Effect of Variation of Concrete Properties on the Accuracy of Estimated Deflections of Reinforced Concrete Slabs

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ABSTRACT: The simplified procedures contained in CSA A23.3-M94 for calculating immediate and long-term deflections are inadequate in some situations, especially for deflection sensitive elements such as floor slabs. The calculated deflection is sometimes significantly less than the actual deflection causing serviceability problems. The final deflection of a slab is affected by the uncertainty of the concrete properties, the construction procedure, the degree of curing, and several other parameters that make it difficult for designers to predict deflections with confidence. This paper emphasizes the importance of considering clear margins for the error anticipated in the calculated deflections of reinforced concrete slabs. The theory of errors is applied to the mean curvature method of calculating deflections. A mathematical model is developed to be used for studying the effect of variation of concrete properties on the accuracy of the calculated deflections.

1. INTRODUCTION

Prediction of immediate and long-term deflections is important in design of concrete members for satisfactory performance during their use. Canadian Standard CSA A23.3-M94 (1994) and ACI 318-99 (1999) contain simplified procedures for predicting deflections of beams and slabs. These procedures have proven inadequate in some situations, especially in the case of reinforced concrete floor slabs. These members are usually thin relative to their spans and are therefore deflection sensitive. The calculated deflections are sometimes less than the actual deflections, and serviceability problems resulting from excessive deflections are not uncommon for slabs designed with the codes. The final deflection of a slab is affected by the uncertainty of the concrete properties such as modulus of elasticity, modulus of rupture, creep coefficient, in addition to shrinkage. It is also affected by the construction procedure including load history, curing method and period, temperature gradients in the first few weeks after casting, etc. All these parameters make it difficult for designers to predict deflections with confidence.

This paper emphasizes the importance of considering specific margins for the error anticipated in the calculated deflections of reinforced concrete slabs. These margins would enable designers to determine how much buffer they have to the deflection limits specified by the codes. Here, the theory of errors is applied to the mean curvature method of calculating deflections. A mathematical model is developed to be used for studying the effect of variation of concrete properties on the accuracy of the calculated deflections.

2. METHODS OF PREDICTING DEFLECTIONS OF REINFORCED CONCRETE SLABS

2.1 Simplified Methods

2.1.1 Immediate deflections

Canadian Standard CSA A23.3-M94 (1994) requires that immediate deflection of reinforced concrete members be calculated by elastic analysis using an effective moment of inertia I_e not greater than I_g, the moment of inertia of the gross concrete cross section about its centroidal axis (Branson 1977)

$$[1] \qquad I_e = I_{cr} + (I_g - I_{cr})\left(\frac{M_{cr}}{M_a}\right)^3 \leq I_g$$

where I_{cr} is the moment of inertia of a cracked section transformed to concrete, neglecting concrete in tension; M_{cr} (= $f_r I_g / y_t$) is the cracking moment; f_r is the modulus of rupture of concrete; y_t is the distance from the centroidal axis of gross section to the extreme fibre in tension; and M_a is the maximum moment in a member at the load stage deflection is calculated.

2.1.2 Long-term deflections

For the additional long-term deflection resulting from creep and shrinkage of flexural members, CSA A23.3-M94 (1994) requires multiplying the immediate deflection due to sustained load by the factor

$$[2] \qquad \frac{S}{1+50\rho'}$$

where S is a time-dependent factor equal to 2.0, 1.4, 1.2, 1.0, respectively, for 5 years or more, 12 months, 6 months, and 3 months; ρ' (= A_s' / bd) is the ratio of compression reinforcement at mid-span for simple span; A_s' is the area of compression reinforcement; b is the width of compression face of member; and d is the distance from extreme compression fibre to the centroid of tension reinforcement.

2.1.3 Deficiencies of simplified methods

Several recent studies have indicated the deficiencies of Eq. 1 (Ghali and Azarnejad 1999, Gilbert 1999, and Sherif and Dilger 1998). For most slabs, M_a may not be much greater than M_{cr}. In such cases, the value of I_e is close to, or perhaps equal to, I_g when in fact the section has already cracked. In addition, the loss of stiffness that inevitably occurs with time due to cracking is not accounted for in the deflection calculations if the value of I_e from Eq. 1 is used.

