Bodie, Kane, Marcus, Perrakis and Ryan, Chapter 7

Answers to Selected Problems

1. What is the beta of a portfolio with $E[r_p] = 18$ percent, if $r_f = 6$ percent and $E[r_M] = 14$ percent?

   Answer: Using the CAPM equilibrium condition,

   $$E[r_p] \;=\; r_f \;+\; \beta_p\big[E[r_M] - r_f\big] \qquad \Rightarrow \qquad \beta_p \;=\; \frac{E[r_p] - r_f}{E[r_M] - r_f} \;=\; \frac{.18 - .06}{.14 - .06} \;=\; 1.5\;.$$

2. The market price of a security is \$50. Its expected return is 14 percent. The risk-free rate is 6 percent and the market risk premium is 8.5 percent. What will be the market price of the security if its covariance with the market portfolio doubles (and all other variables remain unchanged)?

   Answer: The security's risk premium is actually $.14 - .06 = 8\%$. According to the CAPM, doubling β_i will double the security's risk premium since

   $$E[r_i] - r_f \;=\; \beta_i\,(E[r_M] - r_f)\,.$$

   That is, the new risk premium for this security will be 16%, which implies a new expected return of $.16 + .06 = 22\%$.

   Let $E[d_1]$ and $E[p_1]$ denote the expected dividend and the expected price of the security one year from now, and let p_0 denote today's price. Then

   $$E[r_i] \;=\; \frac{E[p_1] + E[d_1] - p_0}{p_0} \qquad \Rightarrow \qquad p_0(1 + E[r_i]) \;=\; E[p_1] + E[d_1].$$

If markets are efficient, then $E[p_1] = p_0$ and thus

$$p_0 \;=\; \frac{E[d_1]}{E[r_i]},$$

which is the result we have obtained had we assumed that the firm were expected pay a constant dividend forever. We know that $p_0 = \$50$ when $E[r_i] = 14\%$, and thus

$$E[d_1] \;=\; 50 \times .14 \;=\; \$7.$$

Assuming that $E[d_1]$ does not change even though the security is more risky, We obtain the new price of the security by dividing $E[d_1]$ with the new expected return, i.e.

$$p_0 \;=\; \frac{7}{.22} \;=\; \$31.82\ .$$

3. You are a consultant to a large manufacturing corporation that is considering a project with the following net after-tax cash flows (in millions of dollars):

Years from now	**After-Tax Cash Flow**
0	-20
1-9	10
10	20

The project's beta is 1.7. Assuming that $r_f = 9$ percent and $E[r_M] = 19$ percent, what is the net present value of the project? What is the highest possible beta estimate for the project before its NPV becomes negative?

Answer: The NPV of the project is given by

$$\text{NPV} \;=\; -\,20 \;+\; \frac{10}{E[r_p]}\left(1 - \frac{1}{(1+E[r_p])^9}\right) \;+\; \frac{20}{(1+E[r_p])^{10}}$$

Using the CAPM equation, we can find the required return on the project, which is

$$E[r_p] \;=\; r_f \;+\; \beta_p\left[E[r_M] - r_f\right] \;=\; .09 \;+\; 1.7(.19 - .09) \;=\; 26\%\ .$$

Using this rate of return, we find that

$$\text{NPV} \;=\; -\,20 \;+\; \frac{10}{.26}\left(1 - \frac{1}{(1.26)^9}\right) \;+\; \frac{20}{(1.26)^{10}} \;=\; 15.64\ .$$

The internal rate of return for this project is 49.55%. That is, the NPV of the project is zero when $E[r_p] = .4955$. Using the CAPM expected-return-beta relationship, this means

$$\beta_p \;=\; \frac{E[r_p] - r_f}{E[r_M] - r_f} \;=\; \frac{.4955 - .09}{.19 - .09} \;=\; 4.055\ .$$

4. Are the following statements true or false?

   a. Stocks with a beta of zero offer an expected rate of return of zero.

   False. If $\beta_i = 0$, then $E[r_i] = r_f$.

   b. The CAPM implies that investors require a higher return to hold highly volatile securities.

   The answer depends on what volatile means here. If volatility comes from the firm-specific risk, then investors won't require a higher rate of return since the firm-specific risk is diversified away in the market portfolio. If, on the other hand, highly volatile means a high covariance with the market portfolio, then yes, investors will require a high rate of return to invest in this security.

   c. You can construct a portfolio with a beta of .75 by investing .75 of the budget in bills and the remainder in the market portfolio.

