

Business 2019

Fundamentals of Finance, Chapter 6

Solution to Selected Problems

8. **Calculating Annuity Values** You want to have \$50,000 in your savings account five years from now, and you're prepared to make equal annual deposits at the end of each year. If the account pays 9.5 percent interest, what amount must you deposit each year?

Answer: Let C denote the amount deposited each year. Payments are made at the end of each period, so the value of the first payment in five years is $C(1.095)^4$, the value of the second payment in five years is $C(1.095)^3$, etc.. That is, C is such that

$$\begin{aligned} \text{FV} = 50,000 &= C(1.095)^4 + C(1.095)^3 + C(1.095)^2 + C(1.095)^1 + C \\ &= C \left[\frac{(1.095)^5 - 1}{.095} \right], \end{aligned}$$

which implies

$$C = \frac{50,000}{((1.095)^5 - 1) / .095} = \$8,271.82 .$$

9. **Calculating Annuity Values** Betty's Bank offers you a \$25,000, seven-year term loan at 11 percent annual interest repayable in equal annual amounts. What will your annual loan payment be?

Answer: The payment is such that

$$\begin{aligned} PV = 25,000 &= \frac{C}{1.11} + \frac{C}{(1.11)^2} + \dots + \frac{C}{(1.11)^7} \\ &= C \left[\frac{1 - \frac{1}{(1.11)^7}}{.11} \right]. \end{aligned}$$

Hence

$$C = \frac{25,000}{\left(1 - \frac{1}{(1.11)^7}\right) / .11} = \$5,305.38 .$$

10. **Calculating Perpetuity Values** Bob's Life Insurance Co. is trying to sell you an investment policy that will pay you and your heirs \$1,000 per year forever. If the required return on this investment is 12 percent, how much will you pay for a fair deal?

Answer: At a 12 percent discount rate, this perpetuity is worth

$$\frac{1,000}{.12} = \$8333.33 .$$

12. **Calculating EAR** Find the EAR in each of the following cases:

APR	Compounded	Effective Rate (EAR)
8%	Quarterly	$\Rightarrow \text{EAR} = \left(1 + \frac{.08}{4}\right)^4 - 1 = 8.24\%$
12%	Monthly	$\Rightarrow \text{EAR} = \left(1 + \frac{.12}{12}\right)^{12} - 1 = 12.68\%$
6%	Daily	$\Rightarrow \text{EAR} = \left(1 + \frac{.06}{365}\right)^{365} - 1 = 6.18\%$
21%	Continuously	$\Rightarrow \text{EAR} = e^{.21} - 1 = 23.37\%$

13. **Calculating APR** Find the APR, or stated rate, in each of the following cases:

APR	Compounded	Effective Rate (EAR)
	Semiannually	6%
	Monthly	9%
	Weekly	17%
	Continuously	24%

Answer: When interest is compounded m times throughout the year, we have

$$\begin{aligned} \text{EAR} &= \left(1 + \frac{\text{APR}}{m}\right)^m - 1 \quad \Rightarrow \quad 1 + \text{EAR} = \left(1 + \frac{\text{APR}}{m}\right)^m \\ &\Rightarrow \quad (1 + \text{EAR})^{1/m} = 1 + \frac{\text{APR}}{m} \\ &\Rightarrow \quad (1 + \text{EAR})^{1/m} - 1 = \frac{\text{APR}}{m} \\ &\Rightarrow \quad m \left((1 + \text{EAR})^{1/m} - 1\right) = \text{APR} . \end{aligned}$$

When interest is continuously compounded, we have

$$\text{EAR} = e^{\text{APR}} - 1 \quad \Rightarrow \quad \text{APR} = \ln(1 + \text{EAR}).$$

Hence

$$\begin{aligned} \text{EAR} = 6\%, \quad m = 2, \quad &\Rightarrow \quad \text{APR} = 2 \left((1 + .06)^{1/2} - 1\right) = 5.91\% \\ \text{EAR} = 9\%, \quad m = 12, \quad &\Rightarrow \quad \text{APR} = 12 \left((1 + .09)^{1/12} - 1\right) = 8.65\% \\ \text{EAR} = 17\%, \quad m = 52, \quad &\Rightarrow \quad \text{APR} = 52 \left((1 + .17)^{1/52} - 1\right) = 15.72\% \\ \text{EAR} = 24\%, \quad m = \infty, \quad &\Rightarrow \quad \text{APR} = \ln(1 + .24) = 21.51\% . \end{aligned}$$

21. **Calculating the Number of Periods** One of your customers is delinquent on his accounts payable balance. You've mutually agreed to a repayment schedule of \$300 per month. You will charge 1.5 percent per month interest on the overdue balance. If the current balance is \$12,054.24, how long will it take for the account to be paid off?

