

Ecuatii cu derivate partiale de ordinul intai

1. Determinati solutia generala pentru urmatoarele sisteme de ecuatii diferentiale:

$$(a) \frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$$

$$(b) \frac{dx}{z+3y} = \frac{dy}{3(z-x)} = \frac{dz}{-x-3y}$$

$$(c) \frac{dx}{y-zx} = \frac{dy}{-x-yz} = \frac{dz}{z^2+1}$$

$$(d) \begin{cases} \frac{dy}{dx} = \frac{z}{(y-z)^2} \\ \frac{dz}{dx} = \frac{y}{(z-y)^2} \end{cases}$$

$$(e) \begin{cases} \frac{dy}{dx} = \frac{1}{z} \\ \frac{dz}{dx} = \frac{1}{y} \end{cases}$$

$$(f) \begin{cases} \frac{dy}{dx} = \frac{z^2}{y} \\ \frac{dz}{dx} = \frac{y}{z} \end{cases}$$

2. Determinati solutia generala pentru urmatoarele ecuatii cu derivate partiale de ordinul intai liniare si omogene:

$$(a) (1+x^2) \frac{\partial u}{\partial x} + xy \frac{\partial u}{\partial y} = 0$$

$$(b) yz \frac{\partial u}{\partial x} + xz \frac{\partial u}{\partial y} + xy \frac{\partial u}{\partial z} = 0 \text{ in the domain } x > 0 \text{ and } y > 0.$$

$$(c) x^2 \frac{\partial u}{\partial x} - xy \frac{\partial u}{\partial y} - y^2 \frac{\partial u}{\partial z} = 0$$

$$(d) xy \frac{\partial u}{\partial x} - y\sqrt{1-y^2} \frac{\partial u}{\partial y} + (z\sqrt{1-y^2} - x) \frac{\partial u}{\partial z} = 0$$

$$(e) x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$$

3. Determinati solutia generala pentru urmatoarele ecuatii cu derivate partiale de ordinul intai cvasiliniare:

$$(a) y^2 \frac{\partial z}{\partial x} - xy \frac{\partial z}{\partial y} = x\sqrt{1+z^2}$$

$$(b) xy \frac{\partial z}{\partial x} - y^2 \frac{\partial z}{\partial y} = x^2$$

$$(c) (x^2 + y^2) \frac{\partial z}{\partial x} + 2xy \frac{\partial z}{\partial y} + z^2 = 0$$

$$(d) y \frac{\partial z}{\partial x} + yz \frac{\partial z}{\partial y} + 1 + z^2 = 0$$

$$(e) (1 + \sqrt{z-x-y}) \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 2$$

$$(f) \tan x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = z$$

$$(g) v \frac{\partial v}{\partial x} - v \frac{\partial v}{\partial y} = y - x$$

4. Rezolvati urmatoarele probleme Cauchy:

- (a) $\frac{\partial u}{\partial x} - y \frac{\partial u}{\partial y} = 0$
 $u|_{x=0} = \sin y$
- (b) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$
 $u|_{x=1} = y - z$
- (c) $\frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + (x + y) \frac{\partial u}{\partial z} = 0$
 $u|_{x=0} = yz$
- (d) $xz \frac{\partial z}{\partial x} + yz \frac{\partial z}{\partial y} + xy = 0$
 $z|_{y=2} = x$
- (e) $\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = \frac{y-x}{z}$
 $z|_{x=1} = y^2$
- (f) $\frac{\partial v}{\partial x} + x \frac{\partial v}{\partial y} = v$
 $v|_{x=0} = e^y$