Comparison between TOF and non-TOF PET using a scan statistic numerical observer

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Motivation

- We want to compare two imaging methods A and B.
  - T is the true task performance (difficult to obtain)
  - M is the measured task performance, replaces T.
- The task performance measuring method M should have the following properties relative to T.
  - **Monotonicity** $M(A) > M(B)$ if $T(A) > T(B)$

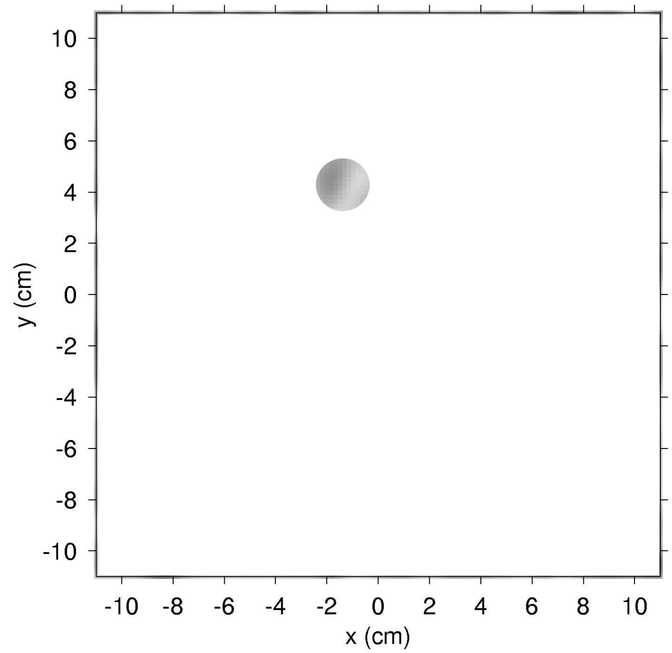
    Preserves the ranking order.
  - **Proportionality** (or the relation with the true performance should be known.)

    It can be used for cost-benefit analysis.
  - **Convenient**, not too difficult so it can be applied.

Small nodule detection task

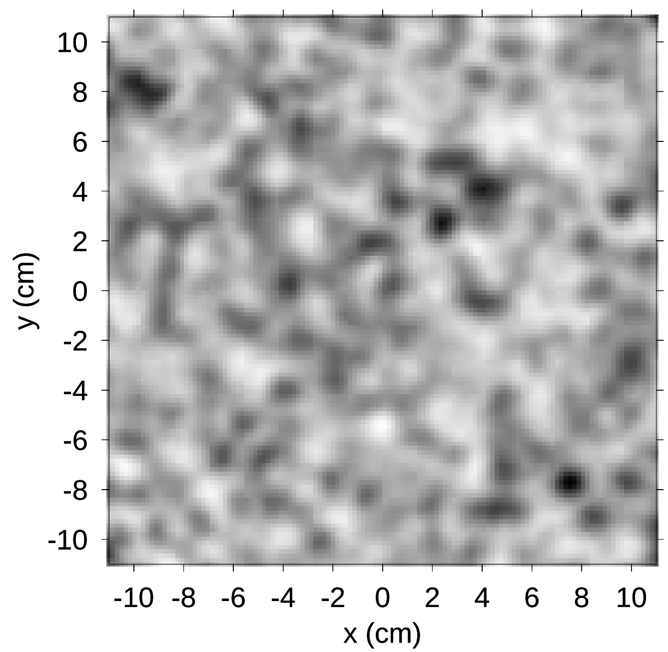
- Localization known

  (fixed position detection)



- Localization unknown

  (area search detection)

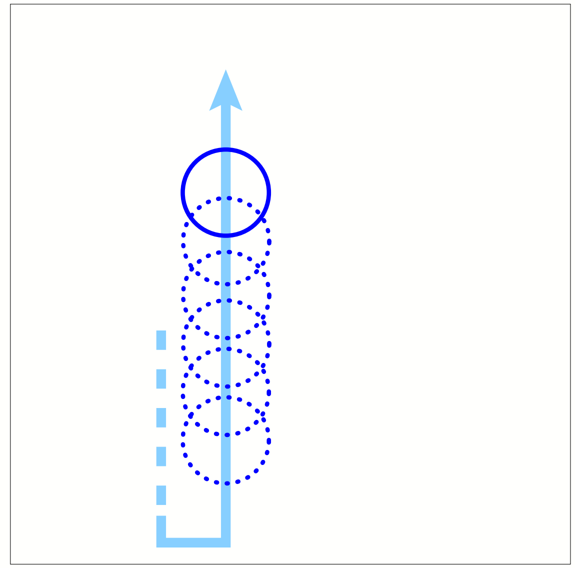


Known vs. unknown localization

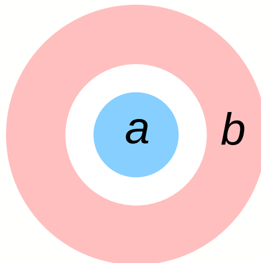
- **The localization known case**
  - We need to know the variations of a reduced region of the image.
  - One can go to great lengths and consider in detail the statistical variations of our image spot, obtaining “ideal” observers.
  - However, the variations with the position, within the same image, of equivalent image spots are more important.
- **The localization unknown case**
  - Signal like features randomly appear in the image background. For a detectability study we need to know:
    - the **frequency** of appearance,
    - and the **distribution** of such features with the variables used also to describe the signal (contrast, size, etc.).
  - We can obtain such information by **scanning the images**.

