

## Lecture 6: The Dornbusch overshooting model

The following notes are adapted from Dr. Saqib Jafarey's course notes on the topic

The famous Dornbusch overshooting model helps explain why exchange rates move so sharply from day to day.

PPP version with a sticky price level.

Assumptions:

- 1) Price level is predetermined at each point in time. Over time, it adjusts in order to satisfy PPP.
- 2) The exchange rate adjusts instantaneously to equate supply and demand for foreign exchange.

In the following equations, in order to make things simple and all linear, logs of the variables are used. To understand the reason why economists often do this, think about the relationship between four variables W, X, Y, Z:  $X/W = Y/Z$ ; taking logs on both sides gives

$$\log X - \log W = \log Y - \log Z.$$

Denoting  $x = \log X$ ,  $w = \log W$ ,  $y = \log Y$ , and  $z = \log Z$ , we have  $x - w = y - z$ , a linear relationship that is easier to deal with than the ratios. In what follows, just accept this idea and don't worry about the details. This way, LM and other curves will be linear, because taken in logs)

Equations:

$$m_t = \alpha y_t - \lambda i_t + p_t \quad (1)$$

(this is LM in logs: m is money supply, p is price, y is output and i the nominal interest rate, all in logs, and index t is for time)

$$i_t = i^* + (e_{(t+1)} - e_t) \quad (2)$$

(This is the uncovered interest parity condition but in logs (the one you already know), e is the nominal interest rate in domestic currency per unit of foreign currency, but in logs, and i and  $i^*$  are the domestic and foreign nominal interest rates, also in logs)

$$p_{(t+1)} - p_t = \phi (p^* + e_t - p_t) \quad (3)$$

This equation represents the short run adjustment (from t to t+1) of prices p (on the market for goods and services).

$$p_t = e_t + p^* \quad (4)$$

This is the PPP condition (but in logs), which is assumed to hold IN THE LONG RUN.

Note that in (3),  $\phi$  is the speed of adjustment of prices; if it tends to infinity, then prices adjust

instantaneously and PPP always holds. (if you divide the right hand side by  $\phi$  and  $\phi$  tends to infinity, you get (4), that is, PPP)

At each time  $t$ ,  $m_t$ ,  $y_t$ ,  $i_t^*$ ,  $p_t^*$ , and  $p_t$  are exogenous. For simplicity, let's even assume the following:

$m_t = m$ ,  $y_t = y$ ,  $i^*(t) = i^*$ ,  $p^*(t) = p^*$ , that is, these four variables are constant over time. But once in awhile, they may change: for example, we're going to study the impact of a one time change in  $m$ )

(by the way, for the variables with \*, my type setting software seems to leave a stupid question mark (upside down, right next to them), please ignore this, it has nothing to do with the equations, it's just Word being silly)

The endogenous variables are:  $e_t$ ,  $i_t$ , and *future* values of  $p_t$  (i.e, at a given period  $t$ ,  $p_t$  is fixed, doesn't adjust instantaneously, but then, over time, it adjusts according to equation (3), and in the long run PPP, equation (4), holds)

Now let's solve the model, using all the above assumptions.

Plugging (2) into (1), you get:

$$m_t - p_t = \alpha y - \lambda (i^* + e_{(t+1)} - e_t) \quad (5)$$

Rearranging, you get:

$$\lambda (e_{(t+1)} - e_t) = p_t - (m + \lambda i^* - \alpha y) \quad (6)$$

Denoting  $\bar{p} = (m + \lambda i^* - \alpha y)$ , you get

$$(e_{(t+1)} - e_t) = \frac{1}{\lambda} (p_t - \bar{p}) \quad (7)$$

According to (7),  $(e_{(t+1)} - e_t) > 0$  if  $(p_t > \bar{p})$  and  $< 0$  otherwise.

We are trying to construct a graph with  $p$  on the horizontal axis and  $e$  on the vertical axis, in order to describe the behavior of  $p$  and  $e$  over time.

Our goal is to describe the behavior of  $e$  over time (and see how it can « overshoot », we'll define this soon). The first step towards the important graph that will allow to do so is to find the equations describing the set of points  $(p, e)$  for which  $(e_{(t+1)} - e_t) = 0$  : according to (7), this is simply given by

$$p_t = \bar{p} \quad (8)$$

So we know how the exchange rate varies over time depending on whether  $(p_t > \bar{p})$  or  $(p_t < \bar{p})$  (and it does not vary when  $(p_t = \bar{p})$ )

Now, to complete the graph, we need to describe how  $p$  varies over time. First, we want the set of points  $(p, e)$  for which  $p$  does not vary over time. From equation (3), that is,

$$p_{(t+1)} - p_t = \phi(p^* + e_t - p_t) \quad , \text{ we get that } p_{(t+1)} - p_t = 0 \quad \text{if} \\ e_t = p_t - p^* \quad (9)$$

(Note that equation (9) is in fact PPP at each time  $t$ )

Equation (9) defines a straight line in the  $(p, e)$  space, with slope +1.

Now, graph (8) and (9) in the  $(p, e)$  space, assuming that  $\bar{p} > p^*$  , so that the two lines intersect. (Remark: the assumption that  $\bar{p} > p^*$  makes it so that, the exchange rate at  $t$  given by  $e_t = p_t - p^*$  (equation (9)) cannot be negative. Negative exchange rates do not make any sense so it's fine to assume this does not happen in the model)

On the graph, the intersection between (8) and (9) define four zones on the graph. In each zone, think about how  $p$  and  $e$  behave (increase or decrease) over time (since  $p$  is constant for points on (9) and  $e$  is constant for points on (8), by definition of these curves).

This gives you what is called a phase diagram, describing the dynamic (i.e. Over time) behavior of  $p$  and  $e$ . The intersection is the steady state (where  $p$  and  $e$  are constant), and only combinations starting on the (unique) « saddle path » lead to the steady state. All other combinations lead to bubbles (i.e  $p$  and  $e$  not converging toward the steady state where they are constant)

**Now it is easy to analyze the effect on the dynamics of a one time permanent increase in the money supply  $m$  (that is, a change in monetary policy)**

Main intuition of the effect of a change in  $m$ :

At the instant of the change in  $m$ ,  $p$  is fixed so  $m - p$  rises. This means that there is initially an excess supply of money on the domestic money market. Given that output is assumed fixed, in order to preserve equilibrium on the money market, the interest rate  $i$  should fall at that instant so as to adjust the demand for money so that it matches the increased supply.

With perfect capital mobility, and given  $i^*$ , this implies that  *$e$  must be decreasing in order for uncovered interest parity to hold* while the domestic interest rate (which has fallen instantly) is below  $i^*$ . (uncovered interest parity has to hold at each instant  $t$ )

But we know that from the old long run equilibrium, to the new one, in fact,  *$e$  must increase!*

How can a long run depreciation ( $e$  increases) be reconciled with the instantaneous appreciation ( $e$  decreases) of the exchange rate?

Well, this is Dornbusch brilliant idea: this is possible if initially, the increase in  $e$  occurring at the time of the increase in  $m$  makes  $e$  temporarily « overshoot » (that is, goes above) its long run value, and then  $e$  begins falling back to its long run value.