Business 3059

Investment Management

Lakehead University

Midterm Exam

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Time allowed: 1 hour 15 minutes.

Instructions: Calculators are permitted.
One 8.5×11 inches crib sheet is allowed.
The marks awarded for each question are in brackets.

Good luck!

Part I: True or False. For FOUR of the following statements, say if the statement is true or false (2 points), and explain why (3 points).

1. Calculating a T-Bill yield using the bank discount yield method necessarily gives a lower value than if it is calculated with the bond equivalent yield method.
   True: Consider a T-bill that matures in n days and let P denote the current price of of the T-Bill, with a face value of \$1,000. For the return on this T-Bill to be positive, we need $P < 1,000$. The bond equivalent yield is then given by
   $$r_{bey} = \frac{1,000 - P}{P} \times \frac{365}{n} .$$
   On the other hand, the bank discount yield for this T-Bill is given by
   $$d = \frac{1,000 - P}{1,000} \times \frac{360}{n} .$$

The bank discount yield can only be lower than the bond equivalent yield since

$$\begin{aligned}
d &= \frac{1,000-P}{1,000} \times \frac{360}{n} \\
&= \frac{1,000-P}{P} \times \frac{P}{1,000} \times \frac{365}{n} \times \frac{360}{365} \\
&= \frac{1,000-P}{P} \times \frac{365}{n} \times \frac{360}{365} \times \frac{P}{1,000} \\
&= r_{bey} \times \frac{360}{365} \times \frac{P}{1,000} \\
&= r_{bey} \times \underbrace{\frac{P}{1,013.89}}_{<1} < r_{bey} .
\end{aligned}$$

2. A mutual fund never sells above its net asset value.

   False: A closed-end fund may either sell above or below net asset value.

3. Let R denote the nominal interest rate and let i denote the inflation rate. Then $r = R - i$ is always a good approximation of the real rate of interest.

   False: This approximation is good only for small values of of R and i. For example, if $R = 80\%$ and $i = 70\%$, then $R - i = 10\%$ whereas the exact real interest rate is $\frac{1+R}{1+i} - 1 = \frac{1.8}{1.7} - 1 = 5.88\%$.

4. The profit made on a short sale is the mirror image of the profit that would have been made had the stock been bought and then sold.

   True: Let p_0 denote the stock price at the time of the short sale and let p_1 denote the stock price when the osition is covered. Let d denotes the dividends that have been distributed to shareholders meanwhile. Since the short seller has to pay dividends to the stock lender, the profit from the short sale is $p_0 - (p_1 + d)$. If, instead, the stock had been bought and sold, the trader's profits would have been $p_1 + d - p_0$, which is indeed the mirror image of the profit from the short sale.

5. The steeper the capital allocation line, the better the investment opportunities to a risk-averse investor.

   True: The steeper the CAL, the better the reward-to-variability ratio, and thus the better the return for any level of risk. That is, the better the investment opportunities.

6. Consider Figure 1. If the curve MVF represents the minimum-variance frontier, then a portfolio such as portfolio A cannot exist.

   False: If there exists a risk free asset and its return, r_f, allows it, then portfolio A can be a combination of a risky portfolio and the risk-free asset. That is, A cannot be a portfolio composed entirely of risky assets, since these portfolios are all to the right of the MVF curve. It can nevertheless can be a combination of portfolio P, for instance, and the risk-free asset, as shown in Figure 2.

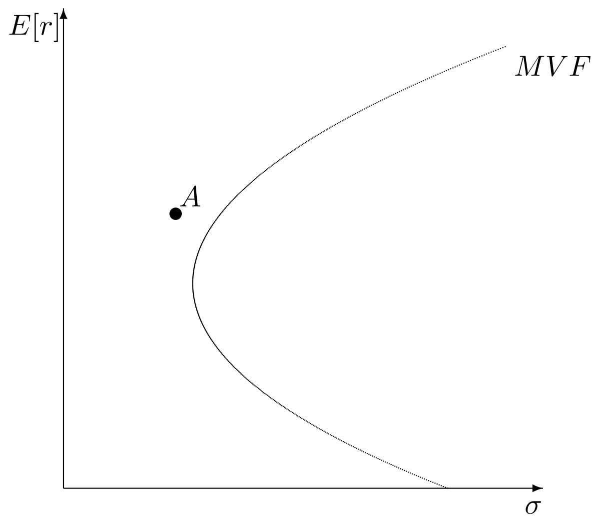

Figure 1: The minimum-variance frontier.

Part II: Problems. Answer TWO of the following problems (10 points each).

1. *Margin Purchase* Suppose you buy on margin 1,000 shares of company XYZ, currently selling at \$30 per share, with an initial margin ratio of 50%. The interest on your broker's loan is 6% annually and the maintenance margin requirement is 30%.

