Weinberg Eigenvalue Analysis and the NN Potential

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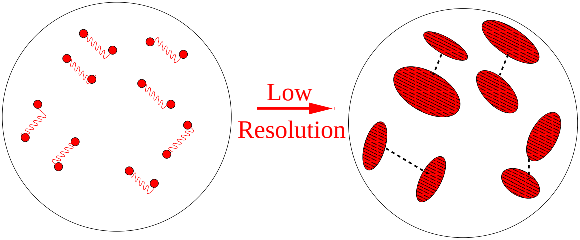
Renormalization Group Approach

- T matrix Eq. in k space for the l^{th} partial wave

$$\mathrm{T}_l(k',k;k^2) = \mathrm{V}_l(k',k) + \frac{2}{\pi}\int_0^\infty q^2 dq \frac{\mathrm{V}_l(k',q)\mathrm{T}_l(q,k;k^2)}{(k^2-q^2)}$$

- Resolution



- Enter R.G. Principle (q,k,k' < cut-off Λ)

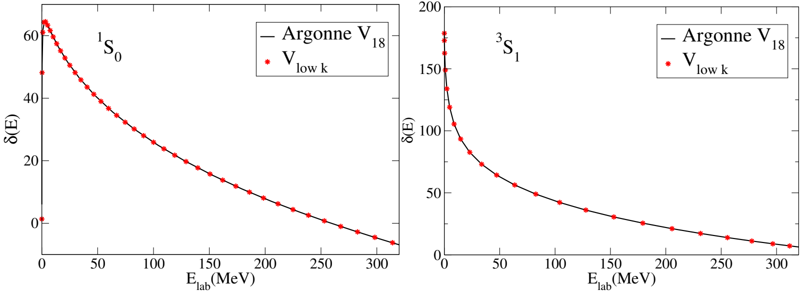
$$\mathrm{T}_l^\Lambda(k',k;k^2) = \mathrm{V}_l^\Lambda(k'k) + \frac{2}{\pi}\int_0^\Lambda q^2 dq \frac{\mathrm{V}_l^\Lambda(k',q)\mathrm{T}_l^\Lambda(q,k;k^2)}{(k^2-q^2)}$$

Renormalization Group Approach

- Physical Observables are independent of the cut-off, therefore:

$$\frac{dT_l^\Lambda(k',k;k^2)}{d\Lambda} = 0.$$

- R.G. EQUATION $\rightarrow$ Differential Equation for V_l^Λ ("$V_{\text{low}\,k}$")



$V_{\text{low}\,k}$ and Many-body Calculations

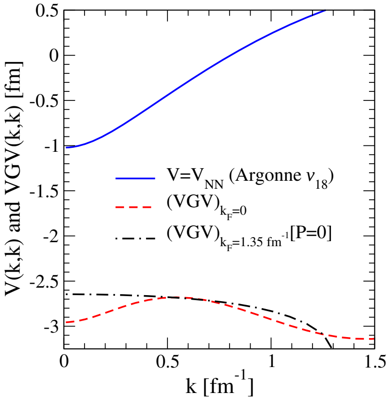
- Sources of non-perturbative physics:
  - Short range repulsion
  - Tensor force from one-pion exchange
  - Shallow bound states (S channel)
- As a result Born Series is non-perturbative

