Investments, Chapter 5

Answers to Selected Problems

8. During a period of severe inflation, a bond offered a nominal HPR of 80 percent per year. The inflation rate was 70 percent per year.

   a. What was the real HPR on the bond over the year?

   Answer: Let r denote the nominal return on the bond and let i denote the inflation rate. Then the real HPR is given by

   $$\frac{1+r}{1+i} - 1 = \frac{1.80}{1.70} - 1 = 5.88\% .$$

   b. Compare this real HPR to the approximation $R = r - i$.

   Answer: With the approximation, we get

   $$R = r - i = 80\% - 70\% = 10\%.$$

   Clearly, $R = r - i$ is not a good approximation for large values of r and i.

Use Table 1 to answer problems 9 through 11.

	Bear Market	**Normal Market**	**Bull Market**
Probability	0.2	0.5	0.3
Stock X	-20%	18%	50%
Stock Y	-15%	20%	10%

Table 1: Table for problems 9 through 11.

9. What is the expected return for stocks X and Y?

   Answer: The expected return for stock X is

$$E[r_X] \;=\; .2 \times (-20\%) \;+\; .5 \times 18\% \;+\; .3 \times 50\% \;=\; 20\%.$$

   The expected return for stock Y is

$$E[r_Y] \;=\; .2 \times (-15\%) \;+\; .5 \times 20\% \;+\; .3 \times 10\% \;=\; 10\%.$$

10. What is the standard deviation of returns on stocks X and Y?

    Answer: The standard deviation of returns on stock X is

$$\begin{aligned} \sigma_X &= \sqrt{.2 \times (-20\% - 20\%)^2 \;+\; .5 \times (18\% - 20\%)^2 \;+\; .3 \times (50\% - 20\%)^2} \\ &= 24.33\%. \end{aligned}$$

    The standard deviation of returns on stock Y is

$$\begin{aligned} \sigma_Y &= \sqrt{.2 \times (-15\% - 10\%)^2 \;+\; .5 \times (20\% - 10\%)^2 \;+\; .3 \times (10\% - 10\%)^2} \\ &= 13.23\%. \end{aligned}$$

11. Assume you invest your \$10,000 portfolio in \$9,000 of stock X and \$1,000 of stock Y. What is the expected return on your portfolio?

    Answer: Let P denote this portfolio, which is 90% invested in stock X and 10% invested in stock Y. The expected return on the portfolio is

$$E[r_P] \;=\; E[.9r_X + .1r_Y] \;=\; .9E[r_X] + .1E[r_Y] \;=\; .9 \times 20\% + .1 \times 10\% \;=\; 19\%.$$

12. Given \$100,000 to invest, what is the expected risk premium in dollars of investing in equities versus risk-free T-bills based on the following information?

$$\text{\$100,000 in equities returns} \begin{cases} \$50,000 & \text{with probability } .6 \\ -\$30,000 & \text{with probability } .4, \end{cases}$$

    whereas \$100,000 invested in T-bills returns \$5,000.

    Answer: The risk premium is

$$.6 \times 50,000 \;+\; .4 \times (-30,000) \;-\; 5,000 \;=\; \$13,000\ .$$

17. Consider a risky portfolio. The end-of-year cash flow derived from the portfolio will be either \$50,000 or \$150,000 with equal probabilities of .5. The alternative risk-free investment in T-bills pays 5 percent per year.

a. If you require a risk premium of 10 percent, how much will you be willing to pay for the portfolio?

Answer: Let X denote the risky portfolio and let p denote its price. Then its expected return is

$$E[r_X] \;=\; \frac{.5 \times 50,000 + .5 \times 150,000 - p}{p} \;=\; \frac{100,000 - p}{p}\;.$$

If you require a risk premium of 10 percent, then you will invest in X only if $E[r_X] - r_F \geq 10\%$, i.e. only if

$$\begin{aligned}
\frac{100,000 - p}{p} \;-\; 0.05 \;\geq\; 0.10 \;\;&\Rightarrow\;\; \frac{100,000 - p}{p} \;\geq\; 0.15 \\
&\Rightarrow\;\; 100,000 - p \;\geq\; 0.15p \\
&\Rightarrow\;\; 100,000 \;\geq\; 1.15p \\
&\Rightarrow\;\; p \;\leq\; \frac{100,000}{1.15} \;=\; \$86,956.52\;.
\end{aligned}$$

b. Suppose that the portfolio can be purchased for the amount you found in (a). What will be the expected rate of return on the portfolio?

Answer: The expected rate of return will be 15%.

c. Now, suppose that you require a risk premium of 15 percent. What is the price that you will be willing to pay?

Answer: In this case, you will purchase the portfolio only if $E[r_X] - r_F \geq 15\%$, i.e. only if

$$\begin{aligned}
\frac{100,000 - p}{p} \;-\; 0.05 \;\geq\; 0.15 \;\;&\Rightarrow\;\; \frac{100,000 - p}{p} \;\geq\; 0.2 \\
&\Rightarrow\;\; 100,000 - p \;\geq\; 0.2p \\
&\Rightarrow\;\; 100,000 \;\geq\; 1.2p \\
&\Rightarrow\;\; p \;\leq\; \frac{100,000}{1.2} \;=\; \$83,333.33\;.
\end{aligned}$$

d. Comparing your answers to (*a*) and (*c*), what do you conclude about the relationship between the required risk premium on a portfolio and the price at which the portfolio will sell?

Answer: The greater the required risk premium, the lower the price at which the portfolio will sell.

23. John's utility function over investment (I) is given by $U(I) = E[r_I] - \frac{1}{2}A\sigma_I^2$, where $A = 4$. Which of the following investments does he prefer? Investment 1, with an expected return of 12% and a standard deviation of 30%; Investment 2, with an expected return of 15% and a standard deviation of 50%; Investment 3, with an expected return of 21% and a standard deviation of 16%; Investment 4, with an expected return of 24% and a standard deviation of 21%.

Answer: The utility John derives from each investment is given by

$$\begin{aligned} U(1) &= 0.12 - 2 \times (.3)^2 &&= -0.06 \\ U(2) &= 0.15 - 2 \times (.5)^2 &&= -0.35 \\ U(3) &= 0.21 - 2 \times (.16)^2 &&= 0.1588 \\ U(4) &= 0.24 - 2 \times (.21)^2 &&= 0.1518\ . \end{aligned}$$

Hence John prefers investment 4.