   (a) (4 points) If the stock price of XYZ is \$35 one year from now, what will the return on the margin purchase be? Will you receive a margin call? If so, how much money will you have to deposit in your account? How does the return on the margin purchase compare with the return on the stock?
   Answer: The initial margin of 50% means that $\frac{\$30\times 1{,}000}{2} = \$15,000$ has initially been invested and \$15,000 has been borrowed from the broker. The return on the margin purchase is then

$$\frac{35 \times 1,000 - 15,000 \times 1.06 - 15,000}{15,000} = 27.33\% \ .$$

   Since the trade was profitable, the margin ratio after can only be higher than what it was initially, i.e. it is necessarily higher than 50% and thus there cannot

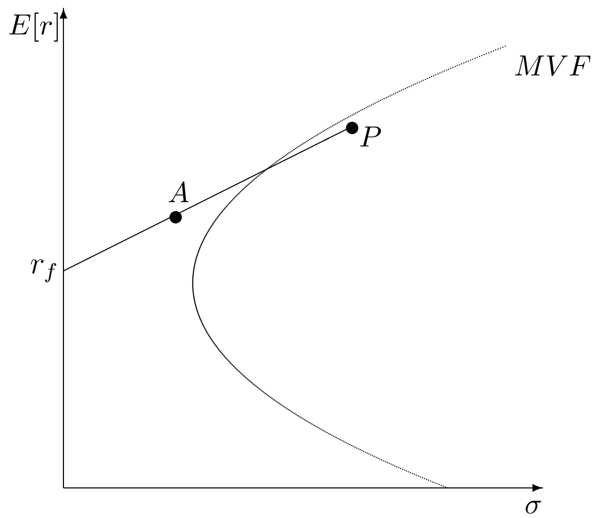

Figure 2: How to obtain portfolio A.

be a margin call. We can nevertheless look at the exact value of the margin ratio after one year, which is

$$\frac{\text{assets} - \text{liabilities}}{\text{assets}} = \frac{35,000 \ - \ 15,000 \times 1.06}{35,000} = 54.57\% \ > \ 30\%.$$

Hence there is no need to deposit any money in the brokerage account. The return on the stock is

$$\frac{35 - 30}{30} = 16.67\% \ .$$

As we can see, buying on margin has magnified the gain.

(b) (4 points) If the stock price of XYZ is \$20 one year from now, what will the return on the margin purchase be? Will you receive a margin call? If so, how much money will you have to deposit in your account? How does the return on the margin purchase compare with the return on the stock?

Answer: The return on the margin purchase is then

$$\frac{20 \times 1,000 - 15,000 \times 1.06 - 15,000}{15,000} = \ - 72.67\% \ .$$

The margin ratio after one year is

$$\frac{\text{assets} - \text{liabilities}}{\text{assets}} = \frac{20,000 \ - \ 15,000 \times 1.06}{20,000} = 20.5\% \ < \ 30\%.$$

Since the margin ratio is less than 30%, some money has to be deposited in the brokerage account. The deposit, D, has to be such that

$$\frac{20,000 \ - \ 15,000 \times 1.06 \ + \ D}{20,000} = 30\% \qquad \Rightarrow \qquad D \ = \ \$1,900 \ .$$

The return on the stock itself is

$$\frac{20 - 30}{30} \; = \; -\,33.33\% \;,$$

and thus buying on margin has magnified the loss.

(c) (2 points) How large has to be the return on XYZ stock over one year for this margin purchase to be profitable?

Answer: Let r denote the return on the stock. That is, let $p_1 = (1+r)30$ denote the stock price after one year. The return on the margin purchase is then

$$\frac{30,000(1+r) \;-\; \overbrace{15,900}^{1.06\times 15,000} \;-\; 15,000}{15,000} = \frac{30,000r \;-\; 900}{15,000}.$$

Having a positive return means

$$\frac{30,000r \;-\; 900}{15,000} \;>\; 0 \quad \Rightarrow \quad r \;>\; 3\%$$

2. *Mutual Funds* Consider the three following funds: Fund A, that charges a front-end load fee of 5 percent and operating expenses of .5% of total assets; Fund B, that charges a back-end load fee of 4% for shares redeemed in the first two years and 1% thereafter, with operating expenses of 1.5% of total assets; Fund C, that has no load fee and operating expenses of 2.5% of total assets. All of these funds have no liabilities.

(a) (6 points) Suppose the assets held by each fund return 10% annually. What is the net return on each fund after (i) 2 years? (ii) 5 years? (iii) 10 years?