Answer: Let T denote the number of months. Then we need

$$\begin{aligned}12,054.24 &= 300 \left(\frac{1 - \frac{1}{(1.015)^T}}{0.015} \right) &\Rightarrow & \frac{12,054.24}{300/.015} = 1 - \frac{1}{(1.015)^T} \\&&\Rightarrow & .6027 = 1 - \frac{1}{(1.015)^T} \\&&\Rightarrow & \frac{1}{(1.015)^T} = 1 - .6027 \\&&\Rightarrow & \frac{1}{(1.015)^T} = .3973 \\&&\Rightarrow & (1.015)^T = \frac{1}{.3973} \\&&\Rightarrow & (1.015)^T = 2.5170 \\&&\Rightarrow & \ln((1.015)^T) = \ln(2.5170) \\&&\Rightarrow & T \ln(1.015) = \ln(2.5170) \\&&\Rightarrow & T = \frac{\ln(2.5170)}{\ln(1.015)} \\&&\Rightarrow & T = 62 \text{ months.}\end{aligned}$$

23. **Valuing Perpetuities** Maybepay Life Insurance Co. is selling a perpetual annuity contract that pays \$625 monthly. The contract currently sells for \$50,000. What is the monthly return on this investment vehicle? What is the APR? the EAR?

Answer: Let r denote the monthly return. Then

$$50,000 = \frac{625}{r} \quad \Rightarrow \quad r = \frac{625}{50,000} = 1.25\%.$$

The APR is then $12 \times 1.25\% = 15\%$, and the EAR is

$$\text{EAR} = (1.0125)^{12} - 1 = 16.08\%.$$

30. **Calculating EAR** You are looking at an investment that has an effective annual rate of 18 percent. What is the effective semiannual return? The effective quarterly return? The effective monthly return?

Answer: Let r_s denote the effective semiannual return. Then

$$(1 + r_s)^2 - 1 = 0.18 \quad \Rightarrow \quad r_s = ((1.18)^{1/2} - 1) = 8.63\%.$$

Let r_q denote the quarterly return. Then

$$(1 + r_q)^4 - 1 = 0.18 \quad \Rightarrow \quad r_q = ((1.18)^{1/4} - 1) = 4.22\%.$$

Let r_m denote the monthly return. Then

$$(1 + r_m)^{12} - 1 = 0.18 \quad \Rightarrow \quad r_m = ((1.18)^{1/12} - 1) = 1.39\%.$$

31. **Calculating Interest Expense** You receive a credit card application from Shady Banks Savings and Loan offering an introductory rate of 5.90 percent per year, compounded monthly for the first six months, increasing thereafter to 19 percent compounded monthly. Assuming you transfer the \$3,000 balance from your existing credit card and make no subsequent payments, how much interest will you owe at the end of the year?

Answer: The interest owed at the end of the year is calculated as

$$3,000 \times \left(1 + \frac{0.059}{12}\right)^6 \times \left(1 + \frac{0.19}{12}\right)^6 - 3,000 = \$394.97 .$$

32. **Calculating the Number of Periods** You are saving to buy a \$100,000 house. There are two competing banks in your area, both offering certificates of deposit yielding 5 percent. How long will take your initial \$75,000 investment to reach the desired level at First Bank, which pays simple interest? How long at Second Bank, which compounds interest monthly?

Answer: Let T_{FB} denote the number of years at First Bank. Then

$$75,000 + 75,000 \times T_{FB} \times .05 = 100,000 \quad \Rightarrow \quad T_{FB} = 6.7 \text{ years.}$$

Let T_{SB} denote the number of years at Second Bank. Then

$$75,000 \times \left(1 + \frac{.05}{12}\right)^{12T_{SB}} = 100,000 \quad \Rightarrow \quad T_{SB} = \frac{\ln(100/75)}{12 \ln(1 + .05/12)} = 5.8 \text{ years.}$$

35. **Calculating Rates of Return** Suppose an investment offers to quadruple your money in 24 months. What rate of return per quarter are you being offered?

