

Effective Field theory and Nuclear Interactions

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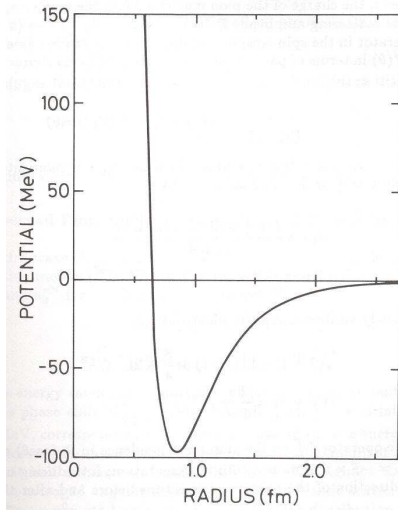
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- *Supported in part by the NSF*

- Conventional NN interaction potential
- QCD and Nuclear Physics - Connection
- Effective Field theories
- Renormalization Group Approach to NN interactions
- Conclusions

Conventional NN interaction

- Potential as a function of inter-particle distance r

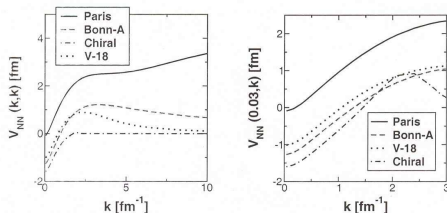


Conventional NN interaction

- In the big picture: Build up a microscopic theory for finite nuclei and nuclear matter. (**Many-Body Systems**)
- Main Features of conventional NN interactions:
 - short-range repulsion
 - intermediate and long-range attraction
 - long-range attraction and of "finite" range
- What do we know about the physics of NN potential?
 - Long-range part of the potential: **One-Pion exchange**
 - Intermediate and short range: **scalar and vector meson exchanges**

High-Precision Potentials

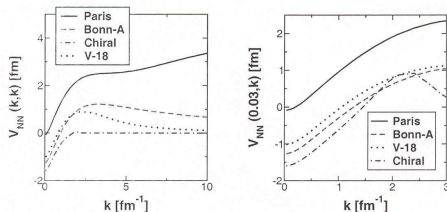
- High-Precision Potentials in momentum space



- Ingredients: One-pion exchange, 2-Pion exchange, short-distance: Different models use different ingredients.

High-Precision Potentials

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- Reproduce two-nucleon data to $(\frac{\chi^2}{d.o.f.} \approx 1.)$

Nuclear Physics and QCD

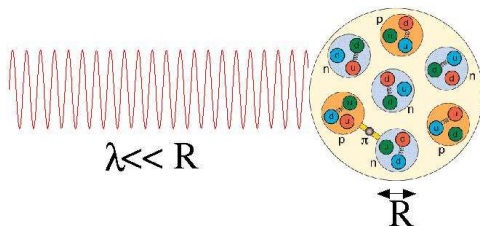
- Theory of strong interactions: **Quantum Chromodynamics (QCD)**
- Interaction between light hadrons: **Nuclear Physics** (momentum scale ≈ 300 MeV)
- different degrees of freedom (**Quarks and gluons** - QCD; **Hadrons and Mesons** - Nuclear Interaction)

What is the connection?

Nuclear Physics and QCD

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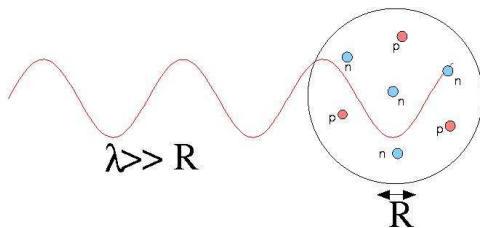
What is the connection? - Enter Effective Field Theories - EFT



Nuclear Physics and QCD

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What is the connection? - **Enter Effective Field Theories - EFT**



Effective Theories - ET

Phenomena at low-momenta cannot probe the details of the high-momentum structure.

- Example: Particle (mass m) at a height h from surface of earth: Gravitational Potential Energy difference

$$dU = mgh$$

- Newton's Gravitation theory - ET - limit of small space-time curvature:

$$dU = \frac{GMm}{r_i} - \frac{GMm}{r_f}$$

- Setting: $r_i = R$; $r_f = R + h$,

$$dU = \frac{GMm}{R^2} \frac{R}{R+h} h$$

$$dU = mgh \left(1 - \frac{h}{R} + \frac{h^2}{R^2} + \dots \right)$$

- Ingredients to set up an EFT
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Nuclear Effective Field Theory - EFT

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- 4 Short-distances in the form of contact interactions

The diagram shows a propagator with a wavy line and four external legs, labeled with $\rho, \omega, \sigma, \dots$. Below it is the expression $\frac{g^2}{q^2 + m^2}$. An arrow points to the right, where the propagator is expanded into a series of contact interactions. The first term is a four-point contact interaction with a black dot and the expression $\frac{g^2}{m^2}$. This is followed by a plus sign, then a four-point contact interaction with a black dot and a ∇^2 operator, with the expression $-\frac{g^2}{m^2} \left(\frac{q^2}{m^2} \right)$. This is followed by a plus sign and an ellipsis \dots .