As mentioned earlier, time-dependent deformations of concrete members depend on many factors including the properties of concrete, geometry of the member, and ambient conditions such as temperature and relative humidity. Ghali and Azarnejad (1999) showed that it is impossible to account for all of these factors using a simple multiplier (Eq. 2) that depends only on time and compression reinforcement. In addition, Gilbert (1999) suggested that this multiplier is poorly calibrated for load periods of one year or less.

2.2 Mean Curvature Method

The mean curvature method is a more accurate and more general method for predicting deflections of concrete members. It has been adopted by the CEB–FIP Model Code 90 (MC–90) (1993). In this method, the deflection of a member can be determined from the values of the curvature ψ at a number of sections (Ghali and Favre 1994). For example, when three sections are used and a linear variation of ψ between these sections is assumed, the mid-span deflection, Δ_{Mid}, of a one-way slab can be given by the geometrical relationship

$$[3] \qquad \Delta_{Mid} = \frac{L^2}{96}\left(\Psi_{Left} + 10\Psi_{Mid} + \Psi_{Right}\right)$$

where

$$[4] \qquad \Psi_i = \frac{M_i}{E_c I_{ei}}$$

L is the span length; M is the bending moment; and E_c is the modulus of elasticity of concrete.

Equation 3, which can be derived by double integration, is exact when the variation of ψ is parabolic. In other practical cases where ψ–variation is not parabolic, the deflection calculated by Eq. 3 is still sufficiently accurate. Increasing the number of sections will increase the accuracy. The mean curvature method accounts for the variation in cross-sectional properties (i.e., the loss of stiffness) that inevitably occurs with time in concrete members due to cracking.

3. SOURCES OF ERROR IN PREDICTING DEFLECTIONS

Jokinen and Scanlon (1985) analyzed several field measurements of identical slabs in a multi-story building. They reported a coefficient of variation between calculated and measured deflection in the range of 30% for both short and long-term deflections. This large variation was attributed to the uncertainty of many variables included in calculating deflection.

Based on Eqs. 3 and 4, the factors that affect the calculation of deflection are loading, flexural rigidity, and time-dependent parameters. The second and the third factors affect deflection calculation directly by affecting the stiffness of the structural element, and indirectly by defining the way moment distribution and redistribution takes place in the structural system. Gardner (1990) showed that neglecting construction loads could result in excessive deflection of concrete slabs. Fling (1992) showed that while part of these parameters is known before construction, a considerable part, including concrete properties, may be uncertain until construction takes place. Not only are concrete properties not known before construction, but also those properties incorporated in deflection calculation (i.e., modulus of elasticity and modulus of rupture) have wide scatter in their values.

Application of probabilistic concepts to deflection serviceability has been rarely addressed (Frangopol 1988 and Holicky 1988). Scanlon and Pinheiro (1992) compared the current deterministic approach to deflection control with design for safety which is based on probability considerations. They suggested that the best practical probability limit can be generated using a measure of the associated damage to serviceability due to excessive deflections.

4. UNCERTAINTIES IN THE PROPERTIES OF CONCRETE

4.1 Role of the Modulus of Elasticity

Design codes provide prediction models for the modulus of elasticity of concrete based on its compressive strength, f'_c. Equations 5 to 11 give the models to predict the modulus of elasticity in MPa as per CSA A23.3-M94 (1994), the Ontario Highway Bridge Design Code (OHBDC) (1992), ACI 318-99 metric and U.S. customary units editions (1999), Standards New Zealand NZS 3101 (1995), Standards Australia AS 3600-1994 (1994), and CEB-FIP MC-90 (1993), respectively. While the Australian standards explicitly state that the predicted value may include a variation of ± 20%, no other standard indicates how much variation to be expected in the predicted modulus of elasticity.

