Prices and Exchange Rates

In frictionless markets, freely tradable goods should have the same price anywhere:

$$P^{\$} \times S \;=\; P^{¥}$$

$$\begin{aligned} P^{\$} &\equiv \text{price in US\$} \\ S &\equiv \text{Exchange rate in yen per dollar} \\ P^{¥} &\equiv \text{Price in Japanese yen} \end{aligned}$$

Purchasing power parity (PPP) exchange rate:

$$S \;=\; \frac{P^{¥}}{P^{\$}}$$

The *hamburger strandard* (April 17, 2001):

Price of a Big Mac: C$ 3.33 and US$2.54.
PPP exchange rate (C$/US$):

$$S = \frac{3.33}{2.54} = 1.31\text{C\$/US\$}$$

Actual exchange rate: 1.56 C$/US$.

According to the PPP theory, Canadian dollar is undervalued by

$$\frac{1.31}{1.56} - 1 = -16\%$$

In a less extreme form, the PPP theory should hold for baskets of goods.

$$
\begin{aligned}
PI^{¥} &\equiv \text{price of a basket of goods in yen} \\
PI^{\$} &\equiv \text{price in US\$ of the same basket}
\end{aligned}
$$

PPP exchange rate between Japan and US:

$$S = \frac{PI^{¥}}{PI^{\$}}$$

Relative Purchasing Power Parity

Relative changes in prices between two countries over a period of time is what determines the changes in exchange rates over the same period.

If the spot exchange rate between two countries starts in equilibrium, any change in the differential rate of inflation between them tends to be offset over the long run by an equal but opposite change in the spot exchange rate.

$PI_t^{\$} \equiv$ US price index at time t
$PI_t^{¥} \equiv$ Japanese price index at time t
$\pi^{\$} \equiv$ US inflation from t to $t+1$
$\pi^{¥} \equiv$ Japanese inflation from t to $t+1$
$S_t \equiv$ exchange rate at time t (¥/\$)

Assuming purchasing power parity,

$$S_t = \frac{PI_t^{¥}}{PI_t^{\$}} \quad \text{and} \quad S_{t+1} = \frac{PI_t^{¥}(1+\pi^{¥})}{PI_t^{\$}(1+\pi^{\$})} = S_t \times \frac{1+\pi^{¥}}{1+\pi^{\$}}$$

Percent change in S from t to $t+1$:

$$\begin{aligned}
\frac{S_{t+1}}{S_t} - 1 &= \frac{S_t \times \frac{1+\pi^{¥}}{1+\pi^{\$}}}{S_t} - 1 \\
&= \frac{1+\pi^{¥}}{1+\pi^{\$}} - 1 \\
&= \frac{\pi^{¥} - \pi^{\$}}{1+\pi^{\$}} \\
&\propto \pi^{¥} - \pi^{\$}
\end{aligned}$$

Example

Honda's price per vehicle: ¥4,000,000
S = ¥125/$
Expected inflation in Japan: 1%
Expected inflation in U.S.: 3%
Assuming PPP, what is the expected spot exchange rate at the end of the year?

$$\begin{aligned} S_{\text{end}} &= S \times \frac{1+\pi^{¥}}{1+\pi^{\$}} \\ &= ¥125/\$ \times \frac{1.01}{1.03} \\ &= ¥122.57/\$ \end{aligned}$$

Real and Nominal Exchange Rates

Suppose PPP holds:

$$\$1.50/\pounds \;=\; \frac{\$1,500/\text{U.S. good}}{\pounds 1,000/\text{British good}}$$

and thus

$$\frac{\$1.50/\pounds}{\$1,500/\pounds 1,000} \;=\; 1 \;\; \text{British good per U.S. good,}$$

i.e. identical goods trade on a one-for-one basis.

Real Exchange Rate

The units of the real exchange rate between U.S. and U.K. is

$$\frac{\text{U.S. goods}}{\text{British goods}} \quad \text{or} \quad \frac{\text{British goods}}{\text{U.S. goods}}$$

When exchange rates change by the same percentage as relative prices, real exchange is constant.

