Report on Analysis of Signal from INSAT 3-A to Determine its Velocity Along the Line of Sight

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1 Introduction

This project is concerned with the analysis of time series. A time series can be represented in time domain or in frequency domain. But these representations, in general, do not completely describe the signal. The frequency domain representation (spectrum) indicates the components present in the time series, but does not say anything about the time when these components were present. If the spectrum of a time series is stable with time, then the spectrum would have described it completely. But in general, the spectrum of signals vary over time. For example, as we speak, the pitch (frequency) of the voice changes over time. These frequencies will be present with different modulations over various stretches of time. A spectrum of such a speech signal over a complete stretch will not contain any information regarding these modulations. But, if spectrum over short intervals (where it is considerably stable) is considered, then the amplitudes associated with these intervals, will vary over the stretch. This will indicate the modulation present in the frequency compnents. This concept is used in Section 3.2 to identify the modulation present in certain tones.

Another aspect of a frequency spectrum is the phase. The phases of a frequency spectrum are those of the original signal with the phase associated with the respective frequency removed. Thus if there is a small shift in the frequency, it can be identified from the time derivative of the phase. This idea is used in Section 3.5 to determine the velocity of the satellite along the line of sight. If a tone is sent to a satellite and received back, the freqency of the received tone would be shifted due to *Doppler effect*. This shift in frequency results in the phase varying with time. The data given to us was derived from such tones transmitted to the satellite INSAT 3-A from Master Control Facility, Hasan and received at Raman Research Institute, Bangalore. A Rubidium oscillator of $10ps$ accuracy was used because of its high stability.

The purpose of our study was to use the power spectra and other relevant data to determine the velocity of the geosynchronous satellite INSAT 3-A along the line of sight. In the following sections, we explain the principles used in our analysis. To illustrate them, we first consider as an example, a particular time series of certain deviations. We explain how inferences about the components present in a time series can be made by looking at its power spectrum.

2 An Example

The time series considered here was that of a certain deviation recorded over a stretch of 55 days at two minute intervals. A local Rubidium oscillator was used to generate a 1 pps pulse. This oscillator has an accuracy of 10 picosecond and is highly stable. A similar pulse was received from GPS satellites, which the local oscillator locked on to. The deviation of the local oscillator pulse from the GPS pulse made up the time series which was analysed.

2.1 Patterns in the time series

2.1.1 Plot of the means

The data was divided into 436 overlapping one-day intervals. The mean over each such interval was calculated and was associated with the central time of each interval. This gave us the mean as a function of time. (Fig.1)

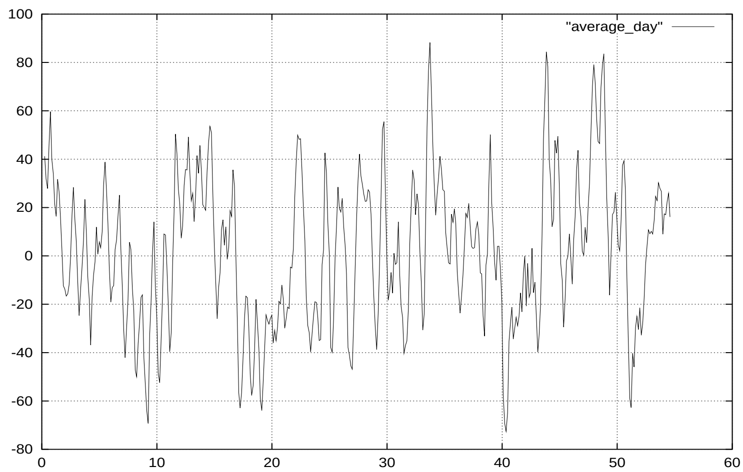

Figure 1: A plot of the mean(ps), taken over 1 day, against time($days$)

No particular pattern was suggested by this mean plot. This prompted us to look at the power spectrum of this new time series (mean vs. time).

2.1.2 The power spectrum

The power spectrum of mean plot is shown in Fig.2. This showed distinct peaks corresponding to periods of 1 day, 2.3 days, 5.5 days and 11 days(from right to left).