   False. Let y denote the fraction of the budget invested in the market portfolio, whose beta is denoted β_p. Then

$$\beta_p \;=\; (1-y)\beta_f \;+\; y\beta_M \;=\; (1-y)\times 0 \;+\; y\times 1 \;=\; y.$$

   That is, if 75% of the budget is invested in M, then the portfolio beta is .75.

5. Consider the following table, which gives a security analyst's expected return on two stocks for two particular market returns:

Market Return	**Agressive Stock**	**Defensive Stock**
.05	.02	.035
.20	.32	.14

a. What are the betas of the two stocks?

Answer: The beta of each stock is its slope with respect to changes in the expected return. Let a denote the aggressive stock and let d denote the defensive stock. For Stock a, we have the following observations for (r_a, r_M): $(.02, .05)$ and $(.32, .20)$. If we draw a line that goes from one point to the other, its slope will be the beta for Stock a, which is given by

$$\beta_a = \frac{.32 - .02}{.20 - .05} = 2.$$

Proceeding in a similar manner for Stock d, we obtain

$$\beta_d = \frac{.14 - .035}{.20 - .05} = 0.7 \ .$$

b. What is the expected rate of return on each stock if the market return is equally likely to be 5 percent or 20 percent?

Answer: The expected return to Stock a is

$$E[r_a] = \frac{1}{2} \times .02 + \frac{1}{2} \times .32 = .17 \ .$$

For Stock d, we have

$$E[r_d] = \frac{1}{2} \times .035 + \frac{1}{2} \times .14 = .0875 \ .$$

c. If the T-bill rate is 8 percent and the market return is equally likely to be 5 percent or 20 percent, draw the SML for this economy. **Answer:** The SML for this economy is

$$\begin{aligned} E[r] &= r_f + \beta\big(E[r_M] - r_f\big) \\ &= .08 + \beta\left(\frac{1}{2} \times .05 + \frac{1}{2} \times .20 - .08\right) \\ &= .08 + \beta\big(.125 - .08\big) \\ &= .08 + .045\beta \end{aligned}$$

d. Plot the two securities on the SML graph. What are alphas of each?

Answer: Using each stock's beta, we find

$$\begin{aligned} E[r] &= .08 + .045 \times \beta_a = .08 + .045 \times 2 = .17 \\ E[r] &= .08 + .045 \times \beta_d = .08 + .045 \times .7 = .1115\ . \end{aligned}$$

Security a is therefore on the SML but Security d is not. Security d's alpha is then $E[r_d] - .1115 = -.024$. Security a's alpha is zero.

If the simple CAPM is valid, which of the following situations in problems 6-12 are possible? Explain. Consider each situation independently.

6. Consider the situation depicted in Table 1. If the CAPM holds, portfolios with higher betas should command higher expected returns, which is not the case in Table 1.

Portfolio	**Expected Return**	**Beta**
A	.20	1.4
B	.25	1.2

Table 1: Question 6.

7. Consider the situation depicted in Table 2. The CAPM prices assets according to their market risk only. Since the standard deviations in Table 2 include firm-specific risk, this situation is possible.

Portfolio	**Expected Return**	**Standard Deviation**
A	.30	.35
B	.40	.25

Table 2: Question 7.

8. Consider the situation depicted in Table 3. According to this table, the market risk premium is $E[r_M] - r_f = .18 - .10 = .08$. If portfolio A is well-diversified, we have

$$\sigma_A^2 \;=\; \beta_A^2 \sigma_M^2 \quad \Rightarrow \quad (.12)^2 \;=\; \beta_A^2 (.24)^2 \quad \Rightarrow \quad \beta_A = .5\ .$$

If the CAPM holds, then

$$E[r_A] \;=\; r_f \;+\; \beta_A\big(E[r_M] - r_f\big) \;=\; .10 \;+\; .5 \times .08 \;=\; .14,$$

which is not the case in Table 3.