$$T = V + VG_0V + VG_0VG_0V + VG_0VG_0VG_0V + \cdots$$

- Sum all terms

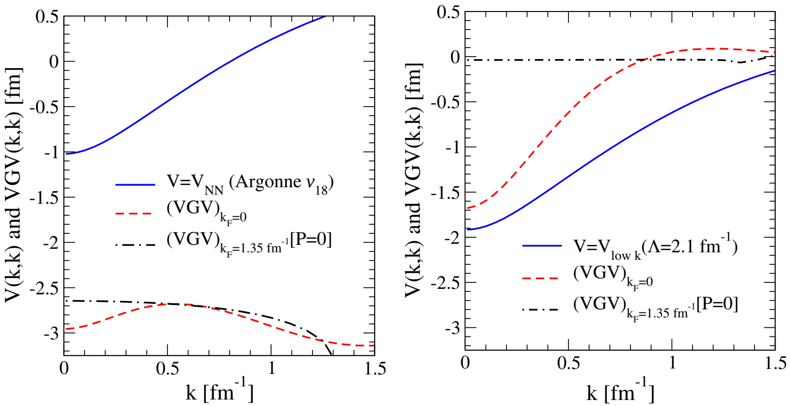
$$T = V + VG_0T$$

- Sources are resolution and/or density dependent



- First and second term in the T matrix expansion for Argonne V_{18} and corresponding $V_{\text{low}\,k}$ (*Bogner et al. nucl-th/0504043*)

$V_{\mathrm{low}\,k}$ and Many-body Calculations



- First and second term in the T matrix expansion for Argonne V_{18} and corresponding $V_{\mathrm{low}\,k}$ (*Bogner et al. nucl-th/0504043*)

Weinberg Eigenvalues [Phys.Rev.**133**(1964) 1589]

- Consider operator $\mathrm{G_0 V}$ then,

$$\mathrm{G_0 V}|\Gamma_\nu(z)\rangle = \eta_\nu(z)|\Gamma_\nu(z)\rangle$$

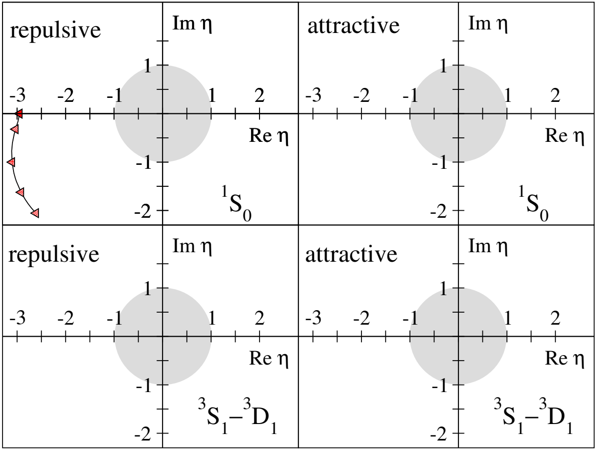
- T matrix operating on the Weinberg eigenstates $\Gamma_\nu(z)$:

$$\mathrm{T}|\Gamma_\nu(z)\rangle = (\mathrm{V} + \mathrm{VG_0V} + \mathrm{VG_0VG_0V} + \cdots)|\Gamma_\nu(z)\rangle$$

$$= \mathrm{V}(1 + \eta_\nu(z) + (\eta_\nu(z))^2 + \cdots)|\Gamma_\nu(z)\rangle$$

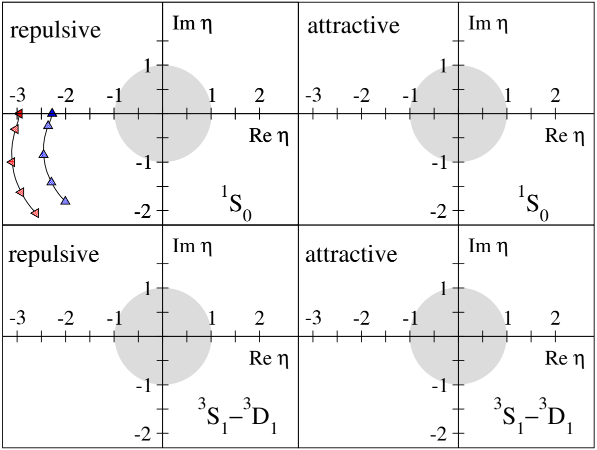
- Converges only if $|\eta_\nu(z)| < 1$
- $\eta_\nu(z)$ Real for $z = E \leq 0$, else $\eta_\nu(z)$ Complex
- For $z < 0$, $\eta_\nu(z) \geq 0$ Attractive eigenvalue and $\eta_\nu(z) < 0$ Repulsive eigenvalue

