Conflicts of Interest
Between Shareholders and Creditors

Here are three examples of actions that can be taken by shareholders to expropriate wealth from debtholders. We assume here that management do exactly what shareholders want.

1 A Leverage-Increasing Recap

Firm's market value today is $V = 100$.

Firm's market value in five years is given by

$$V_5 = \begin{cases} V_5^u = 200 & \text{with probability } p = 0.7, \\ V_5^d = 50 & \text{with probability } 1 - p = 0.3. \end{cases}$$

Expected value of the firm in five years is then

$$E[V_5] = .7 \times 200 + .3 \times 50 = 140 + 15 = 155.$$

To have a present value of 100, the discount rate for the whole firm, denoted r_A, must be 9.16%:

$$V = \frac{E[V_5]}{(1 + r_A)^5} \quad \Rightarrow \quad r_A = \left(\frac{E[V_5]}{V}\right)^{1/5} - 1 = \left(\frac{155}{100}\right)^{1/5} - 1 = 9.16\% .$$

Let D^o denote the value of an original issue of pure discount debt, which pays $X^o = 50$ at time $T = 5$. With $r_f = 5\%$, this means that

$$D^o = \frac{X^o}{(1 + r_f)^5} = \frac{50}{(1.05)^5} = \$39.18 .$$

The value of the firm's equity is then, originally,

$$E^o = V - D^o = 100 - 39.18 = \$60.82 .$$

Suppose that the firm sells a new issue of pure-discount debt that pays $X^n = 40$ at the end of five years. This issue has the same priority as the old debt and the proceeds from the issue are all distributed to shareholders as dividends. We assume that issuing more debt does not affect the value of the firm, i.e. V remains equal to 100, V_5^u remains 200 and V_5^d is still 50. Total promised to debtholders is now $X^o + X^n = 50 + 40 = 90 > 50 = V_5^d$, and thus debt is not risk free anymore.

With the increase in debt, the firm's value of equity in the "down" state will be zero since there won't be enough money to even pay all bondholders, who have priority over shareholders. That is,

$$E_5^n \;=\; \begin{cases} E_5^{u,n} \;=\; V_5^u - (X^o + X^n) \;=\; 200 - 90 \;=\; 110 & \text{if } V_5 = V_5^u, \\ E_5^{d,n} \;=\; \max\left\{\, V_5^d - (X^o + X^n)\ ,\ 0 \,\right\} \;=\; 0 & \text{if } V_5 = V_5^d. \end{cases}$$

We can find the new value of the firm's equity, E^n, by forming a risk-free portfolio, which consists of buying the firm's assets[1] while selling short a fraction δ of the firm's equity, such that

$$V_5^u - \delta E_5^{u,n} \;=\; V_5^d - \delta E_5^{d,n} \quad \Rightarrow \quad \delta \;=\; \frac{V_5^u - V_5^d}{E_5^{u,n} - E_5^{d,n}} \;=\; \frac{200 - 50}{110 - 0} \;=\; 1.3636\ .$$

The present value of this portfolio, $V - \delta E^n$, is then

$$V - \delta E^n \;=\; \frac{V_5^u - \delta E_5^{u,n}}{(1 + r_f)^5},$$

and thus

$$E^n \;=\; \frac{1}{\delta}\left(V - \frac{V_5^u - \delta E_5^{u,n}}{(1 + r_f)^5}\right) \;=\; \frac{1}{1.3636}\left(100 - \frac{50}{(1.05)^5}\right) \;=\; \$44.60\ .$$

Therefore,

$$D^n \;=\; V \;-\; E^n \;=\; 100 \;-\; 44.60 \;=\; \$55.40$$

Since both types of bondholders have equal priority over the firm's assets, the original debtholders will receive a fraction $\frac{5}{9}$ of the firm's assets in the down state whereas the new debtholders will receive a fraction $\frac{4}{9}$. Hence the value of the original debt is now

$$\frac{5}{9} \times 55.40 \;=\; \$30.78,$$

[1] Buying the firm's assets can be done by buying both the debt and equity of the firm.

which is \$8.40 less than before the new issue.

The proceeds from the new issue being paid to shareholders as dividends, their wealth is now

$$44.60 \; + \; \frac{4}{9} \times 55.40 \; = \; \$69.22,$$

an increasing of \$8.40.

2 Increasing the Riskiness of the Firm's Assets

Same original values as in the previous example, $V = 100$, $X^o = 50$, $V_5^u = 200$, $p = 0.7$, $V_5^d = 50$, $r_f = 5\%$, $D^o = \$39.18$ and $E^o = \$60.82$.

Suppose now that the firm's management changes its operational strategy such that possible values in five years are

$$V_5^n \; = \; \begin{cases} V_5^{u,n} \; = \; 300 & \text{with probability } p = 0.7, \\ V_5^{d,n} \; = \; 33.33 & \text{with probability } 1 - p = 0.3. \end{cases}$$

The firm's expected value in five years is now

$$E[V_5] \; = \; .7 \times 300 \; + \; .3 \times 33.33 \; = \; 220,$$

which is a little higher than originally. Assume the risk to be such that the firm's value as of today remains 100, i.e. the new discount rate is 17.08%:

$$V \; = \; 100 \; = \; \frac{220}{(1.1708)^5} \; .$$

We use the same procedure as above to determine the new value of the firm's equity. That is, we form a risk-free portfolio that buys the firm's assets and sells short a fraction δ of the firm's equity, such that

$$V_5^{u,n} \; - \; \delta E_5^{u,n} \; = \; V_5^{d,n} \; - \; \delta E_5^{d,n} \; .$$

