Akerlof's Market for Lemons

A used car is either in bad condition (lemon) or in good condition (jewel)

$p \equiv$ fraction of used cars in bad condition
$1 - p \equiv$ fraction of used cars in good condition

Buyers and sellers valuations:

$$
\begin{aligned}
v_b^l &\equiv \text{maximum price a buyer will pay for a lemon} \\
v_b^j &\equiv \text{maximum price a buyer will pay for a jewel } (v_b^j > v_b^l) \\
v_s^l &\equiv \text{minimum price the seller of a lemon will accept} \\
v_s^j &\equiv \text{minimum price the seller of a jewel will accept } (v_s^j > v_s^l)
\end{aligned}
$$

There is a market for used cars if

$$v_b^l > v_s^l \quad \text{and} \quad v_b^j > v_s^j.$$

Information asymmetry: Only car owners can tell whether they have a lemon or a jewell.

Moral hazard: Since $v_b^j > v_b^l$, lemon owners have an incentive not to reveal the true quality of their car.

If both types of used car are for sale, the maximum price a buyer will pay for a used car is

$$pv_b^l \; + \; (1-p)v_b^j.$$

If this price is below jewel owners' valuation, i.e. if

$$pv_b^l \; + \; (1-p)v_b^j \; < \; v_s^j,$$

then jewel owners won't try to sell their car.

Adverse selection: Information asymmetry and moral hazard have driven jewel sellers out of the market.

Buyers will figure out that only lemons are for sale and thus won't pay more than v_b^l for a used car.

Equilibrium:

- Only lemons are for sale
- Used cars trade at prices between v_s^l and v_b^l.

Market failure: Jewels are not sold even though $v_s^j < v_b^j$.

Example:

$v_b^l = \$1,200\ , \quad v_b^j = \$2,400\ , \quad v_s^l = \$1,000\ , \quad v_s^j = \$2,000\ .$

50% of used cars are lemons.
If both lemons and jewels are for sale, a buyer will pay at most

$$.5 \times 1,200 \ + \ .5 \times 2,400 \ = \ \$1,800$$

for a used car. Jewel owners want at least \$2,000 so none of them will offer his/her car. Buyers will figure that out and thus won't pay more than \$1,200 for a used car.

Equilibrium:

- Only lemons are for sale
- Price of a used car is between \$1,000 and \$1,200.

Adverse Selection in Stock Markets

Suppose the value of a stock is either $v = H$ or $v = L$, where $H > L$.

A trader is either informed (knows what v is) or uninformed.

Uninformed traders believe that $v = H$ with probability p and $v = L$ with probability $1 - p$.

Traders submit orders to a market maker who sets bid and ask prices such that his expected trading profit is zero. The market maker does not know v and has the same beliefs as uninformed traders. All trade orders have the same size.

Let B denote the bid price
Let A denote the ask price

If a trader wants to sell Q shares of the stock, the market maker buys them at the price B/share. The market maker's payoff is

$$Q \times E[v|\text{Market maker's information}] \; - \; Q \times B,$$

where the market maker's information is the occurrence of a sell order.

Market maker's zero-profit condition:

$$B \; = \; E[v|\text{sell order}] \quad \text{and} \quad A \; = \; E[v|\text{buy order}]$$

The market maker observes trades but does not know whether or not a trader is informed.

The market maker believes that a trader is informed with probability q and uninformed with probability $1 - q$.

We can assume that q is the fraction of informed traders in the population.

Notation:

$$\begin{aligned} E[v|I] &= \text{expected value of } v \text{ given information } I. \\ &= \text{Pr}(v = H|I) \times H \ + \ \text{Pr}(v = L|I) \times L \end{aligned}$$

where

$$\begin{aligned} \text{Pr}(v = H|I) &\equiv \text{probability that } v = H \text{ given information } I \\ \text{Pr}(v = L|I) &\equiv \text{probability that } v = L \text{ given information } I \end{aligned}$$

Bayes' rule:

$$\text{Pr}(v = H|I) = \frac{\text{Pr}(v = H)\,\text{Pr}(I|v = H)}{\text{Pr}(v = H)\,\text{Pr}(I|v = H) + \text{Pr}(v = L)\,\text{Pr}(I|v = L)}$$

$$\text{Pr}(v = L|I) = \frac{\text{Pr}(v = L)\,\text{Pr}(I|v = L)}{\text{Pr}(v = H)\,\text{Pr}(I|v = H) + \text{Pr}(v = L)\,\text{Pr}(I|v = L)}$$