Nodule detection by image scanning

- Smoothly slide a scanning window over the area of interest, for each position compute some relevant local statistic



- E.g. local contrast $c = \frac{a}{b}$

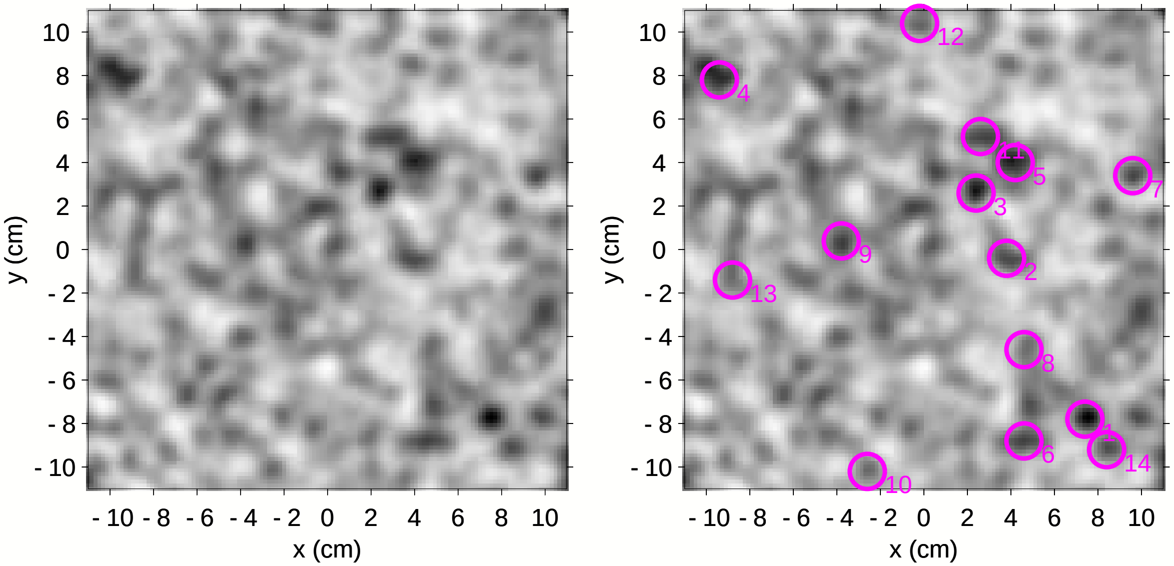


- More complex local statistics can be devised, that use variable sizes, or are matched with the signal profiles, etc.

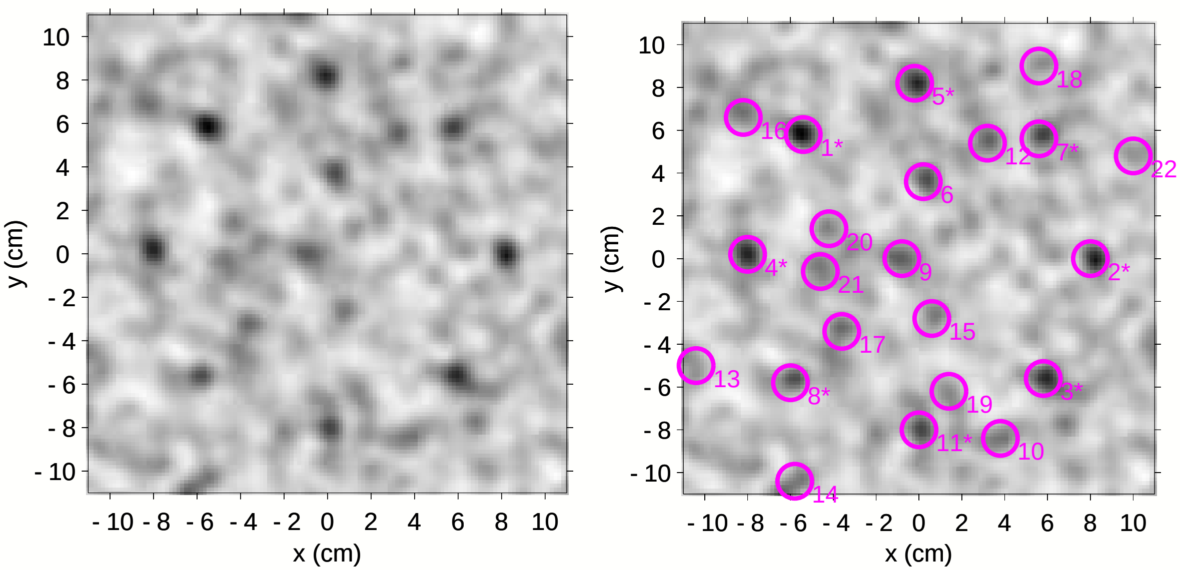
- If we want to find only the most suspicious feature all we need is the **maximum scan** value location.