When saying that a portfolio is well-diversified, we mean that is has no idiosyncratic risk. The actual Beta of Portfolio A is given by

$$\beta_A \;=\; \frac{\text{cov}(r_A, r_M)}{\sigma_M^2} \;=\; \frac{\rho_{A,M}\sigma_A\sigma_M}{\sigma_M^2} \;=\; \frac{\rho_{A,M}\sigma_A}{\sigma_M}\ ,$$

where $\rho_{A,M}$ denotes the correlation between r_A and r_M. Since $\sigma_A = .12$ and $\sigma_M = .24$, this gives us $\beta_A = .5\rho_{A,M}$. Hence the highest possible Beta for Portfolio A is .5, which is when $\rho_{A,M} = 1$. Note that this is the Beta we obtained by assuming that A was well-diversified. Using the highest possible value for β_A, we can find the highest possible expected return for a portfolio with a standard deviation of .12. We have shown above that this expected return was 14%. Hence Portfolio A's actual expected return of 16% contradicts the CAPM.

Portfolio	**Expected Return**	**Standard Deviation**
Risk-free	.10	0
Market	.18	.24
A	.16	.12

Table 3: Question 8.

9. Consider the situation in Table 4. This situation is not possible since Portfolio A has a higher expected return than the market with a lower variance.

Portfolio	**Expected Return**	**Standard Deviation**
Risk-free	.10	0
Market	.18	.24
A	.20	.22

Table 4: Question 9.

Portfolio	**Expected Return**	**Beta**
Risk-free	.10	0
Market	.18	1
A	.16	1.5

Table 5: Question 10.

10. Consider the situation in Table 5. This is not consistent with the CAPM since portfolio A has a beta higher than 1 but an expected return smaller then the market expected return.

11. Consider the situation in Table 6. Here we have

Portfolio	**Expected Return**	**Beta**
Risk-free	.10	0
Market	.18	1
A	.16	.9

Table 6: Question 11.

$$E[r_A] \;=\; r_f \;+\; \beta_A\,(E[r_M] - r_f) \;=\; .10 \;+\; .9 \times .08 \;=\; .172\;.$$

Again, this is inconsistent with the CAPM.

In problems 13-15, assume that the risk-free rate of interest is 6 percent and the expected rate of return on the market is 16 percent.

13. A share of stock sells for \$50 today. It will pay a dividend of \$6 per share at the end of the year. Its beta is 1.2. What do investors expect the stock to sell for at the end of the year?

Answer: Using the CAPM expected-return-beta relationship, we have

$$E[r] \;=\; r_f \;+\; \beta\,(E[r_M] - r_f) \;=\; .06 \;+\; 1.2(.16 - .06) \;=\; .18\;.$$

Therefore,

$$E[r] \;=\; \frac{E[p_1] + d - p_0}{p_0} \;=\; \frac{E[p_1] - 44}{50} \;=\; .18 \qquad \Rightarrow \qquad E[p_1] \;=\; \$53\;.$$

14. I am buying a firm with an expected cash flow of \$1,000 but am unsure of its risk. If I think the beta of the firm is 0.5, when in fact the beta is really 1, how much *more* will I offer for the firm than it is truly worth?

Answer: Believing in a beta of .5, the expected return I require on this investment is

$$E[r] \;=\; r_f \;+\; \beta\,(E[r_M] - r_f) \;=\; .06 \;+\; .5 \times .1 \;=\; 11\%.$$

Hence I am ready to pay $\frac{1{,}000}{1.11} = \$900.90$ for this investment. Using the actual beta of 1, the required expected return is

$$E[r] \;=\; .06 \;+\; 1 \times .1 \;=\; 16\%,$$

which means that this investment is actually worth $\frac{1{,}000}{1.16} = \$862.07$. Hence I am offering $900.90 - 862.07 = \$38.83$ more than what this investment is truly worth.

15. A stock has an expected rate of return of 4 percent. What is its beta?

Answer: Using the CAPM expected-return-beta relationship, we have

$$.04 \;=\; .06 \;+\; .1\beta \qquad \Rightarrow \qquad \beta \;=\; -\,.2\;.$$

16. Two investment advisors are comparing performance. One averaged a 19 percent rate of return and the other a 16 percent rate of return. However, the beta of the first investor was 1.5, whereas that of the second was 1.

a. Can you tell which investor was a better predictor of individual stocks (aside from the issue of general movements in the market)?