Real Exchange Rate

$$\text{Real Exchange Rate} = \frac{\text{Nominal Exchange Rate}}{\text{PPP Exchange Rate}}$$

$$S_R(\$/\pounds) = \frac{S_N(\$/\pounds)}{PI^{\$}/PI^{\pounds}}$$

If $S_R(\$/\pounds) > 1$, US$ is undervalued vis-à-vis the £.

Real Exchange Rate Index

$$\text{Real Exchange Rate Index} = \frac{\text{Nominal Exchange Rate Index}}{\text{PPP Exchange Rate Index}}$$

$$E_R(\mathsf{FC}/\$) = \frac{E_N(\mathsf{FC}/\$)}{C^{\mathsf{FC}}/C^{\$}}$$

If changes in exchange rates offset differential inflation, $E_R = 100$.

PPP and Transaction Costs

Transportation Costs

Helliwell (1998) Canadian provinces are 12 times more likely to trade merchandises and 40 times more likely to trade services among themselves then with closer American states

The border matters

Tests of Purchasing Power Parity

PPP holds well over the very long run but poorly for shorter time periods

The theory holds better for countries with relatively high rates of inflation and underdeveloped capital markets

Example

Businesses and consumers tend to abandon their currency in a hyperinflationary economy.

Suppose a TV set is priced 1,200 pesos in Mexico, reflecting a \$400 TV price and 3 peso/\$.
If the peso devalues to 6 peso/\$, the price of a TV set becomes 2,400 pesos

PPP holds immediately since merchants use PPP to calculate their prices

Exchange Rate Pass-Through

Incomplete exchange rate pass-through could explain why an exchange rate can deviate from its PPP-equilibrium level.

Example: BMW produces automobiles in Germany and pays all production expenses in euros. When esporting to the US, the price should be

$$P^{\$}_{\text{BMW}} = P^{€}_{\text{BMW}} \times S$$

If the euro appreciates by 10% over the US$, then $P^{\$}_{\text{BMW}}$ should also increase by 10%.

If instead $P^{\$}_{\text{BMW}}$ increases by less than 10%, the pass-through is partial.

BMW has an incentive to limit the pass-through.

Example: Suppose exchange rate is \$1/€.

$$P^{\$}_{\text{BMW}} = P^{€}_{\text{BMW}}$$

If the euro appreciates 20% versus the dollar and $P^{€}_{\text{BMW}} =$ €$35,000$,

$$P^{\$}_{\text{BMW}} = 1.2 \times 35,000 = \$42,000.$$

If, instead, $P^{\$}_{\text{BMW}} = \$40,000$, the pass-through is

$$\frac{P^{\$}_{\text{BMW},2}/P^{€}_{\text{BMW},1}}{(S_2 - S_1)/S_1} = \frac{40,000/35,000}{.20} = \frac{.14}{.20} = 70\%$$

Pass-through may depend on the price elasticity of demand

$$\epsilon_p = \frac{\Delta Q_p}{\Delta p}$$

Goods with inelastic demand, i.e. $|\epsilon_p| < 1$, should be associated with a higher degree of pass-through

Example

Honda's price per vehicle is ¥4,000,000, S = ¥125/$
Expected inflation in Japan: 1%, Expected inflation in U.S.: 3%

(a) Assuming PPP and 100% pass through, what is the expected price of a Honda in US$ at the end of the year?

$$P^{\$}_{\text{end}} \times S_{\text{end}} = P^{¥}_{\text{end}}$$

$$P^{\$}_{\text{end}} \times S \times \frac{1+\pi^{¥}}{1+\pi^{\$}} = \left(1+\pi^{¥}\right) P^{¥}_{\text{beg}}$$

$$P^{\$}_{\text{end}} = \frac{\left(1+\pi^{\$}\right) P^{¥}_{\text{beg}}}{S}$$

$$P^{\$}_{\text{end}} = \frac{1.03 \times 4{,}000{,}000}{125} = \$32,960$$

(b) Assuming PPP and 60% pass through, what is the expected price of a Honda in US$ at the end of the year?