[5] (CSA A23.3, 1994) $E_c = 4500\sqrt{f'_c}$

[6] (OHBDC, 1992) $E_c = 5000\sqrt{f'_c}$

[7] (ACI 318 metric, 1999) $E_c = 4700\sqrt{f'_c}$

[8] (ACI 318 U.S., 1999) $E_c = 4730\sqrt{f'_c}$

[9] (NZS 3101, 1995) $E_c = 4700\sqrt{f'_c}$

[10] (AS-3600, 1994) $E_c = \gamma^{1.5}(0.043\sqrt{f'_c})$

[11] (CEB-FIP, 1993) $E_c = 21500\ \kappa \left(\frac{f'_c}{10}\right)^{1/3}$

where γ in Eq. 10 is the concrete density (in kg/m^3), and κ in Eq. 11 is a parameter that equals 0.85 to account for the initial plasticity of concrete when elastic analysis of the structure is to be performed. Figure 1 shows the variation in the modulus of elasticity of concrete with respect to its compressive strength for the different design codes. For concrete strengths from 20 MPa to 80 MPa, there exists a difference in the predicted modulus of elasticity in the range of 5%, 11%, and 15% between CSA A23.3-M94 (1994) and ACI 318-99 (1999), Australian standards AS 3600-1994 (1994), and CEB–FIP MC-90 (1993) respectively. It is worth mentioning that the difference between the two Canadian standards, CSA A23.3-M94 (1994) and the OHBDC (1992) is 9%.

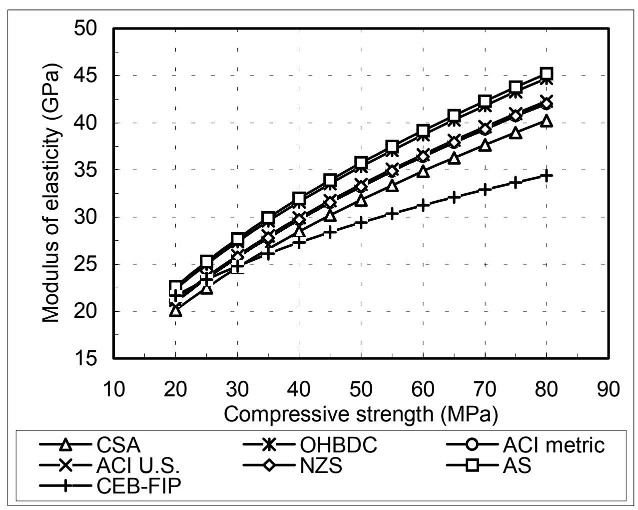

Figure 1. Comparison between the modulus of elasticity models in the different design codes.

The introduction of new types of concrete such as high-performance concrete (HPC) has increased the uncertainty of the concrete properties. HPC has a higher elastic modulus and a more brittle post-peak behaviour compared to normal-strength concrete (NSC). In addition, HPC responds to elastic stresses in a different manner than NSC. The weak aggregate / cement paste transition zone in NSC allows the cement paste to dominate the response to stresses within the elastic range. In HPC, the transition zone is much stronger than in NSC, and failure usually occurs within the aggregate particles. Therefore, more stress transfer to the aggregate takes place within the elastic range, and the aggregate shares a relatively large portion of the elastic response (Aïtcin 1998). Experimental work by Baalabaki *et al.* (1991) proved the possibility of producing two HPC mixes with the same compressive strength and very different moduli of elasticity. Consequently, when HPC is used, the above models are totally violated, and an assumption of an error of even ± 20% will probably be on the unconservative side. Further research on HPC and its modulus of elasticity is needed to provide accurate prediction models.

4.2 Role of the Modulus of Rupture

The modulus of rupture is a measure of the tensile strength of concrete under pure bending moment. Design codes provide different prediction models for the modulus of rupture. Most of these models relate the modulus of rupture to the square root of the concrete compressive strength in order to reflect the disproportional increase of the modulus of rupture with respect to the increase in the compressive strength. Equations 12 to 17 give the models to predict the modulus of rupture of concrete in MPa as per CSA A23.3-M94 (1994), OHBDC (1992), ACI 318-99 metric and U.S. customary units editions (1999), NZS 3101 (1995), and AS 3600-1994 (1994), respectively. Figure 2 shows the variation in the modulus of rupture of concrete with respect to its compressive strength for the different design codes.

[12] (CSA A23.3, 1994) $f_r = 0.6\sqrt{f'_c}$

[13] (OHBDC, 1992) $f_r = 0.5\sqrt{f'_c}$

[14] (ACI 318-metric, 1999) $f_r = 0.7\sqrt{f'_c}$

[15] (ACI 318-U.S. 1999) $f_r = 0.62\sqrt{f'_c}$

[16] (NZS 1301, 1995) $f_r = 0.6\sqrt{f'_c}$

[17] (AS-3600, 1994) $f_r = 0.6\sqrt{f'_c}$

Some design codes require reducing the modulus of rupture from the previously mentioned values when calculating deflections to account for concrete cracking. For example, CSA A23.3-M94 (1994) requires the reduction of the value of the modulus of rupture to half of the value given by Eq. 12 when deflection of two-way slabs is calculated. This reduction is considered to account for the effect of cracking due to restrained shrinkage of two-way slabs based on the research work by Thompson and Scanlon (1988).