Image scan examples

background only image



signals present image

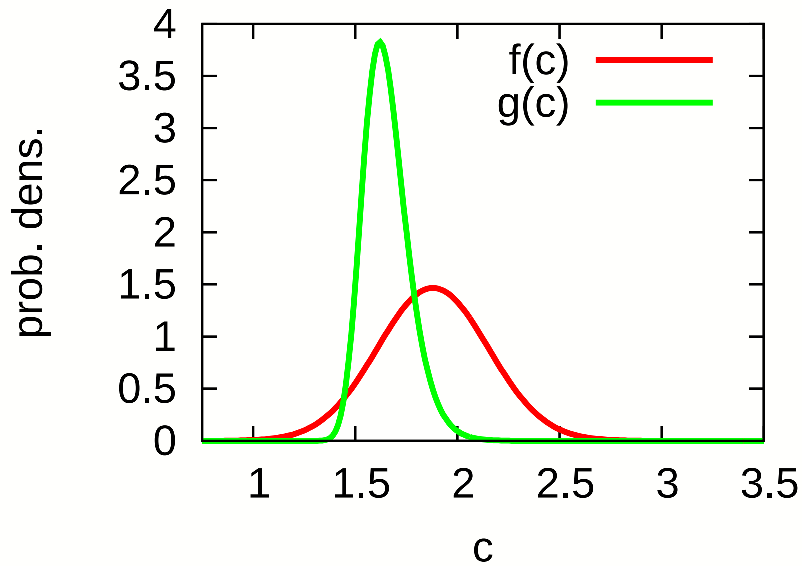


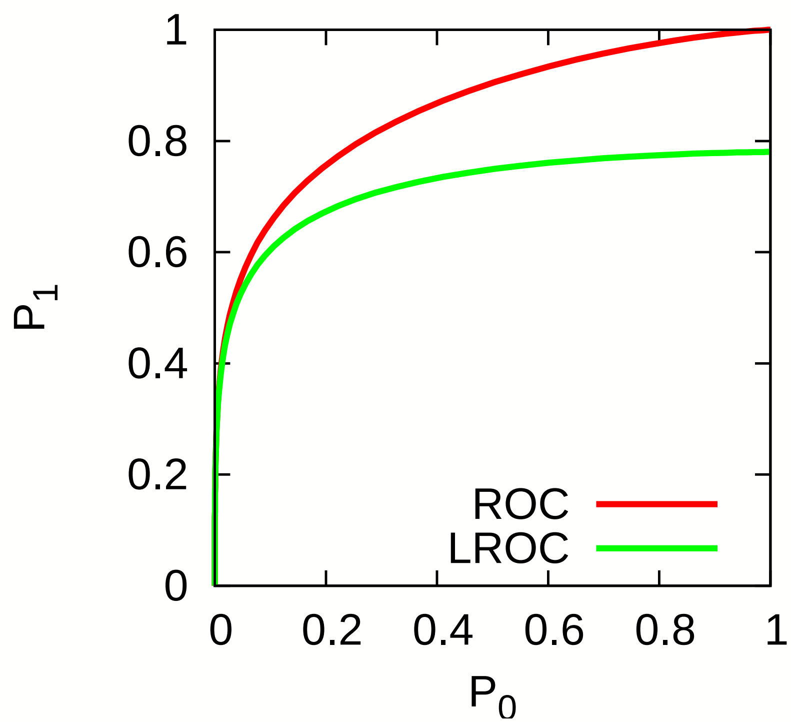
c	signal
2.10	√
1.97	√
1.83	√
1.81	√
1.75	√
1.64	
1.63	√
1.61	√
1.57	
1.52	
1.51	√
1.49	
1.49	
1.48	
1.46	
1.42	

Detectability evaluation using scan statistic

$g(c)$ – the distribution of max-scan results for background only images

$f(c)$ – signal distribution





(Swensson, *Med. Phys.*, 1996)

- The scan statistic techniques are often used when random fields must be searched for local abnormalities (epidemiology, astronomy, etc.).

Simulation study

- 2D PET images (no attenuation, no scatter)

  100–400 image samples for each case.

- non-TOF and various TOF resolutions considered

- data sets containing low, medium and high number of counts.

- Signals: uniform disks, diameter 1 cm, 3:1 contrast.

- Scan using local contrast for a disk of the same size.

  Were determined:

  - max-scan distribution $g(c)$ for different search area sizes:

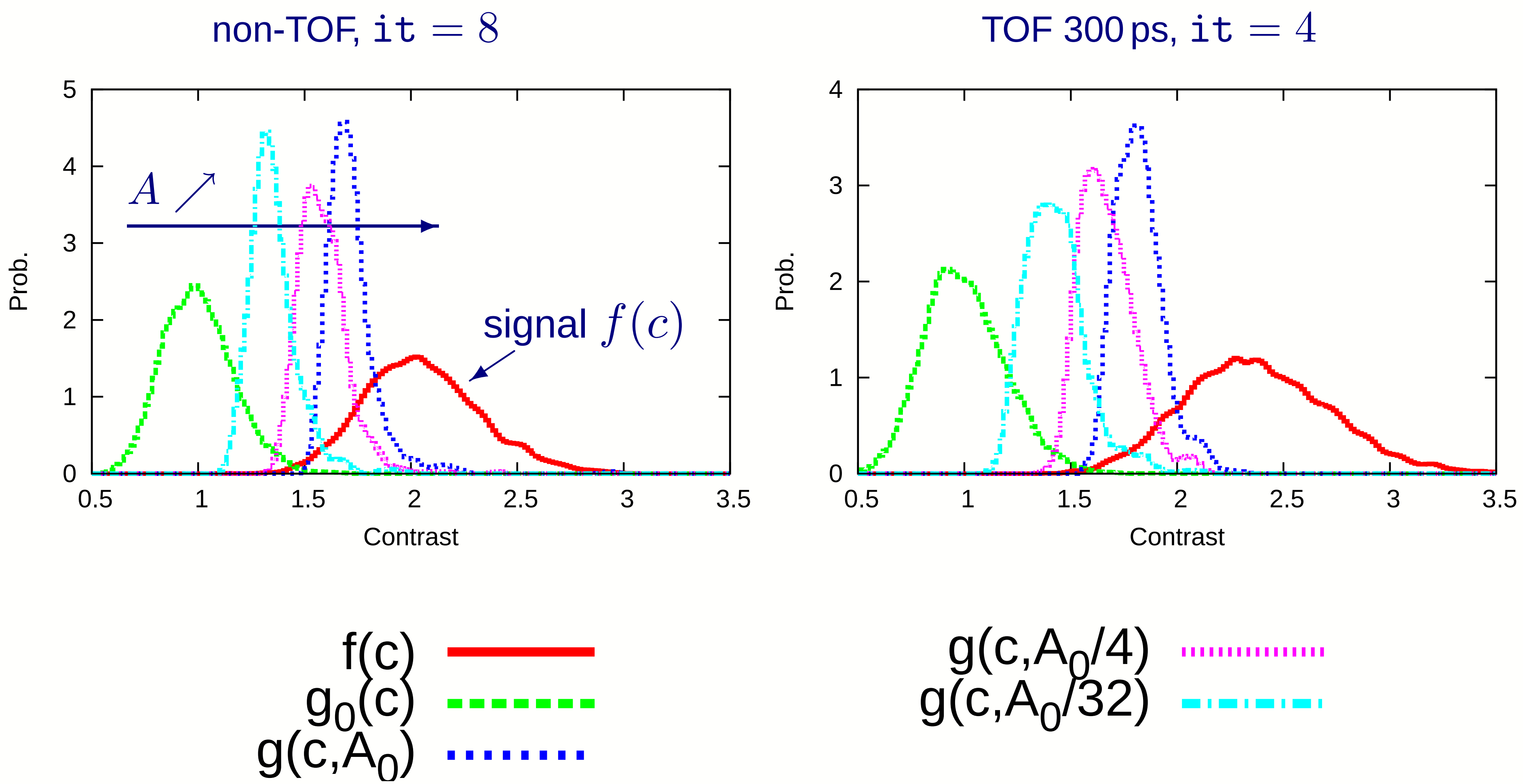
    $A_0 = 22 \times 22\,\text{cm}^2$, $A_0/2$, $A_0/4$, $A_0/16$, $A_0/32$.