$$S_{\text{beg}} = ¥125/\$ \,, \qquad S_{\text{end}} = ¥122.57/\$$$

Japanese yen appreciates

$$\frac{\frac{1}{S_{\text{end}}} - \frac{1}{S_{\text{beg}}}}{\frac{1}{S_{\text{beg}}}} = \frac{S_{\text{beg}} - S_{\text{end}}}{S_{\text{end}}} = \frac{125 - 122.57}{125} = 1.944\%$$

versus the dollar. With a 60% pass through, $P^{\$}_{\text{end}}$ will increase by

$$.6 \times 1.944\% = 1.1664\%$$

Interest Rates and Exchange Rates

The Fisher Effect

$$1 + r = \frac{1+i}{1+\pi} \quad \Rightarrow \quad i = r + \pi + r\pi$$

$r \equiv$ real exchange rate
$i \equiv$ nominal exchange rate
$\pi \equiv$ inflation rate

The International Fisher Effect

$i^{\$} \equiv$ nominal interest in the US
$i^{¥} \equiv$ nominal interest rate in Japan
$S_1 \equiv$ exchange rate (in ¥/\$) at the beginning of the period
$S_2 \equiv$ exchange rate at the end of the period
Return should be the same in both markets

$$\frac{S_1}{S_2}(1 + i^{¥}) \;=\; 1 \;+\; i^{\$} \qquad \Rightarrow \qquad \frac{S_1}{S_2} \;=\; \frac{1 + i^{\$}}{1 + i^{¥}}$$

Subtracting one on both sides:

$$\frac{S_1 - S_2}{S_2} \;=\; \frac{i^{\$} - i^{¥}}{1 + i^{¥}}$$

Empirical tests provide some support to this thesis.
This model does not take into account foreign exchange risk.
Speculation may also creates distorsions in currency markets.

The Forward Rate

The forward rate is an exchange rate quoted for settlement at some future date: 30, 60, 90, 180, 270, 360 days

Invest \$1 in the U.S. a the rate $i^{\$}$ and enter a forward contract that exchanges US\$ for Swiss Francs (SF) at the rate $F^{\mathsf{SF}/\$}$.

Exchange US\$ for SF today at the spot rate $S^{\mathsf{SF}/\$}$ and invest it is Switzerland at the rate i^{SF}.

Should end up with the same number of SF:

$$\left(1+i^{\$}\right)F^{\mathsf{SF}/\$} \;=\; \left(1+i^{\mathsf{SF}}\right)S^{\mathsf{SF}/\$}$$

$F_{90}^{\mathsf{SF}/\$} \equiv$ 90-day forward rate for the SF/$ exchange rate

$i^c \equiv$ annual interest rate in (currency c)-denominated deposits

$$F_{90}^{\mathsf{SF}/\$} \;=\; S^{\mathsf{SF}/\$} \times \frac{1 + i^{\mathsf{SF}} \times \frac{90}{360}}{1 + i^{\$} \times \frac{90}{360}}$$

Interest rate parity

Spot and forward rates are considered to be at interest rate parity if

$$\frac{F_n^{\mathsf{SF}/\$}}{S^{\mathsf{SF}/\$}} = \frac{1 + i^{\mathsf{SF}} \times \frac{n}{360}}{1 + i^{\$} \times \frac{n}{360}}$$

Forward premium:

$$f_n^{\mathsf{SF}/\$} = \frac{S^{\mathsf{SF}/\$} - F_n^{\mathsf{SF}/\$}}{F_n^{\mathsf{SF}/\$}} = \frac{\left(i^{\$} - i^{\mathsf{SF}}\right) \times \frac{n}{360}}{1 + i^{\mathsf{SF}} \times \frac{n}{360}}$$

Currency with a lower interest rate sells forward at a premium

Covered Interest Arbitrage (CIA)

If spot and forward rates are not at interest rate parity, this creates *covered interest arbitrage* opportunities.

These opportunities will continue until spot and forward rates reach interest rate parity.