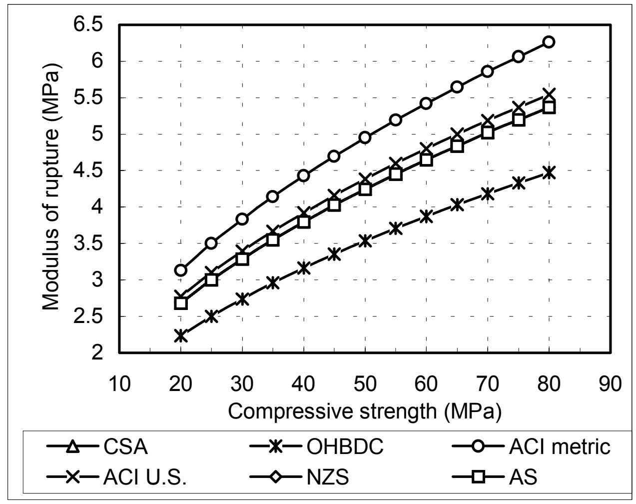

Figure 2. Comparison between the modulus of rupture models in the different design codes.

CEB-FIP MC-90 (1993) is the only code to abandon the modulus of rupture as a criterion for flexural cracking of concrete. It uses the mean tensile strength to represent the limit of tensile strength of concrete. It also requires incorporating the effect of the size of the concrete member on its modulus of rupture.

Researchers have shown a wide scatter in the value of the modulus of rupture. It was suggested that most of the models developed to predict the modulus of rupture of concrete based on its compressive strength are inaccurate. Carrasquillo *et al.* (1981) proposed a more accurate prediction model presented in Eq. 18.

[18] $f_r = 0.94\sqrt{f'_c}$ (MPa)

Raphael (1983) was the first to propose abandoning the square root models for predicting the modulus of rupture of concrete based on its compressive strength. He proposed the following expression

[19] $f_r = 0.7\left(f'_c\right)^{2/3}$ (kg/cm^2)

Recent attempts to predict the concrete splitting tensile strength and its modulus of rupture using the compressive strength have been reported by Légeron and Paultre (2000), and Oluokun (1991). They revealed similar trends presented in Eqs. 20 and 21, respectively. It is worth mentioning that the CEB-FIP model for predicting the tensile strength of concrete also uses the power of 2/3 as shown in Eq. 22.

$$[20] \quad f_r = 0.5\left(f_c^{'}\right)^{2/3} \quad \text{(MPa)}$$

$$[21] \quad f_t = 0.214\left(f_c^{'}\right)^{0.69} \quad \text{(MPa)}$$

$$[22] \quad f_t = 1.4\left(\frac{f_c^{'}}{10}\right)^{2/3} \quad \text{(MPa)}$$

The insistence on using the square root of the concrete compressive strength to predict its tensile strength generally and its modulus of rupture specifically hinders the attempts to develop more accurate prediction models. Figure 3 compares the two models for predicting the modulus of rupture of concrete using Eqs. 18 and 20. Although the two models have much lower coefficients of variation than those reported for models using the square root of the compressive strength, the variation between the predicted modulus of rupture from both models is still in the range of $\pm$ 15%.

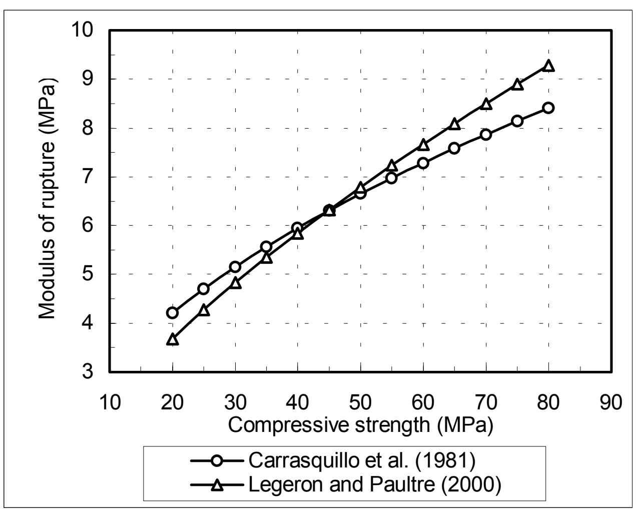

Figure 3. Comparison between the modulus of rupture predicted by Eq. 18 and that predicted by Eq. 20.