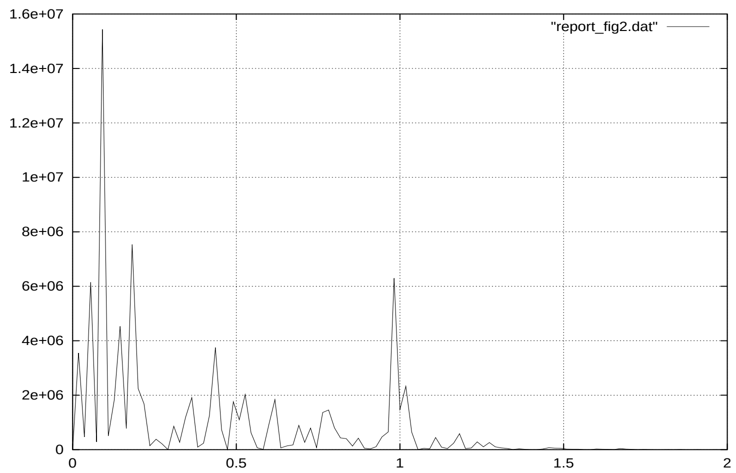

Figure 2: Power spectrum of the mean-plot vs frequency($perday$)

Well defined peaks in a power spectrum indicate that some trends are present in the time series, which are not stray ocurrences. The 11-day peak could not be relied upon much because we had only 5 periods in the data. The 5.5-day peak was a harmonic of it. The 1-day and 2.3-day plots seemed worth examining.

First, to look at the 1-day peak, the power spectra of two-day stretches were taken over the stretch of 55 days and added. This is equivalent to ensemble averaging if it is assumed that each of the two-day stretches is an independent realization. This would be valid if the signal in each stretch is in phase with that in the other. The resultant plot is shown in red in the colour plot.

The high power value at 0 is due to the d.c component of the time series. Distinct peaks were observed at $46.3\mu Hz$ and $115.6\mu Hz$. These correspond to periods of $6hours$ and $2.4hours$ respectively. The same was done for 2-day, 3-day, 5-day and 11-day stretches. The resulting spectra are shown in the colour plot. Each of these showed peaks at the same frequencies mentioned above.

The fact that a peak present in the shorter stretches is also present in the spectra of the longer ones validates the assumption made during the addition(ensemble averaging) mentioned above.

2.2 Inference

The points presented so far suggests that something happened to the signal at periods of $6 hours$ and $2.4 hours$. With the given data this is all we could infer. Additional information on the process of obtaining the data would have helped to associate these observations with physical phenomena.

The idea here was to illustrate how a power spectrum can be used to analyse signals present in the time series. In general, presence of a frequency profile indicates the presence of a signal.

3 Data from INSAT 3-A

The objective of the analysis of the data from INSAT 3-A was to determine the velocity of the geosynchronous satellite along the line of sight. The data size was $117MB$, consisting of a time series sampled at $10MHz$ over a stretch of $12.268392s$. Each byte corresponded to a sample point.

3.1 Frequency profile of the data

To begin with, the power spectrum of the time series,with its mean subtracted, was observed. The power spectrum was a 16384 point DFT. This was done using the FFTW-3 algorithm. As in Section 2.1.2, ensemble averaging was done over the whole data. The resulting power spectrum is shown in Fig.3. Two prominent peaks at $4.218750MHz$ and $4.4140625MHz$ are

