A Note on Bond Prices

Consider a bond with a par value B and an annual coupon rate i. The bond matures in T years and coupon payments are made m times throughout the year, which means that each coupon payment is $\frac{iB}{m}$. Let P denote the price of the bond, and take the annualized yield on the bond, y, as given. The discount rate each period is then $\frac{y}{m}$, and P is given by

$$\begin{aligned} P &= \frac{iB}{m}\left(\frac{1-\left(\frac{1}{1+y/m}\right)^{mT}}{y/m}\right) + \frac{B}{(1+y/m)^{mT}} \\ &= B\left(\frac{i}{y} - \frac{i}{y}\left(\frac{1}{1+y/m}\right)^{mT} + \left(\frac{1}{1+y/m}\right)^{mT}\right). \end{aligned}$$

Using this price function, you can answer the following questions:

(a) What happens when the annual coupon rate is increased?

Answer: The annual coupon rate is represented by i. To see what happens when i increases, we can rewrite the price function as

$$P = B\left(\frac{i}{y}\left(1-\left(\frac{1}{1+y/m}\right)^{mT}\right) + \left(\frac{1}{1+y/m}\right)^{mT}\right).$$

Since $\left(\frac{1}{1+y/m}\right)^{mT} < 1$, we have $1-\left(\frac{1}{1+y/m}\right)^{mT} > 0$ and thus P increases as i increases.

(b) What happens when the yield to maturity is increased?

Answer: The yield to maturity is represented by y. To have an increase in y when everything else remains constant, we need P to decrease. This can be seen by writing P as follows:

$$P = B\left(\frac{i}{y} + \left(1-\frac{i}{y}\right)\left(\frac{1}{1+y/m}\right)^{mT}\right).$$

When y increases, $\frac{i}{y}$ decreases, $1-\frac{i}{y}$ increases and $\left(\frac{1}{1+y/m}\right)^{mT}$ decreases. Note that the decrease in $\left(\frac{1}{1+y/m}\right)^{mT}$ offsets the increase in $1-\frac{i}{y}$ and thus this results in a decrease in P.

(c) What happens when the number of payments per year is increased?

Answer: If we leave the annual coupon rate constant, this means that m only increases. We can see what happens by writing the price function as in the previous exercise, i.e.

$$P = B\left(\frac{i}{y} + \left(1-\frac{i}{y}\right)\left(\frac{1}{1+y/m}\right)^{mT}\right).$$

As m increases, $\left(\frac{1}{1+y/m}\right)^{mT}$ decreases, but the effect on P depends on the sign of $1-\frac{i}{y}$. If $i > y$, then $1-\frac{i}{y} < 0$ and P increases as m increases. If $i = y$, then $1-\frac{i}{y} = 0$ and an increase in m has no effect on P. If If $i < y$, then $1-\frac{i}{y} > 0$ and P decreases as m increases.

(d) What happens when the face value is increased?

Answer: The face value is represented by B. Clearly, P increases when B increases.

(e) What is the relationship between the price of a par bond and time to maturity?

Answer: A par bond is such that $i = y$, which implies $P = B$. This relationship does not depend on T.

(f) What happens when the annual coupon rate is increased to the point that it equals the yield to maturity? What happens when it is increased further?

Answer: The annual coupon rate is represented by i here. We can see what happens to P by writing the price function in the following manner:

$$P \;=\; B\left(\frac{i}{y}\left(1-\left(\frac{1}{1+y/m}\right)^{mT}\right) \;+\; \left(\frac{1}{1+y/m}\right)^{mT}\right).$$

Since $\left(\frac{1}{1+y/m}\right)^{mT} < 1$, we have $1-\left(\frac{1}{1+y/m}\right)^{mT} > 0$ and thus P increases as i increases. When $i = y$, the bond price is equal to its par value, i.e. $P = B$. When $i > y$, then the bond price becomes greater than its par value, i.e. $P > B$.