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A New Definition of Parallelism

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The fifth postulate of Euclid, the parallel postulate, was formulated in 300 B.C., and is still not proved as a theorem, although many have tried to do that. The reason for this is **not** to be found in the postulate per se, but in the definition of parallelism. The most common definition states that **parallel lines never meet.** It will be demonstrated here why this definition is **wrong**, and why parallelism is hard to define.

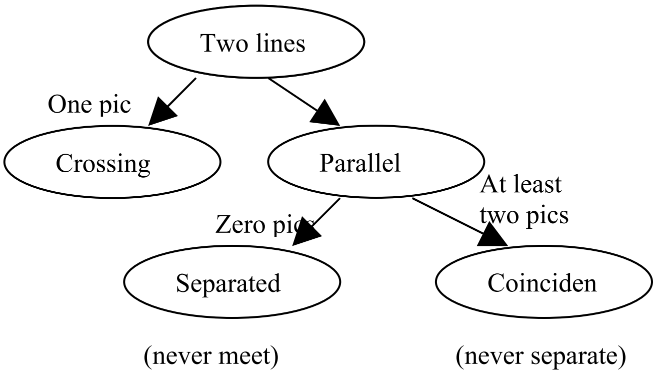

Fig. 1 Relation between two lines. The number of 'points in common' is referred to as 'pics'.

The Old Definition

The relations between two **straight, unlimited lines in a plane** are analysed in Fig. 1, where we can see that the common definition covers **separated** lines instead of **parallel** lines. (Separated lines and coincident lines are two subsets of parallel lines.) A consequence of this is that coincident lines are defined as being **not** parallel, since in geometry we use a binary logic which is 'excluding the middle'. In contrast to the accepted definition we find here, that **parallel lines either never meet or never separate.** This fact (including Fig. 1) is earlier mentioned in [1].

About Definitions

We do not want to use alternatives (either… or…) in a definition and we want a short definition. A definition shall, in a simple expression, cover both kinds of parallelism (separated and coincident). We do not want to use negation and the definition should not contain an unlimited distance to a point of possible intersection. (If we consider a finite distance instead, then the common definition becomes only approximate.) The distance between two lines can not be used, because that distance is not defined until **after** parallelism is proved. Therefore: We need a better definition.

The New Definition

Two theorems together state that we can always draw a line perpendicular to a given line and through a given point independent of if the given point is on or outside the given line. We can therefore define the distance from a point to a line as **the distance along such a perpendicular line**. This distance is not dependent on parallelism, and can therefore serve as base for our definition of parallelism. Since this distance can take the value of zero, coincident lines are also included in the definition. A postulate states, that **two points define a unique line.** Parallelism can now be defined in the following way: Two lines are parallel **if, and only if, two points on one of the lines, lying on the same side of the other line, have equal distances to the other line.** This simple and natural definition avoids an infinite component in the form of the distance to a possible point of intersection. This is done by the use of **two** points, and the mistake of defining **a subset** instead of the correct thing is avoided. It is an amazing fact that this definition has not been tested earlier.

Applying the Definition

A theorem states, that **if equals add to equals, then the results are equal.** Together with our new definition of parallelism it follows, that **if two lines are both parallel to a third line, then the two lines are parallel to each other.** (If the two lines are separated two distances, a and b, to the third line, then the two lines are separated either a+b or a-b to each other.) If these two parallel lines also have a common point, then, as a corollary, they have at least two common points and are coincident. Coincident lines are equal to one line, and this line is unique. After starting with two lines we end up with one unique line. This is what we wanted to prove. Euclid and others could not draw this conclusion, since coincident lines were defined as not parallel. We conclude, that **for every point and for every line there exists exactly one unique line parallel to the given line and passing the given point.** This is the fifth postulate. Therefore: We now longer have to fill our knowledge gap by such a magic invention as the bending of nothing. We can also conclude that the restriction 'the given point should not lie on the given line' is not necessary with this new definition.

Repetition

The statement 'parallel lines never meet' should be changed to '**separated** lines never meet and **coincident** lines never separate'. The common definition ignores coincident lines thereby defining them as **not** parallel. Two parallel lines through a common point are coincident, and therefore, according to the common definition, not parallel. However, according to our **new** definition, we have seen that **a line and a point define a unique line** together. In other words: the fifth postulate.

Conclusions

An incomplete definition of parallelism has had a scandalous effect for the fifth postulate. A better definition of parallelism can be based on the perpendicular distance from a point to a line. This definition can help us to understand a problem that is 2300 years old and to find a proof for this unhappy postulate.

Post Scriptum (August 2007)

This article rendered only one feedback from the conference. Perhaps it was not well formulated. It is important to point out that the article is about defining a concept, and not about proving a postulate. This is declared in the title.

An interesting book about this subject is [2]. The book starts with a declaration about the importance of checking one's premises. On page 19 parallelism is defined by means of intersections, and it is stated that this is the most important definition in the book. Equidistance between lines is considered useless, obviously due to the fact that it is dependent on parallelism. But this is not stated in the book.

However, the real essence of parallelism is equidistance, and the number of intersections is only one of its properties. Checking premises reveals two other kinds of equidistance. Callahan treated equidistance between points. (See [1], ref. 3) His proof was complex, and its validity was difficult to judge. This article suggests a third kind of equidistance, namely between points and a line. This idea appears to render an easier way to a proof.

Definition: The distance between a point and a line is the distance along a perpendicular line (independent of if the point is on the line or not).
Definition: A line l is parallel to a line m if two points on l (laying on the same side of m) are on equal distances (≥ 0) to m.

Although not done here it is assumed that a probable demonstration (based on the definitions above) could be something like the following.
Show that if l is parallel to m, then m is parallel to l.
Show that if l and m are both parallel to s, then l and m are parallel to each other.
Show that if l and m are parallel to each other (according to s) and have a common point, P, then they have another common point, Q, and are coincident and one unique line (defined by s and P).

Reference

[1] John-Erik Persson, 'The Hard-to-Define Concept of Parallelism', *Galilean Electrodynamics* **12** (2), 39-40 (2001)

[2] Marvin Greenberg, **Euclidean and non-Euclidean geometry** 3 rd ed, 1994