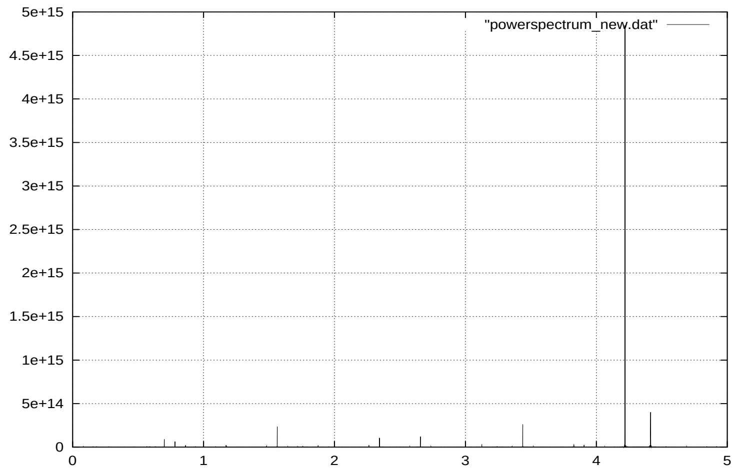

Figure 3: Power spectrum of the time-series vs frequency (MHz)

seen in the figure. This indicates that the signal has two prominent tones. Larger point FFTs also resulted in same frequency profiles, supporting the fact that there are no other tones.

To check for modulations in the signal, the power of each of the two tones was examined over the whole data. This resulted in new time series corroesponding to the power variation of each tone over the $12.268392s$ stretch.

3.2 The new time series

The power of the tones at $4.218750MHz$ and $4.4140625MHz$ were extracted for every 1024 points of the data. This new time series consisted of the power of the tone versus the central time of each stretch. The new time series of both the tones is shown in Fig.4 and Fig.5 respectively.

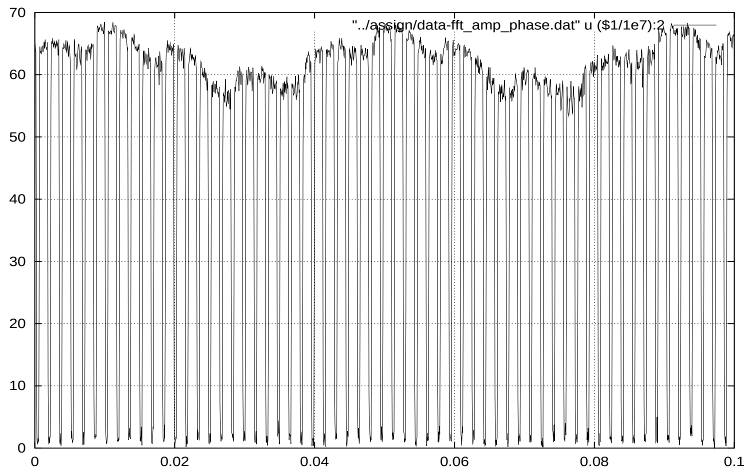

Figure 4: Plot of time series of $4.218750MHz$ tone over a stretch of $100ms$ (vs time in $seconds$)

The new time series had a pattern that repeated every $1s$. For the $4.218750MHz$ tone, this pattern consisted of a series of pulses of period $1.6384ms$ which lasted for around $0.82s$. For the next $0.1768s$, it remained high as shown in Fig.6. Then for $3.2ms$, the signal remained off. The $4.4140625MHz$ tone showed the complimentary pattern, being high whenever the $4.218750MHz$ was low and vice versa. But during the $3.2ms$ of 'off' time, this tone also remained off. The combined plot is shown in another coloured plot in the next page.

The exact periods were obtained from the power spectrum of the time series of each tone. The power spectra obtained is shown in Fig.7,8 and 9. These spectra indicated peaks at $610Hz(1.6384ms)$, $25Hz$ $(40ms)$ and $1Hz(1s)$. The $25Hz$ peak corresponded to the small sinusoid riding over the pulses as seen in Fig.4,5 and 6.

This revealed the modulations (in the shape of a pulse) present on the $4.218750MHz$ and $4.4140625MHz$ tones.