Political risk: Application of capital controls

Uncovered Interest Arbitrage (UIA)

Money is borrowed from countries with low interest rates and invested in countries with high interest rates, without covering with a forward contract.

Expected profits calculated using expected spot rates: foreign exchange risk.

Example: The "yen carry trade"

Borrow yen at extremely low interest rate (0.4%)
Change for dollars, invest the dollars at 5%
Change dollars for yen to repay loan.
If the Yen strenghtens by more than the interest rate differential, investors lose money

Forward Rate as an Unbiased Predictor of the Future Spot Rate

Hypothesis:

$$E_t[S_{t+k}] \;=\; F_{t,t+k}$$

Future spot rates should be distributed around the forward rate if hypothesis is true.

If markets are not efficient, then the forward rate is not an unbiased predictor of the future spot rate.

Asset Market Approach to Forecasting

Exchange rates depend not only on differences in interest rates but also on a country's outlook for economic growth and profitability.

Exchange rates depends on the willingness to hold some currencies.

Problem 1. Polish Zloty

On year ago the exchange rate between Polish zloty and the U.S. dollar was Z3.8000/$. Since then the zloty has fallen 14% against the dollar. Price level in the United States have not changed, but Polish prices have gone up 7%.

a. What is the nominal exchange rate today?

b. What should be the exchange rate today, based on purchasing power parity and assuming last year was normal?

c. Did the Polish zloty depreciate or appreciate in real terms relative to the dollar? By what percentage?

Problem 2. Pinot Noir

Pinot Noir wine is produced in the states of California (U.S.A.) and New South Wales (Australia). Equivalent bottles of Pinot Noir sell in the United States for US$22 and in Australia for A$34.

a. According to the PPP theory, what whould be the US$/A$ spot rate of exchange?

b. Suppose the price of Pinot Noir is expected to rise to $27 over the next year while the price of a comparable bottle of Australian wine is expected to rise to A$44. What should be the one-year forward US$/A$ exchange rate?

c. Given your answers to (a) and (b) above, and given that the current interest rate in the U.S. is 5% for notes of a one-year maturity, what would you expect current Australian interest rates to be?

Problem 4. Covered Interest Arbitrage (¥/$)
Ayako Miyake is a currency arbitrager for Atsugi Bank in Kanagawa. The spot rate this morning is ¥110.60/$, and early indications are the 90-day interest rates in the United States will rise from their current level of 4.25% per annum. The U.S. Federal Reserve Bank, the central bank, is worried about an unjustified level of the U.S. stock market and has been publicly considering raising interest rates 50 basis points (.5%). The 90-day forward exchange rate quoted to Miyake by other Japanese banks are all about the same, ¥109.80/$. The current 90-day yen interest rate is 0.5% per annum. Miyake has ¥400,000,000 at her disposal.

a. How much profit can Miyake make through covered interest arbitrage?

b. What could Miyake do with her ¥400 million if she wished to speculate through uncovered interest arbitrage? How much profit could she expect to make?

Problem 5. London and New York

Money and foreign exchange markets in London and New York are very efficient. You have the following information:

	London	**New York**
Spot exchange rate	\$1.6000/£	£0.6250/\$
One-year treasury bill rate	5.00%	6.00%
Expected inflation rate	2.00%	unknown

Assume parity conditions hold:

a. Estimate inflation in the United States next year.

b. Estimate today's one-year forward exchange rate between pounds and dollars.

c. Diagram how the United Kingdom and the United States are interrelated with respect to exchange rates, interest rates, and inflation rates. Define on the diagram each of the parity conditions which cause these theoretical linkages.

Problem 6. Interest Arbitrage

Henri Jacque, an arbitrageur with Bank of Montreal, faces the following Canadian dollar/U.S. dollar quotes:

Spot rate:	C$1.4900/$
Six-month forward rate:	C$1.5100/$
Six-month Canadian dollar interest rate	7.50%
Six-month U.S. dollar interest rate:	5.00%

Henri is authorized to use C$20,000,000 or its U.S. dollar equivalent. The ending profit, if any, should be realized in Canadian dollars. How can he complete covered interest arbitrage? What will be his profit?