Research work also proved the existence of significant differences between the modulus of rupture of concrete specimens cured under standard testing conditions, and the modulus of rupture of specimens cured under site conditions. The differences varied between 18% to 30% for NSC (Carrasquillo *et al.* 1981) and 35% to 100% for HPC (Légeron and Paultre 2000). The later proposed a mathematical model relating the real modulus of rupture, $f_{r\text{-real}}$, to the modulus of rupture determined under standard condition, f_r, as shown in Eq. 23.

$$[23] \quad f_{r-\text{real}} = \mu f_r$$

where

$$[24] \quad \mu = \frac{1}{1.09 + 0.0034 f_c^{'}}$$

The best curing regime for HPC is 7 days of wet curing followed by dry curing. Due to tight construction schedules, these requirements are hardly met in most cases, despite the widely known adverse effects on the performance of HPC when inadequate curing is practiced (Aïtcin 1998 and Whiting *et al.* 2000). HPC is more susceptible to shrinkage cracks than NSC. Moreover, shrinkage cracks of HPC are more harmful to the modulus of rupture than they are in NSC. Légeron and Paultre (2000) and Raphael (1983) showed how the moisture gradient due to drying induces tensile stresses at the concrete surface causing microcracks to develop, and therefore reducing the modulus of rupture of NSC when it is improperly cured. Although this behaviour applies to HPC, it only constitutes part of the picture. On the one hand, the tensile stresses at the surfaces of HPC members are expected to be higher than in NSC members due to the significant increase of the cementitious material content (e.g., Portland cement, silica fume, and/or fly ash). On the other hand, the increased autogenous shrinkage of HPC is expected to cause homogeneous tensile shrinkage strains in the whole member mass. Thus, several microcracks can develop in the whole HPC member if not properly cured rather than at the skin only as in the case of NSC. Application of fracture mechanics principles to concrete (Shah *et al.* 1995) can prove that the existence of such cracks in the tension zone of a concrete member would significantly reduce the modulus of rupture, but would not have a significant effect on the compressive strength. This explains why curing has a more significant effect on the modulus of rupture of HPC than on that of NSC, and why a wide scatter should be expected and accounted for when the modulus of rupture of HPC is being determined. It also explains why it is more difficult to link the modulus of rupture of HPC to its compressive strength, as both properties are not affected by the same factors in a similar manner. It is therefore recommended to measure the modulus of rupture of HPC when needed rather than using irrelevant models to predict it (Aïtcin 1998).

Based on this review, it can be concluded that the value of the modulus of rupture specified in any design code is a best guess rather than a fixed material property. Therefore, even with the best construction procedures, the estimated modulus of rupture would probably have an error of up to ± 30% from its real value.

4.3 Role of the Time-Dependent Parameters

Several methods in the form of charts and/or mathematical expressions are provided by different design codes to estimate the expected shrinkage strain, ε_{sh}, and creep coefficient, Φ_c. However, it is widely accepted that the predicted shrinkage and creep parameters would incorporate large variations. The effect of shrinkage is to develop internal tensile stress in concrete, f_{sh}, that would detract from its modulus of rupture as indicated in Eq. 25.

$$[25] \quad f_{cr} = f_r - f_{sh}$$

where f_{cr} is the cracking strength of concrete. The stress f_{sh} can be evaluated using the following expression suggested by Gilbert (1999)

$$[26] \quad f_{sh} = \left(\frac{3.5\,\rho\,(1+0.8\,\Phi_c)\,E_s}{E_c + 3\,\rho\,(1+0.8\,\Phi_c)\,E_s} \right) E_a\,\varepsilon_{sh}$$

where

$$[27] \quad E_a = \left(\frac{E_c}{1+0.8\,\Phi_c} \right)$$

The effect of creep as presented in Eq. 27 is to reduce the concrete stiffness with time. Thus, the age-adjusted modulus of elasticity, E_a, is a function of the creep coefficient Φ_c. It is therefore possible to consider the error in Φ_c and ε_{sh} to be incorporated in the errors assumed in the modulus of elasticity and the modulus of rupture, respectively.