Weinberg Eigenvalues for $V_{\text{low}\,k}$



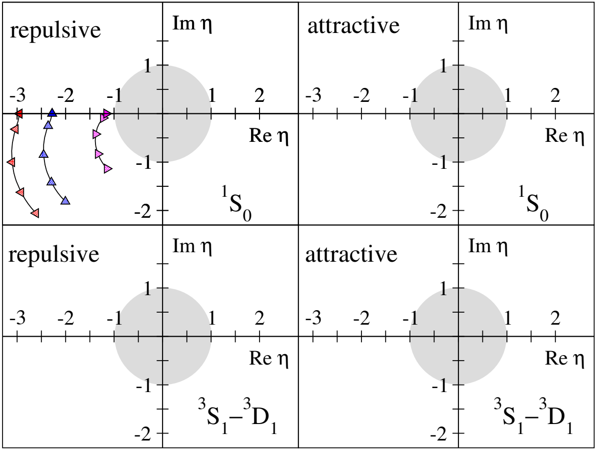
- $\eta_\nu(E)$ for E $\geq$ 0 as a function of cut-off $\Lambda = 10 fm^{-1}$,

Weinberg Eigenvalues for $V_{\text{low } k}$



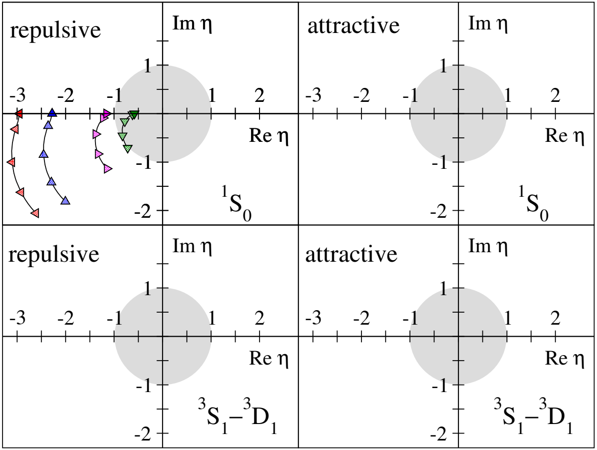
- $\eta_\nu(E)$ for E $\geq$ 0 as a function of cut-off $\Lambda = 10fm^{-1}, 7fm^{-1}$,

Weinberg Eigenvalues for $V_{\text{low}\,k}$



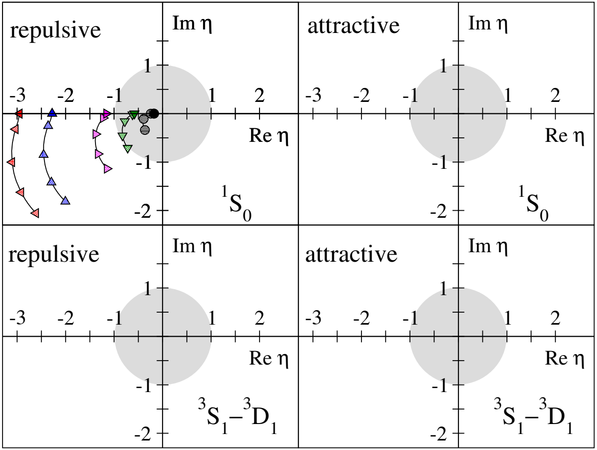
- $\eta_\nu(E)$ for E $\geq$ 0 as a function of cut-off Λ = $10fm^{-1}$, $7fm^{-1}$, $5fm^{-1}$,