Note that $E_5^{d,n} = 0$, since

$$V_5^{d,n} \; = \; 33.33 \; < \; 50 \; = \; X^o,$$

whereas

$$E_5^{u,n} \;=\; V_5^{u,n} \;-\; X^o \;=\; 300 \;-\; 50 \;=\; 250.$$

Hence

$$V_5^{u,n} - \delta E_5^{u,n} \;=\; V_5^{d,n} - \delta E_5^{d,n} \qquad \Rightarrow \qquad \delta \;=\; \frac{V_5^{u,n} - V_5^{d,n}}{E_5^{u,n} - E_5^{d,n}} \;=\; \frac{300 - 33.33}{250 - 0} \;=\; 1.0667 \;.$$

This portfolio being risk free, its present value is given by

$$V \;-\; \delta E^n \;=\; \frac{V_5^{u,n} - \delta E_5^{u,n}}{(1 + r_f)^5} \;,$$

and thus

$$E^n \;=\; \frac{1}{\delta}\left(V - \frac{V_5^{u,n} - \delta E_5^{u,n}}{(1 + r_f)^5}\right) \;=\; \frac{1}{1.0667}\left(100 - \frac{33.33}{(1.05)^5}\right) \;=\; \$69.26 \;,$$

and the new value of the firm's debt is

$$D^n \;=\; V \;-\; E^n \;=\; 100 \;-\; 69.26 \;=\; 30.74 \;.$$

That is, this riskier strategy has increased the firm's equity by

$$E^n \;-\; E^o \;=\; 69.26 \;-\; 60.82 \;=\; \$8.44$$

and has decreased the firm's debt value by

$$D^o \;-\; D^n \;=\; 38.18 \;-\; 30.74 \;=\; \$8.44 \;.$$

3 Paying an Immediate Dividend to Shareholders

Values are the same as in the first example, i.e. $V = 100$, $X^o = 50$, $V_5^u = 200$, $p = 0.7$, $V_5^d = 50$, $r_f = 5\%$, $D^o = \$39.18$ and $E^o = \$60.82$.

Suppose that management has decided to pay a dividend $Q = \$25$ to the firm's shareholders. We will see two cases here, one where the money comes from idle cash and one where some assets are sold to finance the dividend.

3.1 Dividend Paid out of Idle Cash

Suppose that Q would earn the risk-free rate if it were not paid as a dividend. The firm's value in five years is now either

$$V_5^{u,n} \ = \ 200 \ - \ 25 \times (1.05)^5 \ = \ \$168.09 \quad \text{or} \quad V_5^{d,n} \ = \ 50 \ - \ 25 \times (1.05)^5 \ = \ \$18.09 \ .$$

As before, we find the new value of the firm's equity, E^n, by forming a risk-free portfolio that buys the firm's assets and sells short a fraction δ of its equity, such that

$$V_5^{u,n} \ - \ \delta E_5^{u,n} \ = \ V_5^{d,n} \ - \ \delta E_5^{d,n} \ .$$

In this case, $E_5^{u,n} \ = \ 168.09 - 50 = 118.09$ and, as before, $E_5^{d,n} = 0$, which gives us

$$\delta \ = \ \frac{V_5^{u,n} - V_5^{d,n}}{E_5^{u,n} - E_5^{d,n}} \ = \ \frac{168.09 \ - \ 18.09}{118.09} \ = \ 1.2702 \ .$$

The firm's value being now $V^n = 100 - 25 = 75$, the firm's equity is now worth

$$E^n \ = \ \frac{1}{\delta}\left(V^n - \frac{V_5^{u,n} - \delta E_5^{u,n}}{(1+r_f)^5}\right) \ = \ \frac{1}{1.2702}\left(75 - \frac{18.09}{(1.05)^5}\right) \ = \ \$47.89 \ ,$$

and the new value of the firm's debt is

$$D^n \ = \ 75 \ - \ 47.89 \ = \ \$27.11 \ .$$

That is, shareholders' wealth went up by

$$E^n \ + \ Q \ - \ E^o \ = \ 47.89 \ + \ 25 \ - \ 60.82 \ = \ \$12.07$$

and bondholders' wealth went down by the same amount.

3.2 Selling Assets to Pay the Dividend

Suppose the dividend $Q = \$25$ is obtained by selling 25% of the firm's assets, which means that we now have

$$V_5^{u,n} \ = \ .75 \times 200 \ = \ 150 \quad \text{and} \quad V_5^{d,n} \ = \ .75 \times 50 \ = \ 37.5 \ .$$

Our risk-free portfolio is now such that

$$\delta \;=\; \frac{V_5^{u,n} - V_5^{d,n}}{E_5^{u,n} - E_5^{d,n}} \;=\; \frac{150 - 37.5}{100} \;=\; 1.125 \;,$$

and thus

$$E^n \;=\; \frac{1}{\delta}\left(V^n - \frac{V_5^{u,n} - \delta E_5^{u,n}}{(1+r_f)^5}\right) \;=\; \frac{1}{1.125}\left(75 - \frac{37.5}{(1.05)^5}\right) \;=\; \$40.55 \;.$$

Shareholders' wealth has then increased by

$$E^n \;+\; Q \;-\; E^o \;=\; 40.55 \;+\; 25 \;-\; 60.82 \;=\; \$4.73 \;,$$

whereas bondholders' wealth has decreased by the same amount.