If different areas of tension reinforcement and compression reinforcement are used in a slab, shrinkage would induce a curvature, Ψ_{sc}, of the slab that can be approximated using Eq. 28 (Gilbert 1999).

$$[28] \quad \Psi_{sc} = \Gamma\,\frac{\varepsilon_{sh}}{D}\left(1-\frac{A_s'}{A_s}\right)$$

The factor Γ is taken as 0.7 for uncracked and 1.2 for cracked sections, respectively. D is the overall depth of the member, and A_s is the area of tension reinforcement. The effect of creep is incorporated in reducing the age-adjusted modulus of elasticity used to calculate the shrinkage stress as shown in Eqs. 26 and 27, and in increasing the curvature induced by the sustained load as shown in Eq. 29 (Gilbert 1999).

$$[29] \quad \Psi(t) = \Psi_i \left(1+\frac{\Phi_c}{\alpha}\right)$$

where $\Psi(t)$ is the load-induced curvature at any time t due to a sustained service moment; Ψ_i is the initial curvature due to the sustained service moment; and α is a term that accounts for the effect of cracking and the braking action of the reinforcement, and is a function of the tension and compression reinforcement ratios. Gilbert (1999) provided simplified expressions for α for uncracked and cracked cross sections.

5. APPLICATION OF THE THEORY OF ERRORS TO DEFLECTION CALCULATION

It is assumed herein that the variation in concrete properties (i.e., modulus of elasticity and modulus of rupture) is the only source of error in deflection calculations. All other factors including ambient conditions, loading conditions and time-dependent parameters are assumed to have no errors. The large number of factors affecting the final deflection of a slab makes it prohibitive to consider all of them simultaneously. An extensive research program would be required to examine all of these factors separately, and a more sophisticated technique than the direct application of the theory of errors would be needed to examine their interaction.

To examine the effect of the variation of concrete properties on the accuracy of calculated deflections of slabs, a method of deflection prediction should be selected. The theory of errors (Kennedy and Neville 1986) would then be applied to the selected method to derive the expected error in the deflection function based on the errors incorporated in its parameters. The method selected here for predicting deflections is the mean curvature method (Eq. 4). Interpolation between the uncracked and cracked section stiffnesses is performed using Branson's approach (Eq. 1). Combining these two approaches would provide a rational method for predicting deflections, while keeping the method utilized by most design codes to estimate member stiffness. Time-dependent parameters are incorporated by using Eqs. 25 through 29.

Applying the theory of errors to Eq. 4, the standard deviation of the curvature, σ_Ψ, is evaluated as follows

[30] $$\sigma_{\Psi}^2 = \sigma_{I_e}^2 \left(\frac{\partial\Psi}{\partial I_e}\right)^2 + \sigma_{E_c}^2 \left(\frac{\partial\Psi}{\partial E_c}\right)^2 + \sigma_{M_s}^2 \left(\frac{\partial\Psi}{\partial M_s}\right)^2$$

Considering equations 1, 25, 26 and 27:

[31] $$\sigma_{I_e}^2 = \sigma_{E_c}^2 \left(\frac{\partial I_e}{\partial E_c}\right)^2 + \sigma_{f_r}^2 \left(\frac{\partial I_e}{\partial f_r}\right)^2$$

Therefore,

[32] $$\sigma_{\Psi}^2 = \sigma_{f_r}^2 \left(\frac{\partial\Psi}{\partial I_e}\right)^2 \left(\frac{\partial I_e}{\partial f_r}\right)^2 + \sigma_{E_c}^2 \left(\frac{\partial\Psi}{\partial E_c}\right)^2 + \sigma_{E_c}^2 \left(\frac{\partial\Psi}{\partial I_e}\right)^2 \left(\frac{\partial I_e}{\partial E_c}\right)^2$$

where

[33] $$\frac{\partial\Psi}{\partial I_e} = \frac{-M_s}{E_c\, I_e^{\,2}}$$, and

[34] $$\frac{\partial\Psi}{\partial E_c} = \frac{-M_s}{E_c^{\,2}\, I_e}$$

M_s is the service moment. Note that σ_M in Eq. 30 will vanish since loading conditions are assumed to have no errors as discussed earlier. The final deflection at the mid-span of a simply-supported, one-way slab is determined by double integration of the curvature along the span (Ghali and Favre 1994). The curvature used should be the summation of the curvatures induced by loads including the effects of shrinkage (Eq. 28) and creep (Eq. 29). Immediate and long-term deflections are evaluated based on the immediate and the long-term curvatures, respectively using Eq. 3.

