Shareholders versus Creditors, Part II
The Underinvestment Problem

The following examples illustrate the *underinvestment problem* that may arise due to conflicts of interest between shareholders and bondholders. That is, profitable investments may not be undertaken by management because they are costly to shareholders. Throughout this handout, management is assumed to act in the best interest of shareholders.

Two cases will be analyzed: Either the project, if undertaken, is financed with internal equity (idle cash) that would otherwise be paid to shareholders as dividends, or the project is financed with external equity capital. These problems will be solved using the binomial pricing model.

1 Idle Cash is Used Either to Finance a Project or to Pay an Immediate Dividend

Consider a firm that has an initial value $V = 100$, a debt issue that pays $X = 99$ in five years, time at which the value of the firm will be

$$V_5 = \begin{cases} V_5^u = 150 & \text{with probability } p = 0.8, \\ V_5^d = 66.67 & \text{with probability } 1 - p = 0.2 \end{cases}$$

The risk-free rate of interest is $r_f = 5\%$. Since there is a risk of default on the debt, we will find the original value of the firm's equity using the binomial pricing model in order to determine the market value of the firm's debt.

In five years, the firm's equity will be either $E_5^u = 150 - 99 = 51$ or $E_5^d = 0$ since in the down state the firm's assets are not sufficient to repay debtholders in full. Since we can use the risk-free rate with risk-free securities only, we have to form a risk-free portfolio to determine the firm's original value of equity. This can be done by buying the firm's assets while selling short a fraction δ of the firm's equity, where δ is such that

$$V_5^u - \delta E_5^u = V_5^d - \delta E_5^d .$$

This gives us

$$\delta \;=\; \frac{V_5^u - V_5^d}{E_5^u - E_5^d} \;=\; \frac{150 - 66.67}{51} \;=\; 1.6339 \;.$$

The original value of the firm's equity, denoted E, can then be found using the equation

$$V \;-\; \delta E \;=\; \frac{V_5^u - E_5^u}{(1 + r_f)^5} \;,$$

which yields

$$E \;=\; \frac{1}{\delta}\left(V \;-\; \frac{V_5^u - E_5^u}{(1 + r_f)^5}\right) \;=\; \frac{1}{1.6339}\left(100 \;-\; \frac{66.67}{(1.05)^5}\right) \;=\; \$29.23 \;.$$

The original value of the firm's debt is therefore

$$D \;=\; V \;-\; E \;=\; 100 \;-\; 29.23 \;=\; \$70.77 \;.$$

Suppose the firm has $Q = \$25$ in idle cash currently invested at the risk-free rate. Hence the above firm values include the return on Q, i.e. $Q(1 + r_f)^5 \;=\; \$25(1.05)^5 \;=\; \$31.91$. Management has to decide whether to pay Q as a dividend to shareholders or invest it in a project.

1.1 The Idle Cash is Used to Pay a Dividend

If Q is used to pay a dividend, the firm's current value becomes $V^n = 100 - 25 = 75$, its value in five years is either

$$V_5^{u,n} \;=\; 150 \;-\; 31.91 \;=\; 118.09 \quad \text{or} \quad V_5^{d,n} \;=\; 66.67 \;-\; 31.91 \;=\; 34.76,$$

and the firm's equity in five years is either $E_5^{u,n} = 118.09 - 99 = \$19.09$ or $E_5^{d,n} = \$0$. To find the new current value of equity, E^n, we use the same procedure as above, which gives

$$\delta \;=\; \frac{V_5^{u,n} - V_5^{d,n}}{E_5^{u,n} - E_5^{d,n}} \;=\; \frac{118.09 - 34.76}{19.09} \;=\; 4.3651,$$

and thus

$$E^n \;=\; \frac{1}{\delta}\left(V^n \;-\; \frac{V_5^{u,n} - E_5^{u,n}}{(1 + r_f)^5}\right) \;=\; \frac{1}{4.3651}\left(75 \;-\; \frac{34.76}{(1.05)^5}\right) \;=\; \$10.94 \;.$$

Hence shareholders' wealth following the dividend payment is

$$E^n \; + \; Q \; = \; 10.95 \; + \; 25 \; = \; \$35.95 \; = \; E \; + \; \$6.71,$$

whereas the market value of debt has decreased by the same amount.

1.2 The Idle Cash Is Invested in a Profitable Project

Suppose that the firm has access to a project that requires an initial outlay of \$25 and that pays \$40 in either state u or d. The return on this risk-free project is greater than the return on the risk-free security since the latter only pays off \$31.91 after five years. The project being risk-free, its payoff can be discounted at the risk-free rate, which gives us a net present value of

$$-25 \; + \; \frac{40}{(1.05)^5} \; = \; \$6.34 \; .$$

Hence the current value of the firm if the project is undertaken is $V^n = 106.34$, and its value in five years is either

$$V_5^{u,n} \; = \; 150 \; - \; 31.91 \; + \; 40 \; = \; 158.09 \quad \text{or} \quad V_5^{d,n} \; = \; 66.67 \; - \; 31.91 \; + \; 40 \; = \; 74.76 \; ,$$

which gives future equity values of $E_5^{u,n} = 158.09 - 99 = \$59.09$ and $E_5^{d,n} = 0$. The risk-free portfolio in this case is such that

$$\delta \; = \; \frac{V_5^{u,n} - V_5^{d,n}}{E_5^{u,n} - E_5^{d,n}} \; = \; \frac{158.09 - 74.76}{59.09} \; = \; 1.4102,$$

so

$$E^n \; = \; \frac{1}{\delta}\left(V^n \; - \; \frac{V_5^{u,n} - E_5^{u,n}}{(1+r_f)^5}\right) \; = \; \frac{1}{1.4102}\left(106.34 \; - \; \frac{74.76}{(1.05)^5}\right) \; = \; \$33.87 \; .$$

The new value of equity is then higher than the original one by \$4.64, and the new value of debt is

$$D^n \; = \; 106.34 \; - \; 33.87 \; = \; \$72.47,$$

\$1.70 greater than initially.

Note that the overall increase in wealth, $1.70 + 4.64 = \$6.34$ is equal to the net present value of the project. However, shareholders' wealth is greater when Q is paid as a dividend instead of being invested in the project. Thus if management does what is best for shareholders, the project won't be undertaken, which represents a deadweight loss of \$6.34.

2 Shareholders Must Contribute External Equity Funds to Finance the Project

Using the same parameter values as above, suppose now that the project has to be funded with external equity. If the project is undertaken, the value of the firm becomes $V^n = 106.34 + 25 = 131.34$, $V_5^{u,n} = 150 + 40 = 190$ and $V_5^{d,n} = 66.67 + 40 = 106.67$. Note that $V_5^{d,n} > X$ and thus the value of debt, which is now risk free, is given by

$$D^n = \frac{99}{(1.05)^5} = \$77.57,$$

an increase of $77.57 - 70.77 = \$6.80$. The new value of equity is therefore

$$E^n = V^n - D^n = 131.34 - 77.57 = \$53.77.$$

In this case, the value of equity rises by $53.77 - 29.23 = \$24.54$, which is less than the \$25 provided by shareholders. Hence if management has to choose between doing nothing and raising equity to undertake the project, the best for shareholders is to do nothing.

In this example, shareholders lose money on their investment because the increase in bondholders' wealth, \$6.80, is greater than the net present value of the project. The difference between what bondholders get and the project's NPV, $6.80 - 6.34 = \$0.46$, is equal to what shareholders lose.