Answer: The yearly return to a fund with a load fee F, an operating expense ratio e and a return i to its assets is given by

$$\begin{aligned} r &= \frac{\text{Assets}_1 - \text{Liabilities}_1 - (\text{Assets}_0 - \text{Liabilities}_0) - e \times \text{Assets}_1}{\text{Assets}_0 - \text{Liabilities}_0} \\ &= \frac{(1+r) \times \text{Assets}_0 - \text{Assets}_0 - e \times (1+r) \times \text{Assets}_0}{\text{Assets}_0} \\ &= 1 \;+\; r \;-\; e \;-\; er \end{aligned}$$

Note that er is very small and thus $1 + r - e - er \approx 1 + r - e$. However, the exact rate of return will be used hereafter. Let $r_{t,i}$ denote the return to Fund

Fund	2-year return	5-year return	10-year return
A	13.8%	49.2%	134.4%
B	12.7%	47.8%	120.8%
C	15.0%	41.9%	101.4%

Table 1: Compounded returns to funds A, B and C.

$i = A, B, C$ after t years. Then

$$r_{t,A} = .95(1 + .10 - .005 - .0005)^t - 1 = .95(1.0945)^t - 1;$$

$$r_{t,B} = \begin{cases} .96(1 + .10 - .015 - .0015)^t - 1 = .96(1.0835)^t - 1 & \text{if } t \leq 2, \\ .99(1 + .10 - .015 - .0015)^t - 1 = .99(1.0835)^t - 1 & \text{if } t > 2; \end{cases}$$

$$r_{t,C} = (1 + .10 - .025 - .0025)^t - 1 = (1.0725)^t - 1.$$

The compounded returns are given in Table 1.

(b) (2 points) Suppose the annual return on Fund A's assets is 10%. What has to be the annual return on Fund B's assets for the two funds to yield the same net return after 5 years?

Answer: Let r_b denote the annual return to Fund B's assets. Then

$$.95(1.0945)^5 - 1 = .99(1 - .015 + r_b(1 - .015))^5 - 1 \quad \Rightarrow \quad r_b = 10.2\% .$$

(c) (2 points) Suppose the annual return on Fund A's assets is 10%. What has to be the annual return on Fund C's assets for the two funds to yield the same net return after 10 years?

Answer: Let Let r_c denote the annual return to Fund C's assets. Then

$$.95(1.0945)^{10} - 1 = (1 - .025 + r_c(1 - .025))^{10} - 1 \quad \Rightarrow \quad r_c = 11.7\% .$$

3. *Portfolio Selection* Consider an investor with the following utility function over portfolios: $U(c) = E[r_c] - \frac{1}{2}A\sigma_c^2$, where c denotes a portfolio, $E[r_c]$ its the expected return and σ_c the standard deviation of the portfolio returns. The investor has access to a risk-free asset that returns 6%, and a portfolio of risky assets, p, with an expected return of 15% and standard deviation of 30%.

(a) (1 point) What is the slope of the capital allocation line for this problem?

Answer: The reward-to-variability ratio is

$$\frac{E[r_p] \; - \; r_f}{\sigma_p} \;=\; \frac{15\% - 6\%}{30\%} \;=\; 30\%.$$

(b) (5 points) What fraction of the investor's optimal portfolio is invested in p if $A = 5$? If $A = 2$? If $A = 1$? Find the value of A for which 10% of the optimal portfolio is invested in p.

Answer: Given A, the fraction of wealth invested in p is given by $y = \frac{E[r_p]-r_f}{A\sigma_p^2}$. That is,

$$y \;=\; \begin{cases} \frac{.15-.06}{5\times.09} \;=\; 20\% & \text{if } A = 5, \\ \frac{.15-.06}{2\times.09} \;=\; 50\% & \text{if } A = 2, \\ \frac{.15-.06}{1\times.09} \;=\; 100\% & \text{if } A = 1. \end{cases}$$

The value of A for which $y = 10\%$ is

$$\frac{.15 - .06}{A \times .09} \;=\; 10\% \qquad \Rightarrow \qquad A \;=\; 10.$$

(c) (4 points) Suppose there exists another risky portfolio, denoted q, available to the investor. If portfolio q has an expected return of 10% and a standard deviation of 10%, what fraction of the optimal portfolio will be invested in p? What fraction of the optimal portfolio will be invested in q?

Answer: Note that the reward-to-variability ratio associated to portfolio q is $\frac{.10-.06}{.10} = .4$, which is higher than the reward-to-variability ratio associated to portfolio p. Hence assuming that we cannot mix p and q together, any risky investment should be made in portfolio q. The fraction of the optimal portfolio invested in p is therefore 0 and the fraction invested in q is

$$\frac{E[r_q] - r_f}{A\sigma_q^2} \;=\; \frac{.10 - .06}{.01A} \;=\; \frac{4}{A}\,.$$