Weinberg Eigenvalues for $V_{\text{low}\,k}$



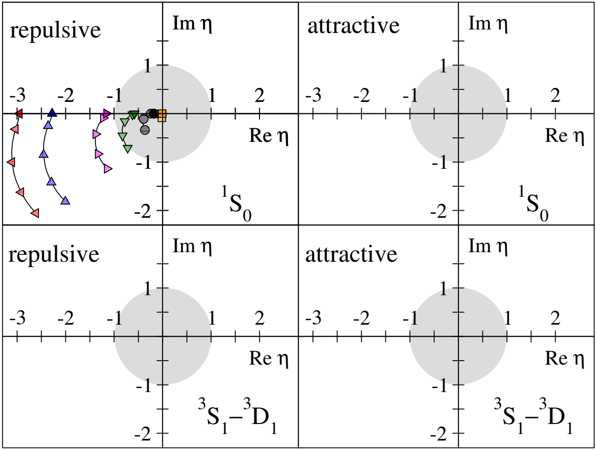
- $\eta_\nu(E)$ for $E \geq 0$ as a function of cut-off Λ = $10fm^{-1}$, $7fm^{-1}$, $5fm^{-1}$, $4fm^{-1}$,

Weinberg Eigenvalues for $V_{\text{low } k}$

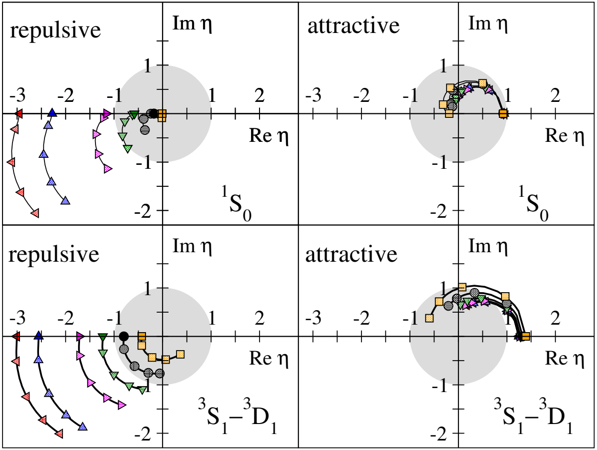


- $\eta_\nu(E)$ for E $\geq$ 0 as a function of cut-off $\Lambda = 10 fm^{-1}$, $7 fm^{-1}$, $5 fm^{-1}$, $4 fm^{-1}$, $3 fm^{-1}$,

Weinberg Eigenvalues for $V_{\text{low } k}$

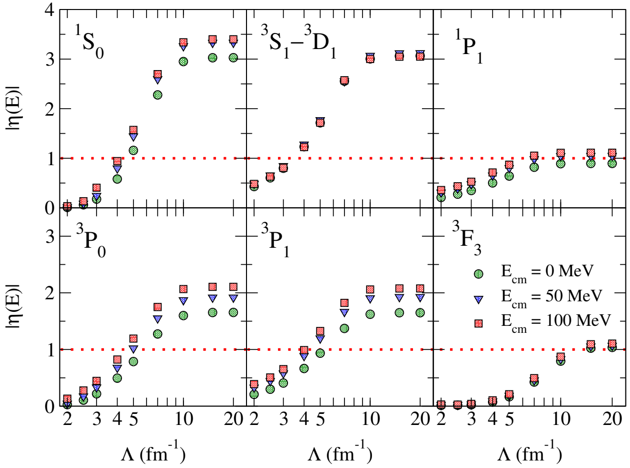


- $\eta_\nu(E)$ for E $\geq$ 0 as a function of cut-off Λ = $10fm^{-1}$, $7fm^{-1}$, $5fm^{-1}$, $4fm^{-1}$, $3fm^{-1}$, $2fm^{-1}$