Nuclear Effective Field Theory - EFT

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The diagram illustrates the expansion of a propagator in Effective Field Theory. On the left, a four-point vertex with a wavy internal line is labeled with $\rho, \omega, \sigma, \dots$ and has a denominator $\frac{g^2}{q^2 + m^2}$. An arrow points to the right, where the propagator is expanded into a series of contact terms: a four-point vertex with a solid black dot and denominator $\frac{g^2}{m^2}$, plus a four-point vertex with a solid black dot and a ∇^2 operator and denominator $-\frac{g^2}{m^2} \left(\frac{q^2}{m^2} \right)$, followed by an ellipsis $+$ \dots .

- Declare a regularization and renormalization scheme.

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$$\frac{g^2}{q^2 + m^2} \longrightarrow \frac{g^2}{m^2} + \frac{g^2}{m^2} \left(\frac{q^2}{m^2} \right) + \dots$$

- 5 Declare a regularization and renormalization scheme.
- 6 Establish a well defined power-counting.

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$$dU = mgh \left(1 - \frac{h}{R} + \frac{h^2}{R^2} + \dots \right)$$

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The diagram shows a propagator on the left, represented by a wavy line between two vertices, with external lines labeled $\rho, \omega, \sigma, \dots$. Below it is the expression $\frac{g^2}{q^2 + m^2}$. An arrow points to the right, where the propagator is expanded into a series of contact terms. The first term is a four-point vertex with a black dot and the expression $\frac{g^2}{m^2}$. The second term is a four-point vertex with a black dot and a ∇^2 operator, with the expression $-\frac{g^2}{m^2} \left(\frac{q^2}{m^2} \right)$. The series continues with $+$ and \dots .

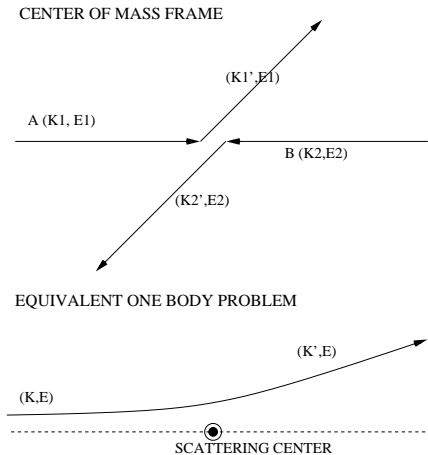
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- Chiral Effective Theory Lagrangian:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi}^2 + \mathcal{L}_{\pi N}^1 + \mathcal{L}_{\pi N}^2 + \mathcal{L}_{\pi N}^3 + \dots$$

- Connection to QCD: **Pions - Pseudo-Goldstone Bosons**

Renormalization Group Approach

- Two-Body Scattering



Renormalization Group Approach

- Lippmann Schwinger Equation:

$$|\Psi\rangle = |\phi\rangle + \frac{1}{E - \hat{H}_0 \pm i\epsilon} \hat{V}|\Psi\rangle$$

- Multiply by \hat{V} :

$$\hat{V}|\Psi\rangle = \hat{V}|\phi\rangle + \hat{V} \frac{1}{E - \hat{H}_0 \pm i\epsilon} \hat{V}|\Psi\rangle$$

- Introduce $\hat{T}^\pm(E)$:

$$\hat{T}^\pm(E)|\phi\rangle = \hat{V}|\Psi\rangle,$$

$$\hat{T}^\pm(E)|\phi\rangle = \hat{V}|\phi\rangle + \hat{V} \frac{1}{E - \hat{H}_0 \pm i\epsilon} \hat{T}^\pm(E)|\phi\rangle,$$

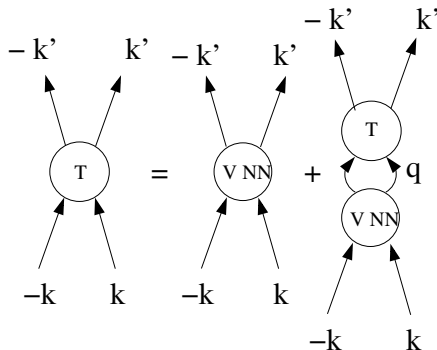
$$\hat{T}^\pm(E)|\phi\rangle = \hat{V}|\phi\rangle + \hat{V}G_0(E \pm i\epsilon)\hat{T}^\pm(E)|\phi\rangle$$

Renormalization Group Approach

- Lippmann-Schwinger equation for the l^{th} partial wave:

$$\langle k' | T_l(k^2) | k \rangle = \langle k' | V_l | k \rangle + \frac{2}{\pi} \mathcal{P} \int_0^\infty q^2 dq \frac{\langle k' | V_l | q \rangle \langle q | T_l(k^2) | k \rangle}{k^2 - q^2}.$$

- Schematically ...