  - distribution of the background at a fixed position $g_0(c)$

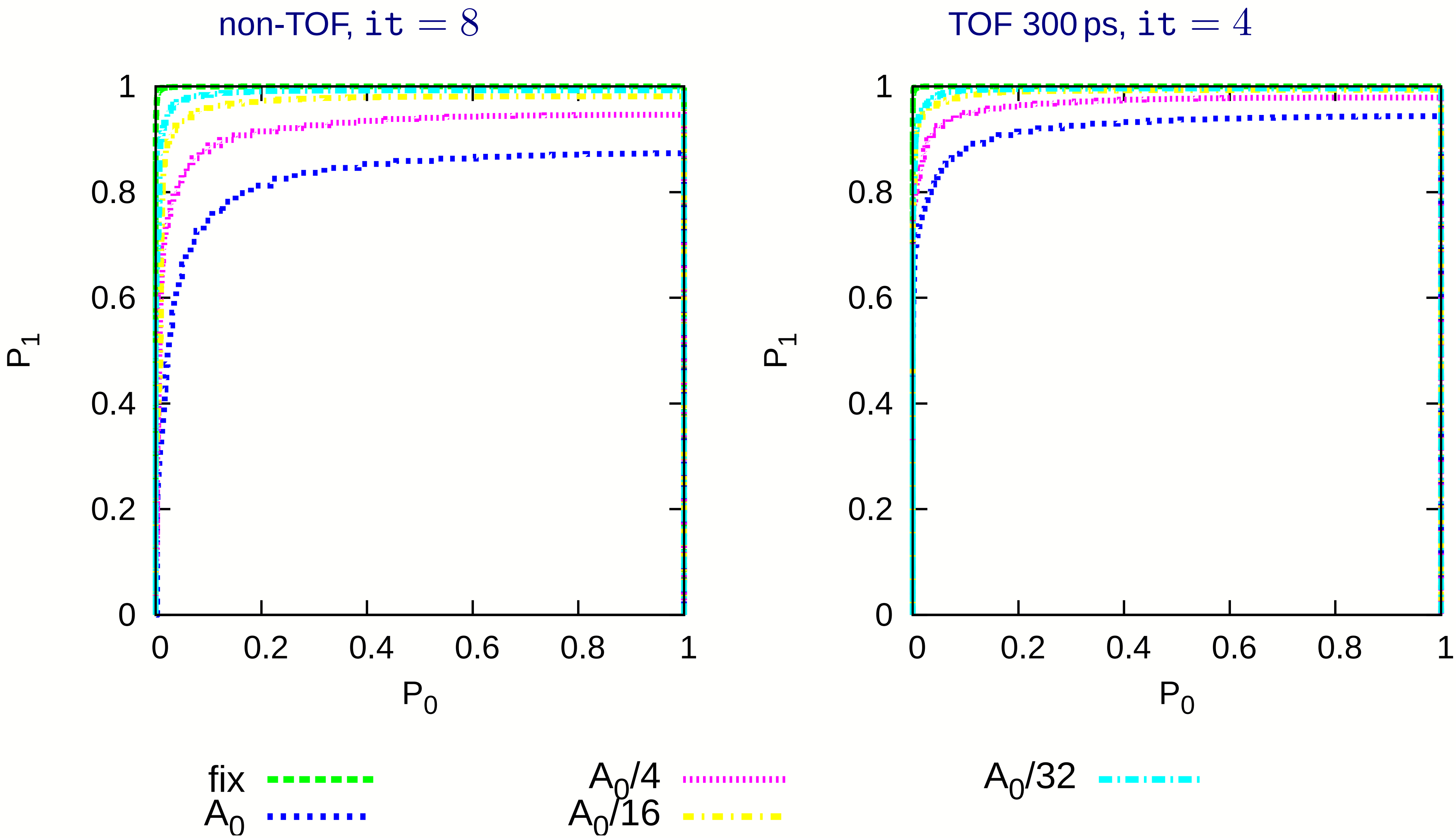
  - signal distribution $f(c)$

