Business 3019

Assignment 2

Due Wednesday January 29, 2003, 4:00pm (beginning of class)

1. A firm's short-run revenue is given by $R(e) = 10e - e^2$, where e is the level of effort by a typical manager (all managers are assumed to be identical). A manager chooses his level of effort to maximize his wage net of effort $w - e$ (the per-unit cost of effort is assumed to be 1). Determine the level of effort and the level of profit (revenue less wage paid) for each of the following wage arrangements. Explain why these different principal-agent relationships generate different outcomes.

   (a) $w = 2$ for $e \geq 1$; otherwise $w = 0$.

   (b) $w = R(e)/2$.

   (c) $w = R(e) - 12.5$.

2. Consider a firm whose total revenue depends on the effort provided by each of its five managers in the following manner:

$$R(e_1, e_2, e_3, e_4, e_5) = \sum_{n=1}^{5} e_n,$$

   where e_n denotes Manager n's effort level and $R(\cdot)$ denotes the firm's total revenue. The dollar cost of effort to a manager is given by the function

$$c(e) = \frac{e^2}{100}.$$

   (a) Suppose that effort can be perfectly monitored and that each manager is paid according to his performance. That is, Manager n's pay if he works e_n is given by

we_n, where w is the wage rate. Calculate the wage rate that maximizes the firm's profits and the effort level, given this wage rate, that maximizes each manager's payoff. What are then the firm's profits?

(b) Suppose now that the firm cannot monitor the effort of all of its managers. That is, the firm can only monitor the effort of one of its managers and each manager has an equal chance of being monitored during the period. Suppose also that each manager can earn a wage of \$6.25 without effort outside the firm if they get fired. What wage must the firm pay to induce an effort level of $e^* = 25$ from each of its managers? What are then the firm's profits? Answer these questions with $e^* = 20$ and $e^* = 10$.

(c) Suppose now that effort levels cannot be monitored at all. To solve this problem, the firm puts in place a revenue-sharing plan that gives each manager one tenth of the firm's total revenue at the end of the period (and no other compensation). What will be the effort provided by each manager under this plan? What are then the firm's profits? (Assume for this question that managers have no outside option.)

(d) Suppose again that effort levels cannot be monitored at all but that the firm wants to solve this problem with a forcing contract as follows: If the firm's total revenue is \$125 or more, then each manager will be paid a wage of \$12.5. If the firm's total revenue is less than \$125, then managers don't get paid at all. Compute at least three possible equilibria for this case and calculate the firm's profits in each of them. (Assume for this question that managers have no outside option.)