Answer: Let r_q denote the quarterly return. There are 8 quarters in two years and thus

$$(1 + r_q)^8 = 4 \quad \Rightarrow \quad r_q = 4^{1/8} - 1 = 18.9\%.$$

36. **Comparing Cash Flow Streams** You've just joined the investment banking firm of Peng, Yi and Lee. They've offered you two different salary arrangements. You can have \$75,000 per year for the next two years, or you can have \$55,000 per year for the next two years, along with a \$30,000 signing bonus today. If the interest rate is 12 percent, compounded monthly, which one do you prefer?

Answer: An annual interest rate of 12% compounded monthly means an effective interest rate of $(1 + .12/12)^{12} - 1 = 12.68\%$. Let S_1 denote the first stream, and let S_2 denote the second. Then

$$\begin{aligned} S_1 &= \frac{75,000}{1.1268} + \frac{75,000}{(1.1268)^2} \\ &= 75,000 \times \left(\frac{1 - \left(\frac{1}{1.1268}\right)^2}{0.1268} \right) = \$125,620.25, \end{aligned}$$

whereas

$$\begin{aligned} S_2 &= 30,000 + \frac{55,000}{1.1268} + \frac{55,000}{(1.1268)^2} \\ &= 30,000 + 55,000 \times \left(\frac{1 - \left(\frac{1}{1.1268}\right)^2}{0.1268} \right) = \$122,128.85. \end{aligned}$$

Thus you should choose the first salary arrangement.

45. **Present and Break-even Interest** Consider a firm with a contract to sell an asset for \$75,000 three years from now. The asset costs \$36,000 to produce today. Given a relevant discount rate on this asset of 12 percent per year, will the firm make a profit on this asset? At what rate does the firm just break even?

Answer: The present value of \$75,000 to be received in three years, with a 12% discount rate, is

$$\frac{75,000}{(1.12)^3} = \$53,383.52 > \$36,000.$$

Yes, the firm will make a profit on this asset. Let r_b denote the discount rate at which the firm breaks even. Then

$$\frac{75,000}{(1+r_b)^3} = \$36,000 \quad \Rightarrow \quad r_b = \left(\frac{75,000}{36,000}\right)^{1/3} - 1 = 27.7\% .$$

49. **Variable Interest Rates** A 10-year annuity pays \$1,500 per month, and payments are made at the end of each month. If the interest rate is 18 percent compounded monthly for the first four years, and 16 percent compounded monthly thereafter, what is the present value of the annuity?

Answer: The monthly interest rate is $\frac{18}{12} = 1.5\%$ for the first 48 months and $\frac{16}{12} = .04/3$ for the last 72 months. Hence the present value factor for each payment received after the 48th month is given by

$$\frac{1}{(1.015)^{48} \times (1 + .04/3)^{t-48}} ,$$

where t represents the month. Hence the present value of this annuity is given by

$$\begin{aligned} \text{PV} &= \frac{1,500}{1.015} + \frac{1,500}{(1.015)^2} + \frac{1,500}{(1.015)^3} + \dots + \frac{1,500}{(1.015)^{48}} \\ &\quad + \frac{1,500}{(1.015)^{48}(1 + .04/3)} + \frac{1,500}{(1.015)^{48}(1 + .04/3)^2} + \dots \\ &\quad \dots + \frac{1,500}{(1.015)^{48}(1 + .04/3)^{71}} + \frac{1,500}{(1.015)^{48}(1 + .04/3)^{72}} \\ &= 1,500 \times \left(\frac{1 - 1/(1.015)^{48}}{.015} + \frac{1}{(1.015)^{48}} \times \frac{1 - 1/(1 + .04/3)^{72}}{.04/3} \right) \\ &= \$84,903.40 . \end{aligned}$$

51. **Calculating Present Value of a Perpetuity** Given an interest rate of 8.5 percent per year, what is the value at date $t = 9$ of a perpetual stream of \$300 payments that begin at date $t = 14$?

Answer: What matters here is when this perpetuity really starts. Since the first payment is made at $t = 14$, its present value as of date $t = 13$ is $\frac{300}{.085}$. Today's date is

9, which is 4 periods ahead of 13. Therefore, the present value of this annuity is

$$PV = \frac{1}{(1.085)^4} \times \frac{300}{0.085} = \$2,546.73 .$$

52. **Calculating the Present Value of a Growing Perpetuity** Harris, Inc., paid a \$3 dividend yesterday. If the firm raises its dividend at 5 percent every year and the appropriate discount rate is 12 percent, what is the price of Harris stock?