  - noise nodule density and contrast distribution

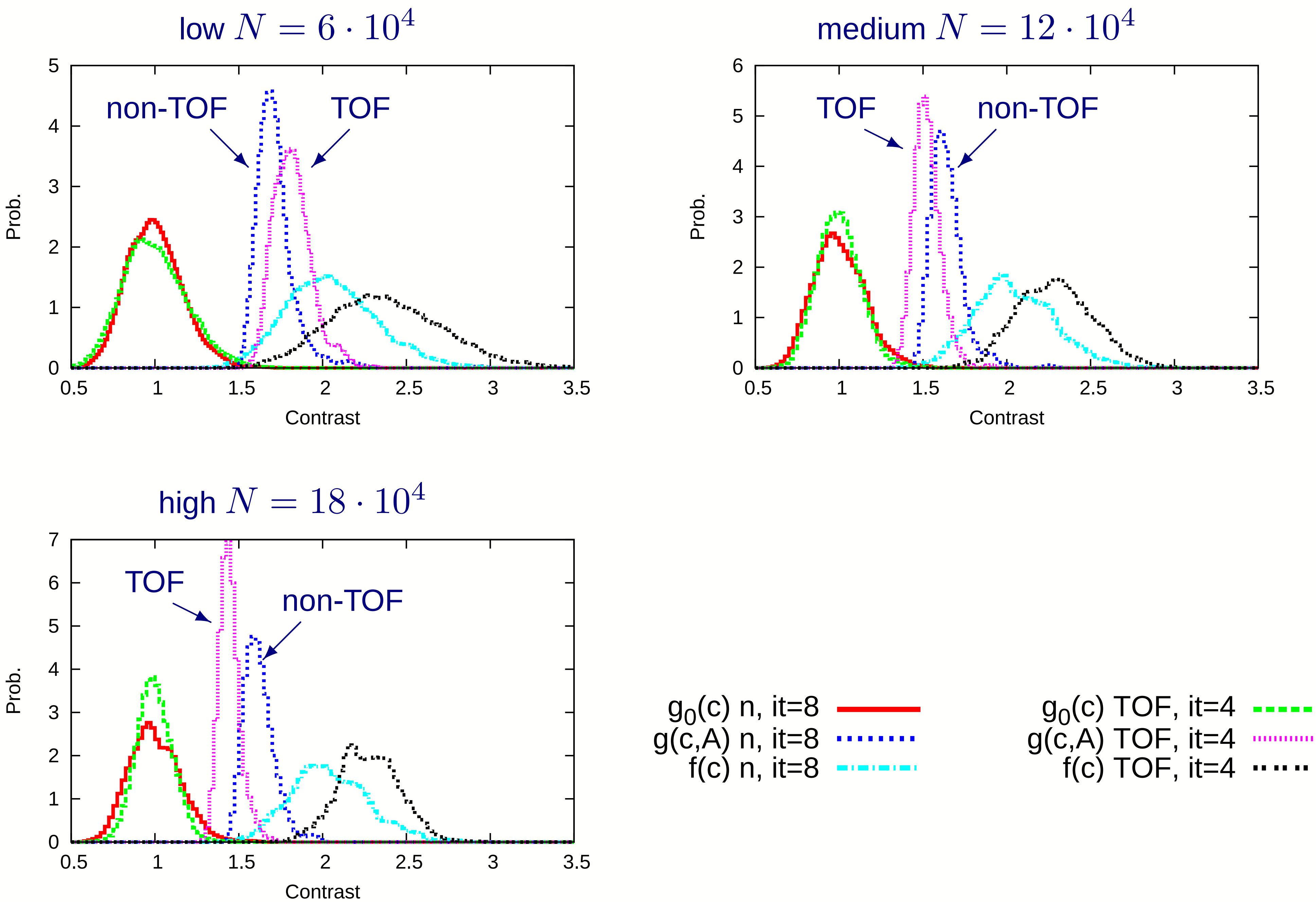
Max-scan distribution as function of the searched area



