Tarea
Teoría del Riesgo.
Procesos estocásticos para la Teoría del Riesgo.
Fecha de entrega el Martes 24-02-2009

1.- Sea X_t el resultado del t-ésimo lanzamiento de un dado. Define el espacio de estados E y el espacio del tiempo T.

2.- Sea Y_t la suma de los **t** resultados obtenidos en los **t** lanzamientos de un dado. Define el espacio de estados E y el espacio del tiempo T.

3.- Un alumno de Actuaría realiza apuestas de $10, con una probabilidad **p** de ganar, una probabilidad **q** de perder y una probabilidad **r** de quedarse igual. Inicia el juego con un capital de $100. Se retira del juego si pierde todo su capital inicial o si lo duplica. Si X_t es el capital del alumno de Actuaría después de la t-ésima apuesta.
Definición: Se dice que un estado es absorbente cuando el proceso no sale ya de ese estado.
a) Define el espacio de estados E y el espacio del tiempo T .
b) ¿ Qué estados son absorbentes?

__ Hacer el problema 8.1, 8.2 , 8.6, 8.7 y 8.8, de E. Çinlar, *Introduction to Stochastic Processes*.

8. Exercises

(8.1) Let $N = \{N_t; t \geq 0\}$ be a Poisson process with rate $\lambda = 15$. Compute
(a) $P\{N_6 = 9\}$,
(b) $P\{N_6 = 9, N_{20} = 13, N_{56} = 27\}$,
(c) $P\{N_{20} = 13 \mid N_6 - 9\}$,
(d) $P\{N_6 = 9 \mid N_{20} = 13\}$.

(8.2) Let N be a Poisson process with rate $\lambda = 2$. Compute

(a) $E[N_t]$, $\mathrm{Var}(N_t)$,

(b) $E[N_{t+s} \mid N_t]$.

(8.3) Arrivals of passengers at a bus stop form a Poisson process N with rate $\lambda = \frac{1}{3}$ per minute. Assume that a bus has left at time $t = 0$ leaving no customers behind. Let T denote the time of arrival for the next bus; then the number of passengers present when it arrives will be N_T. We suppose that T is independent of N and has the distribution φ.

(a) Compute $E[N_T \mid T]$ and $E[N_T^2 \mid T]$.

(b) Compute $E[N_T]$ and $\mathrm{Var}(N_T)$ for

$$d\varphi(t) = \begin{cases} \frac{1}{2}\,dt & \text{if } 9 \leq t \leq 11, \\ 0 & \text{otherwise.} \end{cases}$$

(8.4) A store promises to give a small gift to every thirteenth customer to arrive. If the arrivals of customers form a Poisson process with rate λ,

(a) find the probability density function of the times between the lucky arrivals;

(b) find $P\{M_t = k\}$ for the number of gifts M_t given in the interval $[0, t]$.

(8.5) Arrivals of customers into a store form a Poisson process N with rate $\lambda = 20$ per hour. Find the expected number of sales made during an eight-hour business day if the probability that a customer buys something is 0.30.

(8.6) Consider the road network pictured in Figure 4.8.1. The inputs are Poisson processes with the rates indicated, and the probabilities of a vehicle choosing the indicated directions are written on the arrows. Describe the traffic flow on each branch of the network.

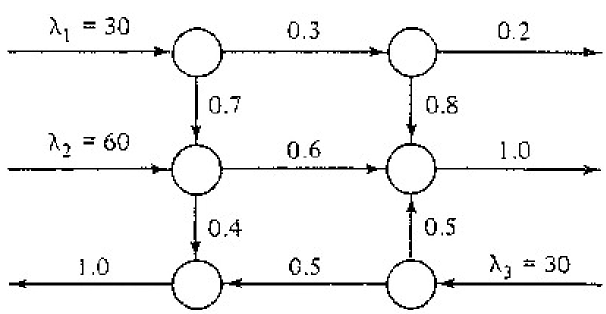

Figure 4.8.1 Network of Exercise (8.6).

(8.7) A department store has 3 doors. Arrivals at each door form Poisson processes with rates $\lambda_1 = 110$, $\lambda_2 = 90$, $\lambda_3 = 160$ customers per hour. 30% of all customers are male. The probability that a male customer buys something is 0.80, and the probability of a female customer buying something is 0.10. An average purchase is worth \$4.50.

(a) What is the average worth of total sales made in a 10 hour day?

(b) What is the probability that the third female customer to purchase anything arrives during the first 15 minutes? What is the expected time of her arrival?

(8.8) Considering the traffic on the road pictured in Figure 4.8.2, the following is known. The number of vehicles passing the point A in an hour follows the Poisson

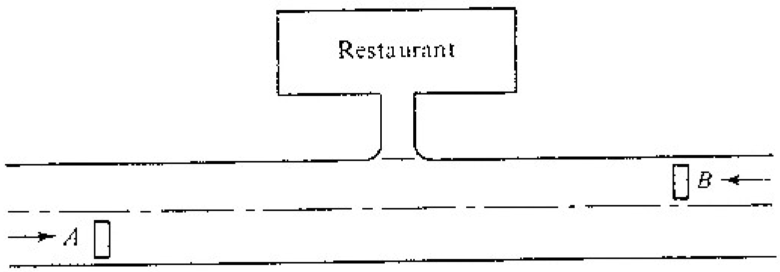

Figure 4.8.2 Picture for Exercise (8.8).

distribution with mean 60; 20% of these vehicles are trucks. The number of vehicles passing B in an hour is also Poisson distributed with mean 80; 30% of these are trucks. In general, 10% of all vehicles stop at the restaurant. The number of persons in a truck is one; the number of passengers in a car is equal to 1, 2, 3, 4, or 5 with respective probabilities 0.30, 0.30, 0.20, 0.10, and 0.10.

(a) Find the expected value $E[Z]$ of the number of persons Z arriving at the restaurant within that one hour.

(b) Compute $E[\alpha^Z]$ for $\alpha \in [0, 1]$.

(8.9) A device is subject to shocks which occur according to a Poisson process N with rate λ. The device can fail only due to a shock, and the probability that a given shock causes failure is p independent of the number and times of previous shocks. Let K be the total number of shocks the device takes before failure, and let $T = T_K$ be the time of failure.

(a) Compute $E[T]$, $\text{Var}(T)$.

(b) Compute $E[T|K]$.

(c) Compute $E[T|K > 9]$.

(8.10) *Mixtures of Poisson Processes.* Let X be a random variable taking values in $E = \{a, b, \ldots\}$ with distribution $\pi = (\pi(a), \pi(b), \ldots)$. For each $i \in E$, let $N(i)$ be a Poisson process with rate $\lambda(i)$. Suppose the processes $N(a), N(b), \ldots$ are independent of each other and of X. Define, for each $t \in [0, \infty)$,

$$N_t = N_t(X).$$

The process $N = \{N_t; t \geq 0\}$ is then called the mixture of $N(a), N(b), \ldots$ according to the distribution π. (This is a misnomer, since N is in fact one of the $N(a), N(b), \ldots$ picked at random; the computation below explains the name better).

(a) Show that for any $t \geq 0$,

$$P\{N_t = k\} = \sum_{i \in E} \pi(i) \frac{e^{-\lambda(i)t}(\lambda(i)t)^k}{k!}, \quad k = 0, 1, \ldots.$$

(b) Show that in general, N does not have independent increments.