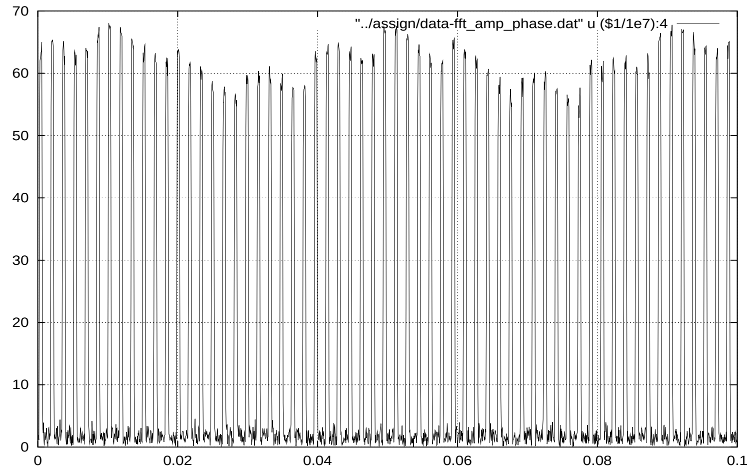

Figure 5: Plot of time series of $4.4140625 MHz$ tone over a stretch of $100 ms$ vs time(s)

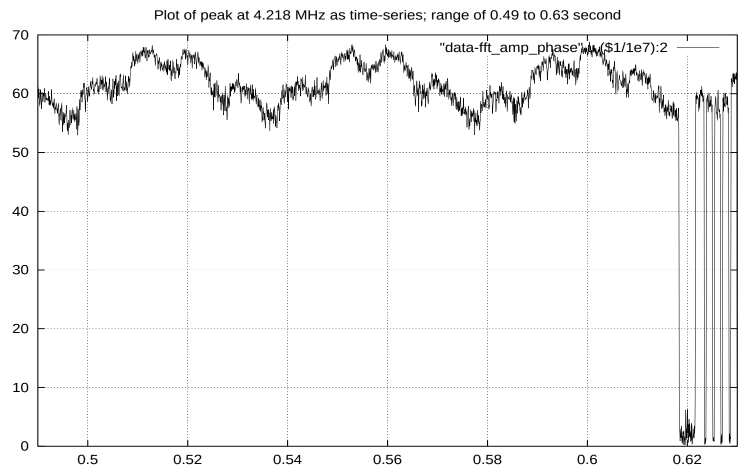

Figure 6: A section of time series of $4.218750 MHz$ tone over a stretch of $630 ms$ vs time(s),showing the 'off' period of $3.2 ms$

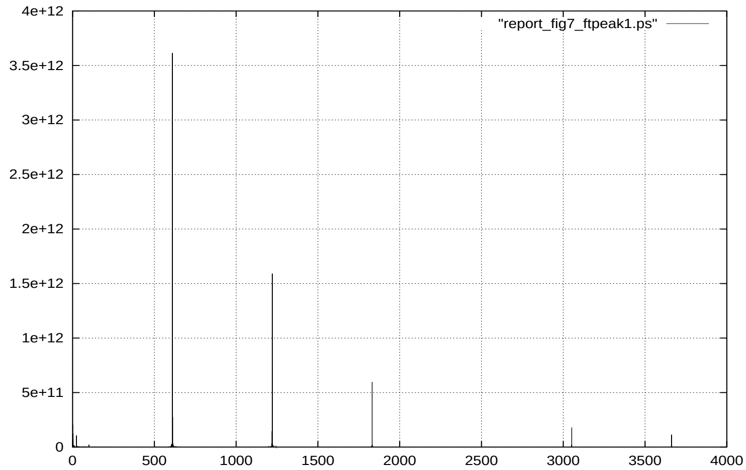

Figure 7: Power spectrum of time series of the $4.218750 MHz$ tone vs frequency (Hz)