Answer: Since the \$3 dividend was paid yesterday, the next dividend will be 3×1.05 , and thus the present value of Harris stock is given by

$$\begin{aligned} PV &= \frac{3 \times (1.05)}{1.12} + \frac{3 \times (1.05)^2}{(1.12)^2} + \frac{3 \times (1.05)^3}{(1.12)^3} + \dots \\ &= 3 \times \left(\frac{1.05}{1.12} + \left(\frac{1.05}{1.12} \right)^2 + \left(\frac{1.05}{1.12} \right)^3 + \dots \right). \end{aligned}$$

In class, we have seen that a series

$$S = q + q^2 + q^3 + q^4 + \dots ,$$

where $0 < q < 1$, is in fact equal to $\frac{q}{1-q}$. Hence if you replace q by $\frac{1.05}{1.12}$ in the parentheses above, the present value of Harris stock can be calculated as

$$PV = 3 \times \frac{1.05/1.12}{1 - 1.05/1.12} = 3 \times \frac{1.05}{1.12 - 1.05} = 3 \times \frac{1.05}{.07} = \$45.$$

Note that this problem differs from what we have seen in class by having a first payment of $(1 + g)C$ instead of C .

53. **Calculating EAR** A local finance company quotes a 12 percent interest rate on one-year loans. So, if you borrow \$20,000, the interest for the year will be \$2,400. Because you must repay a total \$22,400 in one year, the finance company requires you to pay $\$22,400/12$, or \$1,866.67 per month over the next 12 months. Is this a 12 percent loan? What rate would legally have to be quoted? What is the effective annual rate?

Answer: Since the payment each month is \$1,866.67 and the present value of this

loan is \$20,000, this implies a monthly rate, r_m , such that

$$20,000 = 1,866.67 \times \left(\frac{1 - \left(\frac{1}{1+r_m}\right)^{12}}{r_m} \right).$$

Solving by trial and error, we find that r_m is approximately equal to 1.788%. Hence the annual rate that would legally have to be quoted is $1,788 \times 12 = 21.456\%$, and the effective annual rate is $(1.01788)^{12} - 1 = 23.7\%$.

57. **Calculating Annuities Due** You want to lease a new sport car from Muscle Motors for \$42,000. The lease contract is in the form of a 48-month annuity due at 10.5 APR. What will you monthly lease payment be?

Answer: Note that the monthly rate is $\frac{10.5}{12} = 0.875\%$. We know that the present value of a regular annuity paying C over T period with a discount rate of r is

$$\text{PV}_{\text{reg}} = C \left(\frac{1 - \left(\frac{1}{1+r}\right)^T}{r} \right),$$

and we know that the present value of an annuity due is equal to $1 + r$ times its equivalent regular annuity, i.e. $\text{PV}_{\text{due}} = (1 + r)\text{PV}_{\text{reg}}$. Hence we have, in this case,

$$42,000 = (1.00875)C \left(\frac{1 - \left(\frac{1}{1.00875}\right)^{48}}{.00875} \right) = 39.399 \times C \Rightarrow C = \$1,066.02 .$$

64. **Calculating the Number of Periods** Your Christmas ski vacation was great, but it unfortunately ran a bit over budget. All is not lost, because you just received an offer in the mail to transfer your \$10,000 balance from your current credit card, which charges an annual rate of 11.9 percent, to a new credit card charging a rate of 5.9 percent. How much faster could you pay the loan off by making your planned monthly payments of \$200 with the new card? What if there is a 2 percent fee charged on any balances transfer?

Answer: Let T_o denote the number of months it would take to reimburse the \$10,000 with the old credit card. Then T_o is such that

$$10,000 = 200 \times \frac{1 - \left(\frac{1}{1+.119/12}\right)^{T_o}}{.119/12} .$$

Following the same procedure as in Problem 21, this means

$$T_o = \frac{\ln\left(\frac{1}{1-(10,000 \times .119/12)/200}\right)}{\ln(1 + .119/12)} = 69.4 \text{ months.}$$

Let T_n denote the number of months it would take to repay the loan with the new credit card. Assuming that there is no fee on balances transfer, T_n is found by replacing .119 with .059 in the last equation, which gives us

$$T_n = \frac{\ln\left(\frac{1}{1-(10,000 \times .059/12)/200}\right)}{\ln(1 + .059/12)} = 57.53 \text{ months.}$$

If there is a 2% fee on balances transfer, the amount of the loan becomes $10,000 \times 1.02 = \$10,200$, and then the number of months it takes to repay is

$$T_{n,\text{fee}} = \frac{\ln\left(\frac{1}{1-(10,200 \times .059/12)/200}\right)}{\ln(1 + .059/12)} = 58.86 \text{ months.}$$