The ask price A prevails when a buy order comes in.

$$\begin{aligned} A &= E[v|\text{buy order}] \\ &= \Pr(v = H|\text{buy order}) \times H \;+\; \Pr(v = L|\text{buy order}) \times L \end{aligned}$$

From Bayes' rule,

$$\Pr(v = H|\text{buy order})$$
$$= \frac{\Pr(v = H)\Pr(\text{buy order}|v = H)}{\Pr(v = H)\Pr(\text{buy order}|v = H) + \Pr(v = L)\Pr(\text{buy order}|v = L)}$$

and

$$\Pr(v = L|\text{buy order})$$
$$= \frac{\Pr(v = L)\Pr(\text{buy order}|v = L)}{\Pr(v = H)\Pr(\text{buy order}|v = H) + \Pr(v = L)\Pr(\text{buy order}|v = L)}$$

What is Pr(buy order$|v = H$)?

$$\text{Pr(buy order}|v = H) = $$
$$\text{Pr(trader is informed)} \times \text{Pr(an informed trader buys}|v = H) +$$
$$\text{Pr(trader is uninformed)} \times \text{Pr(an uninformed trader buys}|v = H)$$

Let α denote the probability that an uninformed trader submit a buy order, regardless of the value of v (he does not know it anyway). We will figure out what α is later.

This gives us

$$\text{Pr(buy order}|v = H) = q \times 1 + (1 - q)\alpha = q + (1 - q)\alpha.$$

Similarly,

$$\begin{aligned}\text{Pr(buy order}|v = L) \ &= \\ &\text{Pr(trader is informed)} \times \text{Pr(an informed trader buys}|v = L) \ + \\ &\ \text{Pr(trader is uninformed)} \times \text{Pr(an uninformed trader buys}|v = L)\end{aligned}$$

This gives us

$$\text{Pr(buy order}|v = L) \ = \ q \times 0 \ + \ (1 - q)\alpha \ = \ (1 - q)\alpha.$$

Plugging these values into our equation for $\text{Pr}(v = H|\text{buy order})$, we obtain

$$\frac{\overbrace{\text{Pr}(v = H)}^{p}\,\overbrace{\text{Pr}(\text{buy order}|v = H)}^{q+(1-q)\alpha}}{\underbrace{\text{Pr}(v = H)}_{p}\,\underbrace{\text{Pr}(\text{buy order}|v = H)}_{q+(1-q)\alpha} + \underbrace{\text{Pr}(v = L)}_{1-p}\,\underbrace{\text{Pr}(\text{buy order}|v = L)}_{(1-q)\alpha}},$$

i.e.

$$\begin{aligned}\text{Pr}(v = H|\text{buy order}) &= \frac{p(q + (1-q)\alpha)}{p(q + (1-q)\alpha) + (1-p)(1-q)\alpha} \\ &= \frac{p(q + (1-q)\alpha)}{pq + (1-q)\alpha}\end{aligned}$$

Similarly, for Pr($v = L$|buy order), we have

$$\frac{\overbrace{\text{Pr}(v = L)}^{1-p}\,\overbrace{\text{Pr}(\text{buy order}|v = L)}^{(1-q)\alpha}}{\underbrace{\text{Pr}(v = H)}_{p}\,\underbrace{\text{Pr}(\text{buy order}|v = H)}_{q+(1-q)\alpha} + \underbrace{\text{Pr}(v = L)}_{1-p}\,\underbrace{\text{Pr}(\text{buy order}|v = L)}_{(1-q)\alpha}}$$

and thus

$$\begin{aligned}\text{Pr}(v = L|\text{buy order}) &= \frac{(1-p)(1-q)\alpha}{p(q+(1-q)\alpha)+(1-p)(1-q)\alpha} \\ &= \frac{(1-p)(1-q)\alpha}{pq+(1-q)\alpha}\end{aligned}$$

Therefore,

$$\begin{aligned} A &= E[v|\text{buy order}] \\ &= \frac{p(q+(1-q)\alpha)}{pq+(1-q)\alpha} \times H \; + \; \frac{(1-p)(1-q)\alpha}{pq+(1-q)\alpha} \times L \end{aligned}$$

Note that

$$\frac{p(q+(1-q)\alpha)}{pq+(1-q)\alpha} \; > \; p,$$

and thus

$$A \; = \; E[v|\text{buy order}] \; > \; pH \; + \; (1-p)L \; = \; E[v].$$

Since uninformed traders believe that the stock is worth

$$E[v] \;=\; pH \;+\; (1-p)L \;<\; A,$$

they won't submit buy orders ($\alpha = 0$).