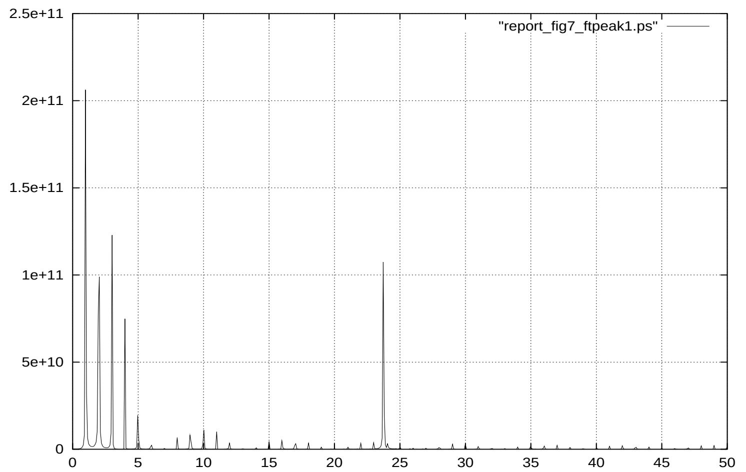

Figure 8: Power spectrum of time series of the $4.218750 MHz$ tone vs frequency (Hz) showing the $1Hz, 25Hz$ peaks

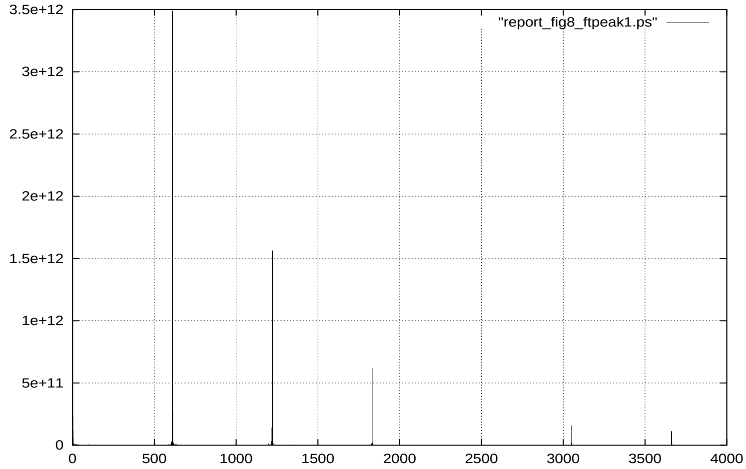

Figure 9: Power spectrum of time series of the $4.4140625 MHz$ tone *vs* frequency (Hz)

3.3 A note on the frequency of the tones

The time series under observation was sampled at $10MHz$. Thus only frequencies upto $5MHz$ could be identified(*Nyquist critereon*). So, nothing could be said about the actual frequency of the tones. The frequencies could have been $(10\mathbf{n} + \mathbf{f})MHz$ where $\mathbf{f}$ denotes the frequency of the tone and $\mathbf{n}$ is a non-negative integer. Since it was known that the frequency transmitted to INSAT 3-A was $24.218750MHz$ and 24.4140625, it was concluded that the frequency of the tones received were actually $24.218750MHz$ and $24.4140625MHz$. These values would have been observed if the sampling was done at more than $50MHz$.

All further references to these tones will correspond to $24.218750MHz$ (tone 1) and $24.4140625MHz$ (tone 2).

3.4 Signal to Noise ratio

Here, we discuss three methods by which the *signal to noise ratio* (SNR) is increased.

1. Coherent averaging
2. Power averaging (Incoherent)
3. From the power spectrum (Incoherent)

3.4.1 Coherent averaging

Since the period of the signal was known, each period $(1.6384ms)$ was folded over the other. One period consists of 32 sample points spaced at $51.2\mu s$.