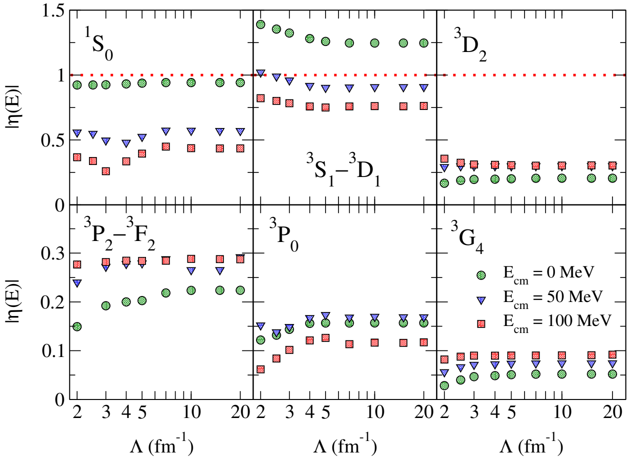
- $\eta_\nu(E)$ for E $\geq$ 0 as a function of cut-off Λ = $10fm^{-1}$, $7fm^{-1}$, $5fm^{-1}$, $4fm^{-1}$, $3fm^{-1}$, $2fm^{-1}$

Weinberg Eigenvalues for $V_{\text{low}\,k}$



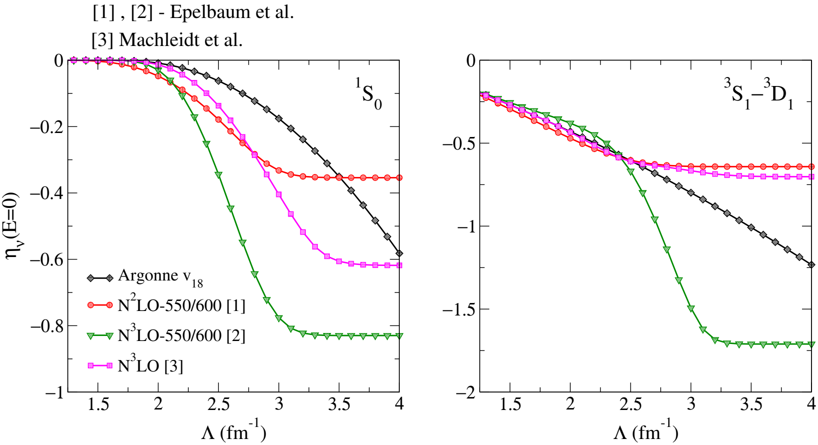
- Repulsive $\eta_\nu(E)$ as a function of Λ.

Weinberg Eigenvalues for $V_{\mathrm{low}\,k}$



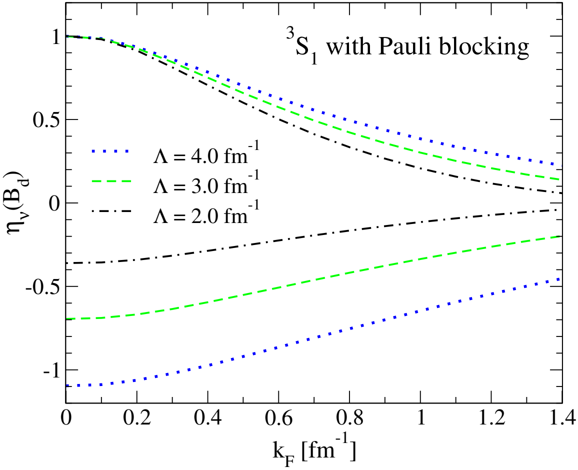
- Attractive $\eta_\nu(E)$ as a function of Λ.

Weinberg Eigenvalues for Chiral Potentials



- Singular potentials at N^3LO tamed by lowering the cut-off

Weinberg Eigenvalues for $V_{low\,k}$ in medium



- In-medium Calculation → perturbative in pp channel! (*Bogner et al. nucl-th/0504043*)

Summary and Outlook

- Summary (*Refer: nucl-th/0602060 for more details*)
  - Renormalization Group based low-momentum potentials simplify many-body calculations
  - Weinberg Eigenvalue Analysis serves as a diagnostic to understand the Physics at different scales for Nucleon-Nucleon interaction potentials
  - Separable Expansions
- Outlook
  - Explore the regulator dependence of eigenvalues
    - Smooth Regulator vs Sharp
  - Explore separable expansions and subsequent regulator dependence
  - Explore the possibility of using the above tool to understand the physics at different scales for other systems