The market maker will figure that out and thus

$$A \;=\; E[v|\text{buy order}] \;=\; H.$$

The same reasoning applies to sell orders:

$$E[v|\text{sell order}] = \Pr(v = H|\text{sell order}) \times H + \Pr(v = L|\text{sell order}) \times L$$

Using Bayes' rule again, we have

$$\begin{aligned} &\Pr(v = H|\text{sell order}) \\ &\qquad = \frac{\Pr(v = H)\Pr(\text{sell order}|v = H)}{\Pr(v = H)\Pr(\text{sell order}|v = H) + \Pr(v = L)\Pr(\text{sell order}|v = L)} \end{aligned}$$

and

$$\begin{aligned} &\Pr(v = L|\text{sell order}) \\ &\qquad = \frac{\Pr(v = L)\Pr(\text{sell order}|v = H)}{\Pr(v = H)\Pr(\text{sell order}|v = H) + \Pr(v = L)\Pr(\text{sell order}|v = L)} \end{aligned}$$

Let γ represent the probability that an uninformed trader submit a sell order. We will find what γ is later.
In this case, we have

$$\begin{aligned} B &= E[v|\text{sell order}] \\ &= \frac{p(1-q)\gamma}{q(1-p)+(1-q)\gamma} \times H \;+\; \frac{(1-p)(q+(1-q)\gamma)}{q(1-p)+(1-q)\gamma} \times L \end{aligned}$$

Note that

$$\frac{p(1-q)\gamma}{q(1-p)+(1-q)\gamma} \;<\; p,$$

and thus

$$B \;=\; E[v|\text{sell order}] \;<\; pH \;+\; (1-p)L \;=\; E[v].$$

Will uninformed traders submit sell orders?

Uninformed traders believe that the stock is worth

$$E[v] \;=\; pH \;+\; (1-p)L \;>\; B$$

so they won't submit sell orders either ($\gamma = 0$).

The market maker will figure that out and will set

$$B \;=\; L.$$

Equilibrium

- Uninformed traders don't trade (adverse selection);
- Ask price is $A = H$;
- Bid price is $B = L$.

Suppose we add *liquidity traders*.

Liquidity traders submit buy orders with probability β and sell orders with probability $1-\beta$ (regardless of v).

$q_i \equiv$ fraction of traders who are informed
$q_u \equiv$ fraction of traders who are uninformed
$1-q_i-q_u \equiv$ fraction of traders who trade for liquidity reasons

In this case,

$$\begin{aligned}\text{Pr(buy order}|v = H) \;&=\; q_i\,\text{Pr(informed buys}|v = H)\\ &\quad+\; q_u\,\text{Pr(uninformed buys}|v = H)\\ &\quad+\; (1 - q_i - q_u)\beta\end{aligned}$$

Let α denote the probability that an uninformed trader sumit a buy order.

Then

$$\text{Pr(buy order}|v = H) \;=\; q_i \;+\; q_u\alpha \;+\; (1 - q_i - q_u)\beta$$

Similarly,

$$\text{Pr(buy order}|v = L) \;=\; q_u\alpha \;+\; (1 - q_i - q_u)\beta,$$

which gives us

$$\begin{aligned} A &= E[v|\text{buy order}] \\ &= \frac{p(q_i \;+\; q_u\alpha \;+\; (1 - q_i - q_u)\beta)}{pq_i \;+\; q_u\alpha \;+\; (1 - q_i - q_u)\beta} \times H \\ &\qquad\qquad + \frac{(1-p)(q_u\alpha \;+\; (1 - q_i - q_u)\beta)}{pq_i \;+\; q_u\alpha \;+\; (1 - q_i - q_u)\beta} \times L \end{aligned}$$

As in the previous case, we have

$$\frac{p(q_i \ + \ q_u\alpha \ + \ (1-q_i-q_u)\beta)}{pq_i \ + \ q_u\alpha \ + \ (1-q_i-q_u)\beta} \ > \ p$$

and thus

$$A \ = \ E[v|\text{buy order}] \ > \ pH + (1-p)L \ = \ E[v],$$

so uninformed traders don't submit buy orders in this case either ($\alpha = 0$).