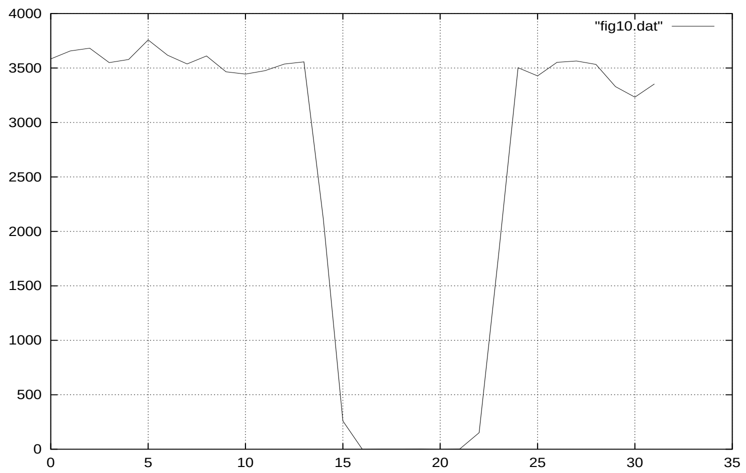

Figure 10: A single pulse

This folding was done by adding each period *in phase*. This ensured that the signal, because of its 1.6384ms periodicity, would highlight itself. This resulted in an SNR of 124dB.

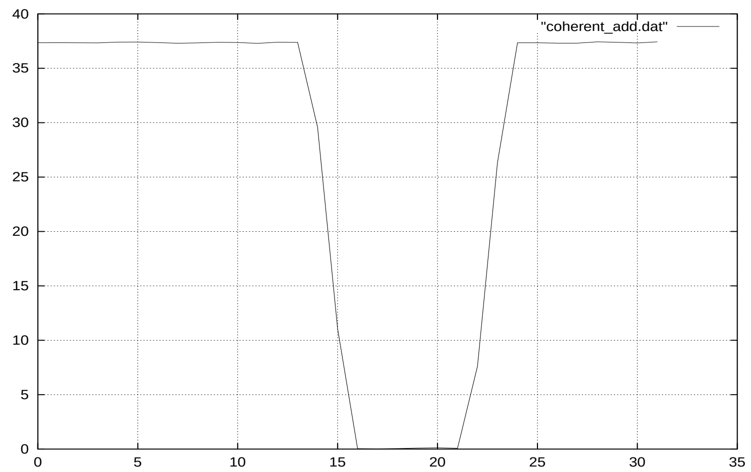

Figure 11: After coherent addition

3.4.2 Power averaging(Incoherent averaging)

It differs from the method described in Section 3.4.1 in the fact that the folding is not done *in phase*, but only the powers are folded.This method gave an SNR of 46dB.

The fact that we are adding powers (incoherent) suggests that noise never gets cancelled. But the point to be noted here is that if the folding is done correctly, the signal in each period exactly folds over the other. Whereas the noise, being random in each period, will not fold exactly on itself. Thus if the folding increases the signal by a factor of **n**, the noise is increased by a factor less than **n**. This results in an increase in the SNR.

In the method described in Section 3.4.1, noise could get cancelled out to a considerable extent because of coherent addition. Thus the SNR obtained by coherent averaging was higher than that obtained by this method.

3.4.3 From the power spectrum(Incoherent averaging)

Here we added up the powers corresponding to $610Hz(1.6384ms)$,$1Hz(1s)$ and their harmonics to get the signal power. The rest of the power in the spectrum was considered as noise. This gave an SNR of $32dB$. This also is incoherent averaging because here again the phase is never considered. Just the power corresponding to concerned frequencies are added up.(In general the power in required frequency band is integrated to get the signal power and the rest is regarded as noise.)

3.5 Determinig the velocity

The frequencies at which signals are transmitted to a geosynchronous satellite are not the same as that received by it. It is known that any relative motion between the source and the receiver, along the line joining them, introduces a *Doppler Shift* given by

$$\frac{\Delta\nu}{\nu} = \frac{v}{c}$$

where $\Delta\nu$ is the *Doppler shift*, ν is frequency of transmission, v is velocity of satellite along line of sight and c is the speed of light. In the case of geosynchronous satellites, due to the effects of the Moon and the Sun, there is always a line of sight component of velocity. This relative motion between the satellite and the source results in a shift in the frequencies of all the components of the signal. The objective of the study was to find this velocity. From the above equation, it is clear that if $\Delta\nu$ is measured then the required velocity is known.