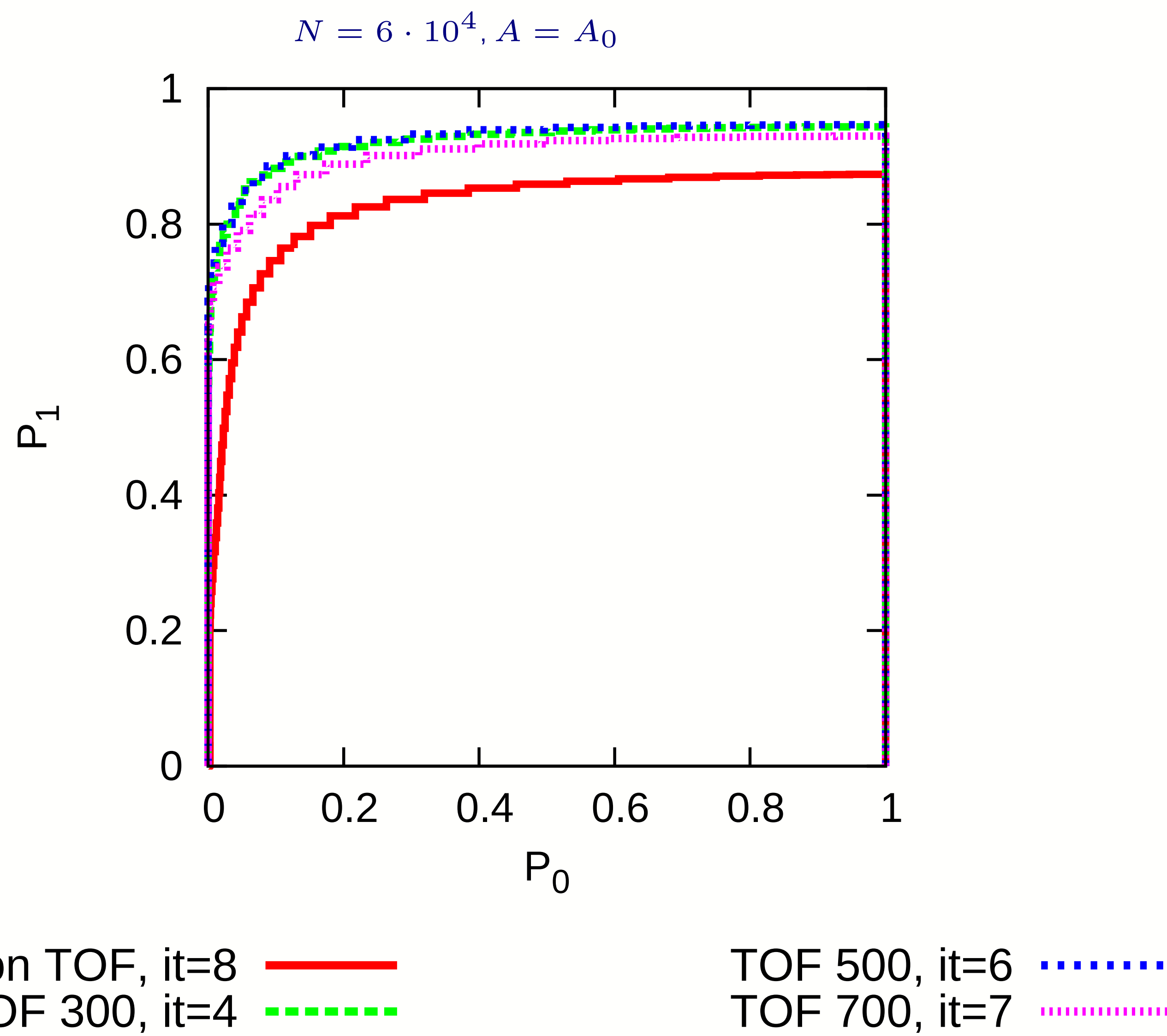
LROCs as function of the searched area



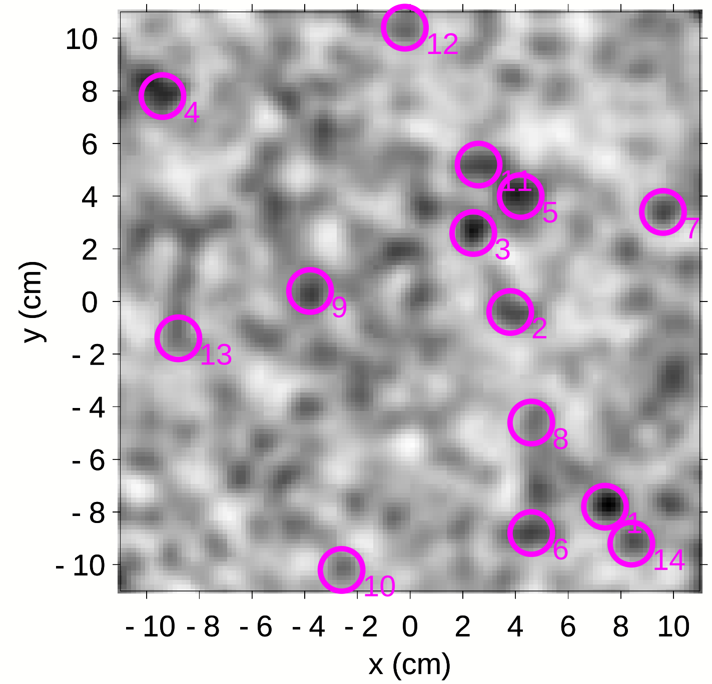
The dependence on the counts number



LROCs comparison for different TOF resolutions



- Instead of picking only the maximum contrast noise nodule, we can obtain a **list of all noise nodules with** $c > c_0$.
- We assume that such noise nodules are a relatively rare occurrence and they are independent of each other.



- A_t total area scanned, N_t total no. of noise nodules with $c > c_0$
  - The **density of noise nodules**: $n_0 = \frac{N_t}{A_t}$.
  - The **noise nodules contrast distribution**: $p(c)$, $\int_{c_0}^{\infty} p(c)\mathrm{d}c = 1$.

For a region of area A we have

- $N = n_0 A$ – the **average total number of noise nodules** (with $c > c_0$).
- $\nu(c) = Nq(c)$ – the **average number of noise nodules with contrast** $> c$,

  $$q(c) = \int_c^\infty p(c')\mathrm{d}c'.$$

- k – the **actual number of noise nodules with contrast** $> c$, a random variable. We assume that it follows the Poisson distribution (P. C. Bunch et al., *J. Appl. Photo. Eng.*, 1978)

$$P(k;c) = \frac{\nu^k}{k!}\mathrm{e}^{-\nu}$$

Similar result are obtained with binomial distribution

$$P(k;c) = \binom{N}{k}[1 - q(c)]^{N-k} q^k(c)$$

Noise nodule distribution model *III*

- The **probability of at least one noise nodule** with contrast greater than c

$$Q(1;c) = 1 - P(0;c) = 1 - \mathrm{e}^{-\nu}$$

- The **likelihood of k nodules** with contrast $c_1, \ldots, c_k$ greater than c

$$L(c_1, \ldots, c_k) = P(k;c) \prod_{j=1}^{k} \frac{p(c_j)}{q(c)} = \frac{N^k}{k!} \mathrm{e}^{-Nq(c)} \prod_{j=1}^{k} p(c_j)$$