Nor will they submit sell orders. To participate, uninformed traders must have beliefs that differ from those of the market maker.

With liquidity traders, equilibrium bid (B) and ask (A) prices are

$$
\begin{aligned}
A &= \frac{p(q_i + (1 - q_i - q_u)\beta)}{pq_i + (1 - q_i - q_u)\beta} \times H \\
&\qquad + \frac{(1-p)(1 - q_i - q_u)\beta}{pq_i + (1 - q_i - q_u)\beta} \times L \\
B &= \frac{p(1 - q_i - q_u)(1-\beta)}{(1-p)q_i + (1 - q_i - q_u)(1-\beta)} \times H \\
&\qquad + \frac{(1-p)(q_i + (1 - q_i - q_u)(1-\beta))}{(1-p)q_i + (1 - q_i - q_u)(1-\beta)} \times L
\end{aligned}
$$

Note that

$$L \leq B < E[v] < A \leq H$$

The greater β, i.e. the greater the probability that liquidity traders submit buy orders, the lower A and B.

For example, suppose $\beta = 1$ (liquidity traders only submit buy orders).
Then

$$B = L \quad \text{and} \quad A = \frac{p(1-q_u)H + (1-p)(1-q_i-q_u)L}{pq_i + 1 - q_i - q_u}$$

Remarks:

- The greater q_i (the fraction of traders who are informed), the greater the difference between A and B.

- Bid-ask spreads also help market makers insure against the risk of holding inventories of stocks.

- Adverse selection also a problem in the insurance market: Bad risks are more likely to buy insurance than good risks.

How to solve the adverse selection problem?

- *Screening*: Group insurance
- *Costly Signaling*: Education
- *Reputation*
- *Contract Enforcement*
- *Guarantees*

Signaling the Quality of a Used Car with a Warranty

Consider a used car market with $v_b^l = \$1,200$, $v_b^j = \$2,400$, $v_s^l = \$1,000$, $v_s^j = \$2,000$ and $p = 0.5$.

Suppose that a bumper-to-bumper warranty would cost, on average, \$100/year to a jewell owner and \$500/year to a lemon owner.

Can a jewel owner effectively signal the quality of his car by offering a one-year bumper-to-bumper warranty?

If buyers believe that all cars with a one-year warranty are jewels, then they are willing to pay up to $2,400 for a car with a warranty.

Knowing that, will lemon owners imitate jewel owners and offer a one-year warranty?

If a lemon owner offers a one-year warranty, he can expect to earn up to

$$2,400 \; - \; 1,000 \; - \; 500 \; = \; \$900,$$

which is greater than $1,200-1,000 = \$200$, his maximum payoff without a warranty.

Lemon ownwers thus have incentives to imitate jewel owners and offer a one-year warranty, which is therefore ineffective at signaling high-quality cars.

What about a two-year warranty?

Assuming that buyers believe that all cars with a two-year warranty are jewels, a lemon owner offering a two-year can expect to earn up to

$$2,400 - 1,000 - 1,000 = \$400,$$

which is again greater than his maximum payoff without a warranty.

A two-year warranty is also ineffective at signaling high-quality cars.

What about a three-year warranty?

Assuming that buyers believe that all cars with a three-year warranty are jewels, a lemon owner offering a two-year can expect to earn up to

$$2,400 \; - \; 1,000 \; - \; 1,500 \; = \; -\$100,$$

and thus is not beneficial for a lemon owner to offer a three-year warranty.

A three-year warranty is therefore effective at signaling high-quality cars.

Note that with a three-year warranty, jewel owners expect to make at most $2,400 - 2,000 - 300 = \$100 > 0$, and thus they can afford this warranty.

Corporate Investment and Capital Structure

The profitability of an existing company is either $\pi = H$ or $\pi = L$, where $0 < L < H$.

The company is owned by an entrepreneur who knows the value of π.

Individuals other than the entrepreneur believe that

$$\pi = \begin{cases} H & \text{with probability } p, \\ L & \text{with probability } 1 - p. \end{cases}$$

Suppose the entrepreneur has access to a profitable project but cannot finance it by himself. The project requires an initial investment I and its payoff is given by

$$R \; > \; (1+r)I,$$

where r is the rate of return required by investors for similar projects.