Under limiting cases if we consider the signal to be a summation of exponentials then the extraction of a particular tone ν_o is done by multiplying the signal by $e^{-j2\pi\nu_o t}$. That is if the signal is

$$f(t) = \sum_{i=0}^{M-1} a_i \mathrm{e}^{\mathrm{j}2\pi\nu_\mathrm{i}\mathrm{t}}$$

(where M is number of tones)

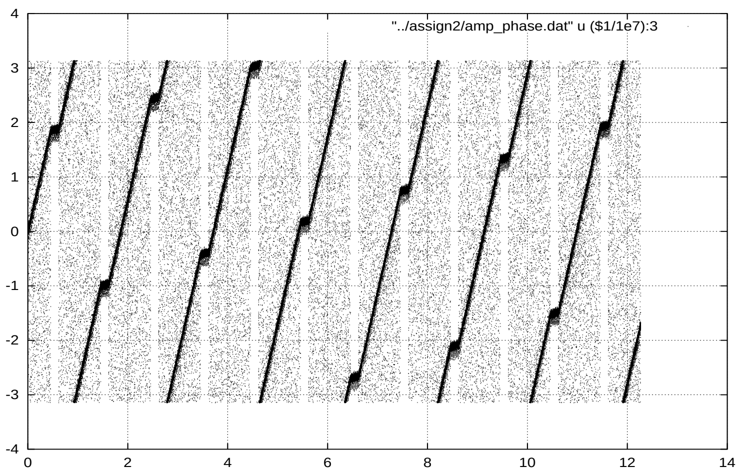

Figure 12: Plot of the phase(rad) of tone 1 vs. time (in s)

the extracted component corresponding to ν_o will be

$$F(\nu_o) = \sum_{i=0}^{N-1} f(t_i)\mathrm{e}^{-\mathrm{j}2\pi\nu_o t_i}$$

(where N is number of sample points)

The transmitted frequency received by the satellite will be given by

$$\nu = \nu_o(1 + v/c)$$

(where v is the velocity of the satellite towards the source) The fact used here to find $\Delta\nu$, which is nothing but ν_oxv/c, is that the phase of the fourier transform will contain the term ν_ox(v/c)xt as the $\nu_o t$ term has already been removed while taking the fourier transform. If this phase is plotted against time (fig.12), its slope will give the required $\Delta\nu$. The sign of the slope will indicate the direction of motion of the satellite. If the slope is positive then the satellite's motion is towards the source.

The analysis gave a slope of $3.32 rad/s$ which corresponds to $.528 Hz$. In our case, there is a *Doppler shift* twice (towards the satellite and back from the satellite). Thus it can be concluded that the line of sight component of velocity of the satellite at that time of observation $(1110 hrs, 16^{th} June, 2005)$ was

$$v = \frac{c\Delta\nu}{2\nu_o} = 3.273 ms^{-1}$$

4 Conclusion

We conclude with comments on two important points which were demonstrated in the above procedure. The first is regarding the SNR. In astronomical observations, little information can be obtained by directly observing the raw data. Even spectral analysis will fail to yield anything of interest. The reason being that the 'signal' is too weak. To increase the SNR to an acceptable level, it is necessary to carry out the Coherent Averaging as was mentioned in Section 3.4.1. To do this, it is essential to know the period over which folding should be done. Improper selection of period will not yield anything useful. In astronomical observations these periods are found out to a desired point of accuracy on a trial and error basis more than precise reasoning. In our analysis, since the data was from a satellite, the SNR was high and these complications did not arise.

The second point concerns the technique used to get the $DopplerShift$. Since the Fourier transform was not calulated over the entire time, the spectrum will have a finite frequency resolution. Thus if a frequency is present in a bin within the resolution limits it will be picked up, but the phase will vary with time owing to the small shift in the frequency. This was the concept used in Section 3.5 to get the $DopplerShift$.

It is of interest to note that the modulations in the signal were identified by looking just at the power spectrum, disregarding the phase completely. Whereas, to determine the velocity of the satellite, only the phase information was used, irrespective of the power.

As a result of our analysis, the velocity of the satellite INSAT 3-A along the line of sight was found to be $3.273ms^{-1}$ at $1110hrs$ on $16^{th}June, 2005$