Bodie, Kane, Marcus, Perrakis and Ryan, Chapter 6

Answers to Selected Problems

You manage a risky portfolio with an expected rate of return of 18 percent and a standard deviation of 28 percent. The T-bill rate is 8 percent.

1. Your client chooses to invest 70 percent of a portfolio in your fund and 30 percent in a T-bill money market fund. What is the expected value and standard deviation of the rate of return on your client's portfolio?

   Answer: Let c denote the client's portfolio, let f denote the money-market fund and let p denote the risky portfolio. Then

   $$E[r_c] \;=\; E[.7r_p + .3r_f] \;=\; .7E[r_p] + .3r_f \;=\; .7 \times .18 + .3 \times .08 \;=\; 15\% \; .$$

   Since $\sigma_f = 0$, the standard deviation of the client's portfolio is given by

   $$\sigma_c \;=\; .7\sigma_p \;=\; .7 \times .28 \;=\; 19.6\% \; .$$

2. Suppose that your risky portfolio included the following investments in the given proportions:

   Stock A: 27 percent

   Stock B: 33 percent

   Stock C: 40 percent

   What are the investment proportions of your client's overall portfolio, including the position in T-bills?

Answer: Since portfolio c is 70 percent invested in p, this means

$$
\begin{aligned}
.7 \times .27 &= 18.9\% \text{ in Stock } A, \\
.7 \times .33 &= 23.1\% \text{ in Stock } B, \\
.7 \times .40 &= 28.0\% \text{ in Stock } C.
\end{aligned}
$$

The fraction invested in T-bills is 30%.

3. What is the reward-to-variability ratio of your portfolio? Your client's?

   Answer: The reward-to-variability ratio of your fund is given by

$$\frac{E[r_p] - r_f}{\sigma_p} = \frac{.18 - .08}{.28} = 35.7\%.$$

   Your client's portfolio offers the same reward-to-variability ratio, that is

$$\frac{E[r_c] - r_f}{\sigma_c} = \frac{.15 - .08}{.196} = 35.7\%.$$

4. Draw the CAL of your portfolio on an expected return-standard deviation diagram. What is the slope of the CAL? Show the position of your client's portfolio on your fund's CAL.

   Answer: The CAL is drawn in Figure 1. The slope of the CAL is the reward-to-variability ratio, i.e. 35.7%. The client's portfolio is the one providing an expected return of $E[r_c]$ and a standard deviation σ_c.

5. Suppose that your client decides to invest in your portfolio a proportion y of the total investment budget so that the overall portfolio will have an expected rate of return of 16 percent.

   a. What is the proportion y?

   Answer: The proportion y is such that $E[r_c] = yE[r_p] + (1 - y)r_f = .16$, which means that

$$y = \frac{.16 - r_f}{E[r_p] - r_f} = \frac{.16 - .08}{.18 - .08} = 80\%.$$

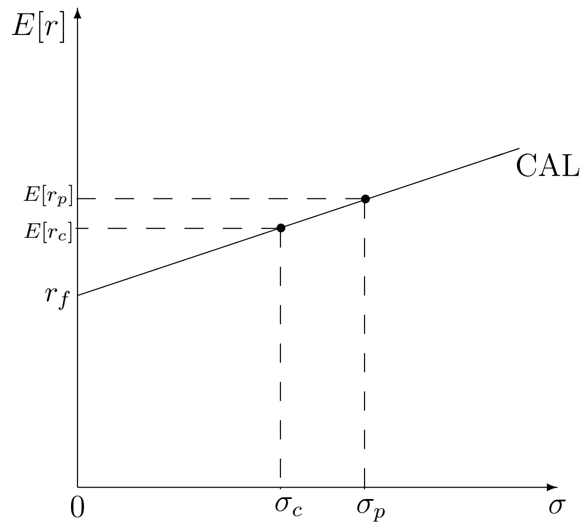

Figure 1: CAL for question 4.

b. What are your client's investment proportions in your three stocks and the T-bill?

Answer: With 80 percent invested in p, we have

$$\begin{aligned} .8 \times .27 &= 21.6\% \text{ in Stock } A, \\ .8 \times .33 &= 26.4\% \text{ in Stock } B, \\ .8 \times .40 &= 32.0\% \text{ in Stock } C. \end{aligned}$$

The fraction invested in T-bills is 20%.

c. What is the standard deviation of the rate of return on your client's portfolio?

Answer: The standard deviation σ_c is now

$$y\sigma_p = 0.8 \times .28 = 22.4\% .$$

6. Suppose that your client prefers to invest in your fund a proportion y that maximizes the expected return on the overall portfolio subject to the constraint that the overall portfolio's standard deviation will not exceed 18 percent.

a. What is the investment proportion y?

Answer: Since expected return increases linearly with risk, the standard deviation of your client's portfolio return has to be 18% in order to be maximizing its expected return. Hence the proportion y is found using

$$\sigma_c \;=\; 0.18 \;=\; y\sigma_p \;=\; y \times .28 \qquad \Rightarrow \qquad y \;=\; \frac{.18}{.28} \;=\; 64.3\% \;.$$

b. What is the expected rate of return on the overall portfolio?

Answer: The expected rate of return is

$$E[r_c] \;=\; .643E[r_p] \;+\; (1 - .643)r_f \;=\; .643 \times .18 \;+\; .357 \times .08 \;=\; 14.43\% \;.$$

7. Your client's degree of risk aversion is $A = 3.5$.

a. What proportion (y) of the total investment should be invested in your fund?

Answer: As we have seen in class,

$$y \;=\; \frac{E[r_p] - r_f}{A\sigma_p^2} \;=\; \frac{.18 - .08}{3.5(.28)^2} \;=\; 36.44\% \;.$$

b. What is the expected value and standard deviation of the rate of return on your client's optimized portfolio?

Answer: The expected return is

$$E[r_c] \;=\; .3644E[r_p] \;+\; .6356r_f \;=\; 11.64\% \;.$$

The standard deviation is

$$\sigma_c \;=\; y\sigma_p \;=\; .3644 \times .28 \;=\; 10.20\% \;.$$

Note: Try problems 37-43 in Bodie, Kane, etc., Chapter 6.