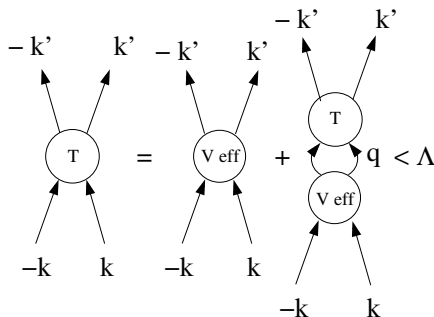


Renormalization Group Approach

- Lippmann-Schwinger equation for the l^{th} partial wave:

$$T_l(k', k; k^2) = V_{\text{eff}}(k', k) + \frac{2}{\pi} \mathcal{P} \int_0^\Lambda q^2 dq \frac{V_{\text{eff}}(k', q) T_l(q, k; k^2)}{k^2 - q^2}.$$

- Schematically ...



Renormalization Group Approach

- Physical Observables are independent of the cut-off, therefore:

$$\frac{dT_I(k', k; k^2)}{d\Lambda} = 0; \quad T_I(k, k; k^2) \propto \sigma_I$$

→ **R.G. EQUATION**

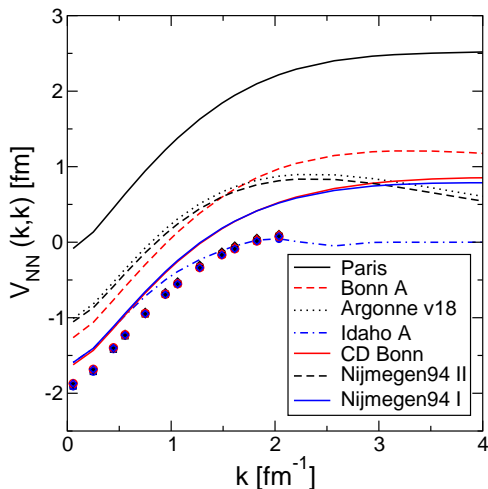
- Differential Equation for V_{eff} known as " $V_{\text{low } k}$ ":

$$\frac{dV_{\text{low } k}(k, k; \Lambda)}{d\Lambda} = \frac{2}{\pi} \frac{V_{\text{low } k}(k, \Lambda) T_{\text{low } k}(\Lambda, k; \Lambda^2)}{1 - \frac{k^2}{\Lambda^2}}.$$

- set of first order coupled differential equation for $V_{\text{low } k}(k', k)$
- Initial Condition: $V_{\text{low } k}(k', k; \Lambda_0) = V_{\text{NN}}(k', k; \Lambda_0)$

Renormalization Group Approach

- Matrix elements of $V_{\text{low } k}$ compared to high-precision potentials

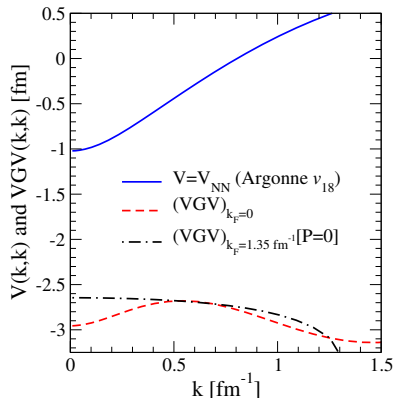


Renormalization Group Approach

- Expanding LS equation

$$T = V + VG_0 T$$

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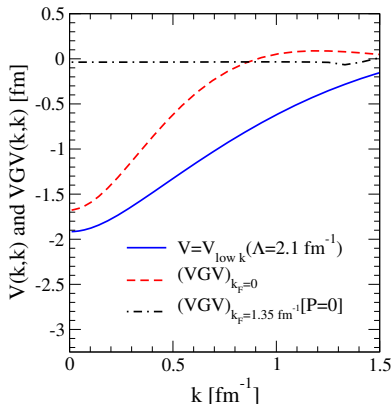
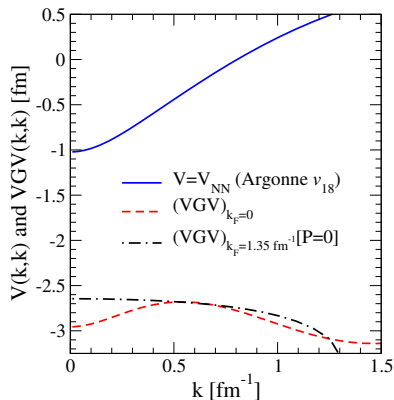


Renormalization Group Approach

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Renormalization Group Approach

- Softer potentials \implies smaller model space in many-body calculation
- Simplifies many-body computations - simple variational calculation for deuteron and triton (Bogner et. al)
- Systematic expansion