Answer: To determine which one is the best, we need to calculate each investor's abnormal return, i.e. the returns above what was expected for the same level of risk. Without knowing the risk free rate and the market risk premium, we cannot determine which investor is better at picking stocks.

b. If the T-bill rate were 6 percent and the market return during the period were 14 percent, which investor would be a superior stock selector?

Answer: The expected return of the investor with a beta of 1.5 was $.06 + 1.5 \times (.14 - .06) = 18\%$, whereas that of the investor with a beta of 1 was $.06 + 1(.14 - .06) = 14\%$. We can measure the quality of each investor by looking at their alpha, as this tells us by how much an investor exceeded the expected return of investments with the same risk. Hence the investor with a beta of 1 seems better at picking stocks since he earned 2 percentage points above expectations whereas the other investor earned only 1 percentage point above expectations.

c. What if the T-bill rate were 3 percent and the market return were 15 percent?

Answer: The expected return of the investor with a beta of 1.5 was, in this case, $.03 + 1.5(.15 - .03) = 21\%$, and that of the investor with a beta of 1 was $.03 + 1(.15 - .03) = 15\%$. The latter is definitely better than the former here since he earned 1 percentage point above his expected return, as opposed to minus two precentage points for the former.

17. In 1999, the rate of return on short-term government securities (perceived to be risk-free) was about 5 percent. Suppose the expected rate of return required by the market for a portfolio with a beta measure of 1 is 12 percent. According to the capital asset pricing model (security market line):

a. What is the expected rate of return on the market portfolio?

Answer: The same as a portfolio with a beta of 1, i.e. 12 percent.

b. What would be the expected rate of return on a stock with $\beta = 0$?

Answer: The same as the risk-free asset, i.e. 5 percent.

c. Suppose you consider buying a share of stock at \$40. The stock is expected to pay \$3 in dividends next year and to sell then for \$41. The stock risk has been evaluated by $\beta = -.5$. Is the stock overpriced or underpriced?

Answer: According to the CAPM,

$$E[r] \;=\; r_f \;+\; \beta\,(E[r_M] - r_f) \;=\; .05 \;+\; (-.5)(.12 - .05) \;=\; .015\ .$$

According to your expectations about the stock,

$$E[r] \;=\; \frac{E[p_1 + d_1] - p_0}{p_0} \;=\; \frac{41 + 3 - 40}{40} \;=\; 10\%.$$

Clearly, this stock is underpriced since it is expected to yield a much higher return than what the CAPM suggests for investments with the same level of risk.

21. The security market line depicts:

    a. A security's expected return as a function of its systematic risk

    b. The market portfolio as the optimal portfolio of risky securities

    c. The relationship between a security's return and the return on an index

    d. The complete portfolio as a combination of the market portfolio and the risk-free asset

    Answer: *a*

22. Within the context of the capital asset pricing model (CAPM), assume:

    · Expected return on the market = 15 percent

    · Risk-free rate = 8 percent

    · Expected rate of return on XYZ security = 17 percent

    · Beta of XYZ security = 1.25

Which *one* of the following is *correct*?

a. XYZ is overpriced.

b. XYZ is fairly priced.

c. XYZ's alpha is -0.25 percent.

d. XYZ's alpha is 0.25 percent.

According to the CAPM, the expected return of a security with a beta of 1.25 is

$$E[r] \;=\; r_f \;+\; 1.25\,(E[r_M] - r_f) \;\;=\;\; .08 \;+\; 1.25(.15 - .08) \;=\; 16.75\%.$$

Since security XYZ has a return of 17%, it is underpriced with an alpha of 0.25%. The correct answer is *d*.

Table 7 shows risk and return measures for two portfolios.

Portfolio	**Average Annual Rate of Return**	**Standard Deviation**	**Beta**
R	11%	10%	0.5
S&P 500	14%	12%	1.0

Table 7: Questions 26-27.

26. When plotting portfolio R on Table 7 relative to the SML, portfolio R lies:

a. On the SML

b. Below the SML

c. Above the SML

d. Insufficient data given

We need to know the risk-free rate in order to derive the SML. We don't have it and thus we cannot determine where portfolio R is relative to the SML. The answer is *d*.

27. When plotting portfolio R relative to the capital market line, portfolio R lies:

   a. On the CML

   b. Below the CML

   c. Above the CML

   d. Insufficient data given

Same as question 26, we can't tell since we don't know the risk-free rate.