Countably generated Hilbert modules, multiplier modules and stable isomorphisms of locally C*-algebras

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Locally C^*-algebras are generalizations of C^*-algebras. Instead of being given by a single norm, the topology on a locally C^*-algebra is defined by a directed family of C^*-seminorms. Such important concepts as Hilbert C^*-module, Morita equivalence, stable isomorphism of C^*-algebras can be defined in the framework of locally C^*-algebras. Brown, Green and Rieffel [1] proved that two C^*-algebras A and B with approximate units are stably isomorphic (that is, the C^*-algebras $A \otimes K$ and $B \otimes K$, where K is the C^*-algebra of all compact operators on a separable Hilbert space, are isomorphic) if and only if they are strongly Morita equivalent (that is, there is an A-B imprimitivity bimodule E). In [2], we proved that the result of Brown, Green and Rieffel is still valid in the context of Fréchet locally C^*-algebras with countable approximate units. Raebrun and Thompson [4] extended the result of Brown, Green and Rieffel and they proved that two C^*-algebras A and B are stably isomorphic if and only if there is an A-B imprimitivity module E which is countably generated in the multiplier module both as an A -module and as a B -module.

In this work, we show that the stabilisation theorem [3] is still valid for Hilbert modules over locally C^*-algebras which have a countable system of generators consisting of the multipliers of modules. Using this result, we show that two locally C^*-algebras A and B are stably isomorphic if and only if there is a full Hilbert A -module E which is countably generated in the multiplier module and such that the locally C^*-algebras B and $K(E)$, the locally C^*-algebra of all compact operators on E, are isomorphic and the Hilbert $K(E)$ -module $K(E, A)$ is countably generated in the multiplier module.

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