[3] $$\Delta_{Mid} = \frac{L^2}{96}\left(\Psi_{Left} + 10\Psi_{Mid} + \Psi_{Right}\right)$$

For a simply-supported slab, Ψ_{Left} and Ψ_{Right} are induced by shrinkage alone. As Eq. 28 shows, shrinkage-induced curvature is not a function of the concrete properties. Therefore, the first and last derivatives of Eq. 3 will vanish, and the standard deviation of the deflection, σ_{Δ}, will be directly proportional to the standard deviation of the mid-span curvature, σ_{Ψ}. Thus,

[35] $$\sigma_{\Delta}^2 = \sigma_{\Psi_{Left}}^2 \left(\frac{\partial\Delta}{\partial\Psi_{Left}}\right)^2 + \sigma_{\Psi_{Mid}}^2 \left(\frac{\partial\Delta}{\partial\Psi_{Mid}}\right)^2 + \sigma_{\Psi_{Right}}^2 \left(\frac{\partial\Delta}{\partial\Psi_{Right}}\right)^2$$

where

[36] $$\frac{\partial\Delta}{\partial\Psi} = \frac{L^2}{9.6}$$

Therefore, the standard deviation of the deflection is

[37] $$\sigma_{\Delta} = \frac{L^2}{9.6}\left\{ \sigma_{f_r}^2 \left(\frac{\partial\Psi}{\partial I_e}\right)^2 \left(\frac{\partial I_e}{\partial f_r}\right)^2 + \sigma_{E_c}^2 \left(\frac{\partial\Psi}{\partial E_c}\right)^2 + \sigma_{E_c}^2 \left(\frac{\partial\Psi}{\partial I_e}\right)^2 \left(\frac{\partial I_e}{\partial E_c}\right)^2 \right\}^{1/2}$$

By examining this mathematical derivation, it can be seen that the factors affecting the standard deviation of the calculated deflection are the modulus of elasticity and the modulus of rupture. In addition, all the factors that affect the effective moment of inertia can also affect the standard deviation of the deflection (e.g., the magnitude of the creep coefficient and the magnitude of the shrinkage strain).

A Mathcad® program was developed using the mathematical model presented in Eq. 37 to predict the error in the calculated deflection of simply-supported, one-way slabs. Parametric studies are being carried out to examine the effect of all the above parameters on the standard deviation of the computed deflection.

6. SUMMARY

Prediction of immediate and long-term deflections is important in design of concrete members for satisfactory performance during their use. Design codes contain simplified procedures for predicting deflections that have proven to be inadequate in some situations. Because of the large number of uncertain parameters affecting the final deflection of a concrete member, it is difficult for designers to predict deflections with confidence. This paper examined the various sources of error associated with deflection calculation of one-way reinforced concrete slabs. The theory of errors was applied to the mean curvature method of calculating deflections. A mathematical model was developed to be used for studying the effect of variation of concrete properties on the accuracy of the calculated deflections.

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8. REFERENCES

ACI Committee 318 (1999) *Building Code Requirements for Structural Concrete (318-99) and commentary (318R-99)*, American Concrete Institute, Farmington Hills, MI, USA.

Aïtcin, P.C. (1998) *High-Performance Concrete*, E&FN Spon, New York, NY, USA.

Baalabaki, W., Benmokrane, B., Chaallal, O., and Aïtcin, P.C. (1991) Influence of Coarse Aggregate on Elastic Properties of High-Performance Concrete, *ACI Materials Journal*, 88: 499-503.

Branson, D.E. (1977) *Deformation of Concrete Structures*, McGraw-Hill Book Co., New York, NY, USA.