To finance the project, the entrepreneur has to raise equity or issue debt.

The Entrepreneur Raises Equity

Suppose the entrepreneur offers an equity stake s to a potential investor, where $0 \leq s \leq 1$.

Upon observing s, the investor decides whether or not to invest in the firm.

If the offer is rejected, the entrepreneur's payoff is π and the investor's payoff $I(1+r)$.

If the offer is accepted, the entrepreneur's payoff is $(1-s)(\pi+R)$ and the investor's payoff is $s(\pi+R)$.

Upon observing s, the investor believes that

$$\pi = \begin{cases} H & \text{with probability } q, \\ L & \text{with probability } 1-q \end{cases}$$

(i.e. the investor updates his belief from p to q).

The investor will invest only if

$$s(qH + (1-q)L + R) \geq I(1+r) \quad \Rightarrow \quad s \geq \frac{I(1+r)}{qH + (1-q)L + R}.$$

The entrepreneur is better off raising equity than not undertaking the project if s is such that

$$(1-s)(\pi+R) \;\geq\; \pi \qquad \Rightarrow \qquad s \;\leq\; \frac{R}{\pi+R}\ .$$

That is,

$$s \;\leq\; \begin{cases} \dfrac{R}{H+R} & \text{if } \pi = H, \\[2ex] \dfrac{R}{L+R} & \text{if } \pi = L. \end{cases}$$

Note that since $L < H$,

$$\frac{R}{H+R} \;<\; \frac{R}{L+R}\ .$$

Were the high-profit entrepreneur able to differentiate himself from the low-profit type, he could offer equity stakes as low as $\frac{I(1+r)}{H+R}$.

The low-profit entrepreneur has an inventive to mimick any offer made by the high-profit type that is accepted by the investor since any share s that is profitable for the high-profit type is also profitable for the low-profit type.

When the investor is convinced that $\pi = L$, i.e. if $q = 0$, then he will accept to invest only if $s \geq \frac{I(1+r)}{L+R}$.

It is always profitable for the low-profit entrepreneur to make an offer that will be accepted by the investor, regardless of q, since

$$\frac{I(1+r)}{qH+(1-q)L+R} \leq \frac{I(1+r)}{L+R} < \frac{R}{L+R} .$$

So it is not in the interest of the low-profit entrepreneur to mimick the high-profit entrepreneur's offer when the latter is rejected by the investor.

A "pooling" equilibrium is such that both types of entrepreneur offer the same equity stake, which is accepted by the entrepreneur. In this case, the investor's updated belief is simply $q = p$ since then s does not contain any information about the firm's quality.

The equity stake being accepted implies that

$$s \;\geq\; \frac{I(1+r)}{pH + (1-p)L + R} \;.$$

For this offer to benefit the high-profit entrepreneur, we must also have $s \leq \frac{R}{H+R}$, and thus a pooling equilibrium exists only if

$$\frac{I(1+r)}{pH + (1-p)L + R} \;\leq\; \frac{R}{H+R} \;.$$

Note that the last condition holds as p approaches one since we have assumed that $R > I(1 + r)$.

In a pooling equilibrium, we have

$$\frac{I(1 + r)}{pH + (1 - p)L + R} \;\leq\; s \;\leq\; \frac{R}{H + R} \;.$$

In this equilibrium, the high-profit type subsidizes the low-profit type: Were the high-profit entrepreneur able to convince the investor that $\pi = H$, the former could offer equity stakes as low as $\frac{I(1+r)}{H+R}$, whereas the low-profit entrepreneur would have to offer stakes at least as large as $\frac{I(1+r)}{L+R}$.

If, on the other hand, we have

$$\frac{I(1+r)}{pH+(1-p)L+R} \; > \; \frac{R}{H+R} \; ,$$

then the high-profit entrepreneur chooses to forego the project.

An equilibrium in this case is such that the high-profit entrepreneur offers $s < \frac{I(1+r)}{H+R}$, which is rejected by the investor, and the low-profit type offers $s = \frac{I(1+r)}{L+R}$, which is accepted by the investor.

This is a “separating” equilibrium.

Conclusion

- If the high-profit entrepreneur finds profitable to raise equity, then the low-profit type mimicks his offer and we have a pooling equilibrium.

- If the high-profit type does not find the pooling equilibrium profitable, then only the low-profit type raises equity, and this at a higher cost of capital.