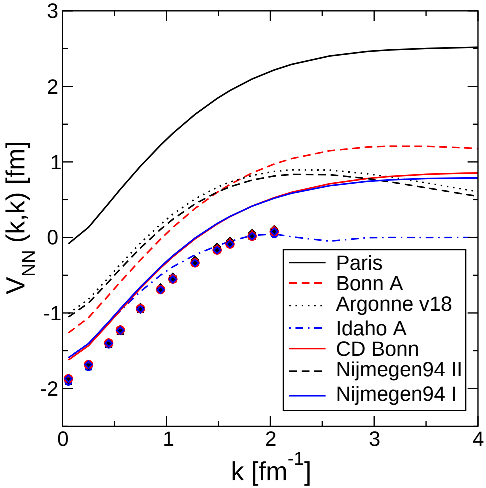
- Differential Equation for V_{eff} known as "$V_{\mathrm{low}\,k}$":

$$\frac{dV_{\mathrm{low}\,k}(k,k;\Lambda)}{d\Lambda} = \frac{2}{\pi}\frac{V_{\mathrm{low}\,k}(k,\Lambda)T_{\mathrm{low}\,k}(\Lambda,k;\Lambda^2)}{1-\dfrac{k^2}{\Lambda^2}}.$$

- set of first order coupled differential equation for $V_{\mathrm{low}\,k}(k',k)$
- Initial Condition: $V_{\mathrm{low}\,k}(k',k;\Lambda_0) = V_{\mathrm{NN}}(k',k;\Lambda_0)$

Renormalization Group Approach

- Matrix elements of $V_{\mathrm{low}\,k}$ compared to high-precision potentials



Renormalization Group Approach

- Expanding LS equation

$$T = V + VG_0T$$

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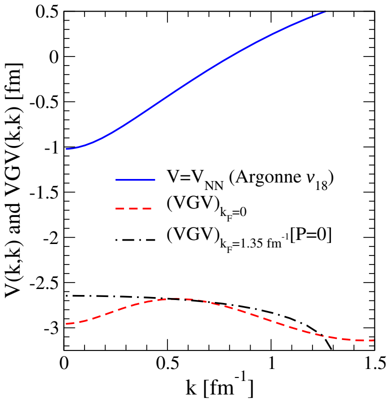

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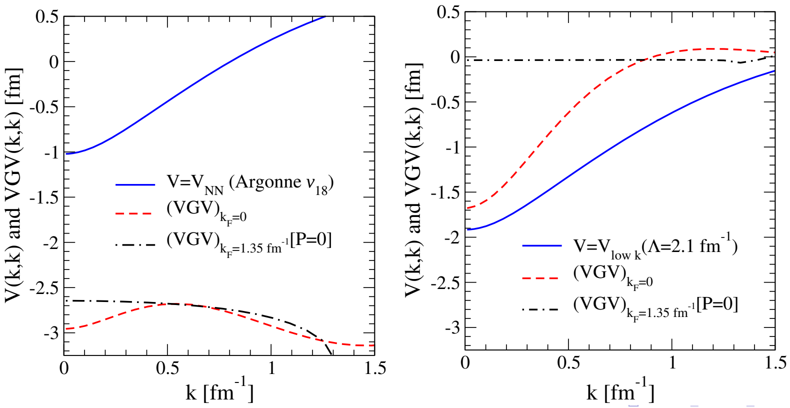

Renormalization Group Approach

- Softer potentials $\Longrightarrow$ smaller model space in many-body calculation
- Simplifies many-body computations - simple variational calculation for deuteron and triton (Bogner et. al)
- Systematic expansion