Carrasquillo, R.L., Nilson, A.H., and Slate, F.O. (1981) Properties of High-Strength Concrete Subjected to Short-Term Loads, *ACI Journal*, 78: 171-178.

CEP–FIP Model Code 90 (1993) *Model Code for Concrete Structures*, Comité Euro-International du Béton (CEB) – Fédération Internationale de la Précontrainte (FIP), Thomas Telford Ltd., London, UK.

CSA A23.3-M94 Technical Committee (1994) *Design of Concrete Structures*, Canadian Standards Association, Toronto, Ontario, Canada.

Fling, R.S. (1992) Practical Considerations in Computing Deflection of Reinforced Concrete, *Designing Concrete Structures for Serviceability and Safety*, ACI SP-133: 69-91.

Frangopol, D.M. (1988) Probabilistic Design for Serviceability Related to Excessive Deformation, *Proceedings of Symposium/Workshop on Serviceability of Buildings*, National Research Council of Canada, Ottawa, Ontario, Canada, 49-61.

Gardner, N.J. (1990) Design and Construction Interdependence, *Concrete International*, 12: 32-38.

Ghali, A. (1993) Deflection of Reinforced Concrete Members: A Critical Review, *ACI Structural Journal*, 90: 364-373.

Ghali, A. and Azarnejad, A. (1999) Deflection Prediction of Members of Any Concrete Strength, *ACI Structural Journal*, 96: 807-816.

Ghali, A. and Favre, R. (1994) *Concrete Structures: Stresses and Deformations*, 2nd ed., E&FN SPON, London, UK.

Gilbert, R.I. (1999) Deflection Calculation for Reinforced Concrete Structures – Why We Sometimes Get It Wrong, *ACI Structural Journal*, 96: 1027-1032.

Holicky, M. (1988) Fuzzy Concept of Serviceability Limit States, *Proceedings of Symposium/Workshop on Serviceability of Buildings*, National Research Council of Canada, Ottawa, Ontario, Canada, 19-31.

Jokinen, E.P. and Scanlon, A. (1985) Field-Measured Two-Way Slab Deflections, *Proceedings of the Annual Conference of the Canadian Society for Civil Engineering*, Saskatoon, Saskatchewan, Canada, 43-58.

Kennedy, J.B. and Neville, A.M. (1986) *Basic Statistical Methods for Engineers and Scientists*, Harper and Row Publishers, New York, NY, USA.

Légeron, F. and Paultre, P. (2000) Prediction of Modulus of Rupture of Concrete, *ACI Materials Journal*, 97: 193-200.

OHBDC (1992) *Ontario Highway Bridge Design Code*, 3rd ed., Ministry of Transportation, Downsview, Ontario, Canada.

Oluokun, F.A. (1991) Prediction of Concrete Tensile Strength from its Compressive Strength: An Evaluation of Existing Relations for Normal Weight Concrete, *ACI Material Journal*, 88: 302-309.

Raphael, J.M. (1983) Tensile Strength of Concrete, *ACI Journal*, 81: 158-165.

Scanlon, A. and Pinheiro, L. (1992) Allowable Deflections: The Other Side of the Equation, *Designing Concrete Structures for Serviceability and Safety*, ACI SP-133: 111-120.

Shah, S.P., Swartz, S.E. and Ouyang, C. (1995) *Fracture Mechanics of Concrete: Applications of Fracture Mechanics to Concrete, Rock and Other Quasi-Brittle Materials*, John Wiley & Sons, Inc., New York, NY, USA.

Sherif, A.G. and Dilger, W.H. (1998) Analysis and Deflections of Reinforced Concrete Flat Slabs, *Canadian Journal of Civil Engineering*, 25:451-466.

Standards Australia AS 3600-1994 (1994) *Concrete Structures*, Standards Association of Australia, Homebush, Australia.

Standards New Zealand NZS 3101 (1995) *The Design of Concrete Structures*, Standards New Zealand, Wellington, New Zealand.

Thompson, D.P. and Scanlon, A. (1988) Minimum Thickness Requirements for Control of Two-Way Slab Deflections, *ACI Structural Journal*, 85: 12-22.

Whiting, D.A., Detwiler, R.J., and Lagergren, E.S. (2000) Cracking Tendency and Drying Shrinkage of Silica Fume Concrete for Bridge Deck Applications, *ACI Materials Journal*, 97: 71-77.