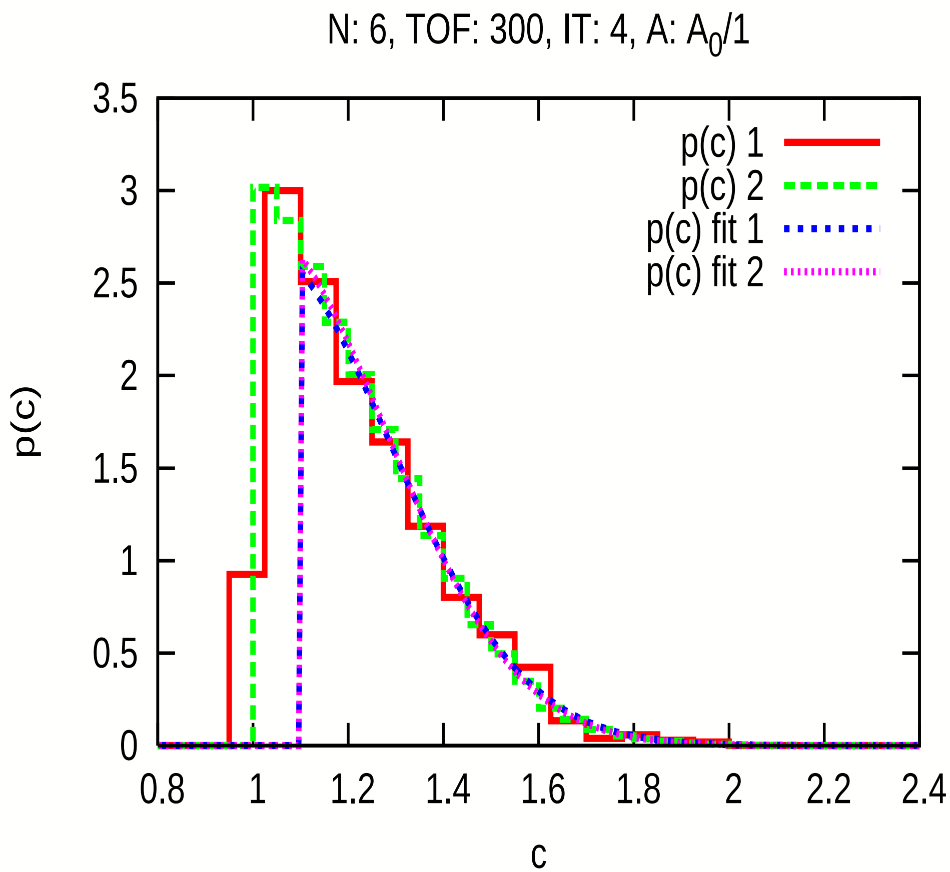
- The **max-scan distribution**

$$g(c) = P(1;c) \frac{p(c)}{q(c)} = Np(c)\mathrm{e}^{-Nq(c)}$$

The cumulative distribution $G(c) = \mathrm{e}^{-Nq(c)}$, $Q(1;c) = 1 - G(c)$.

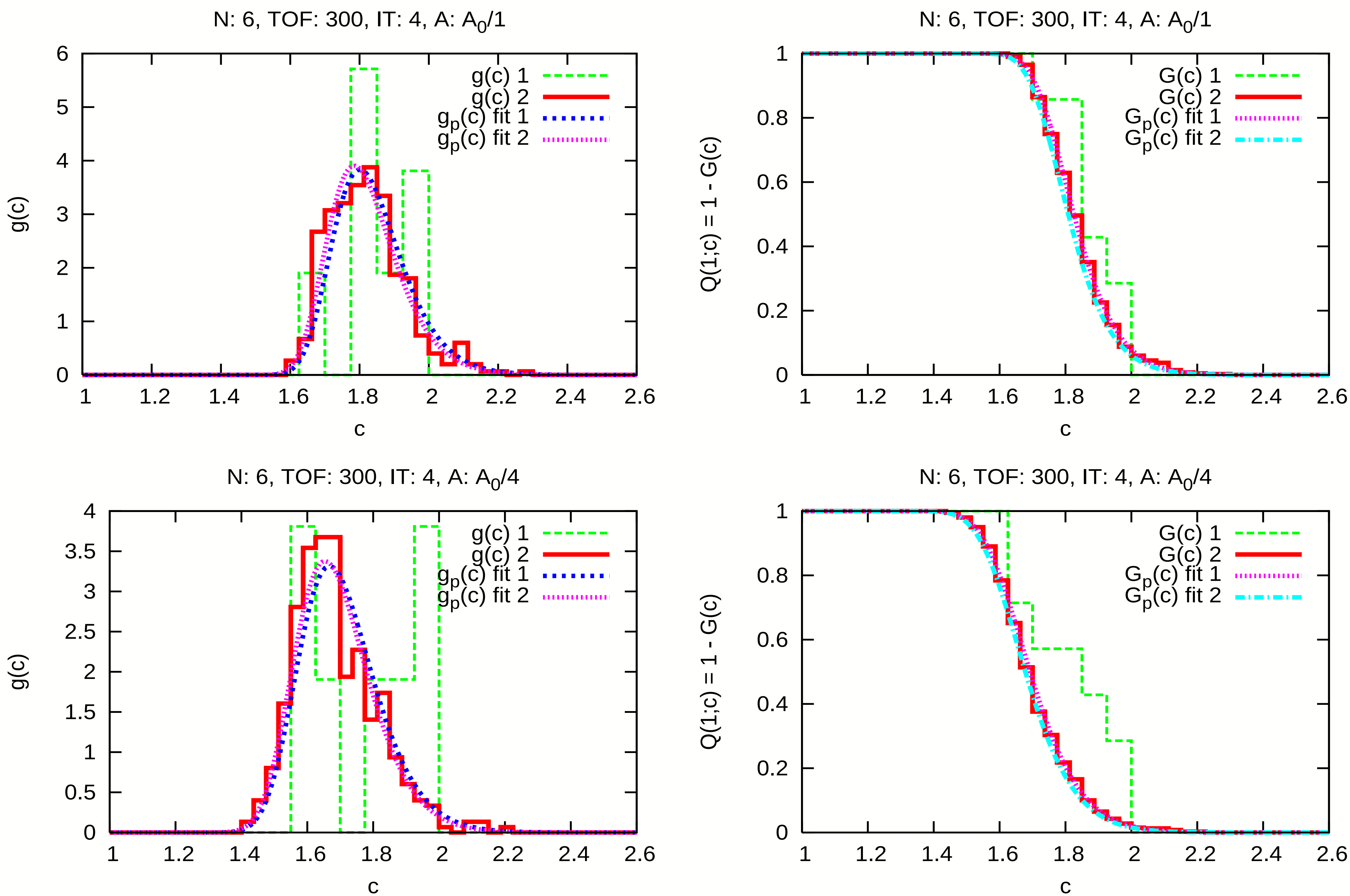
- Note: the **results depend on the searched region area size** ($N = An_0$)

Noise nodule distribution $p(c)$ example

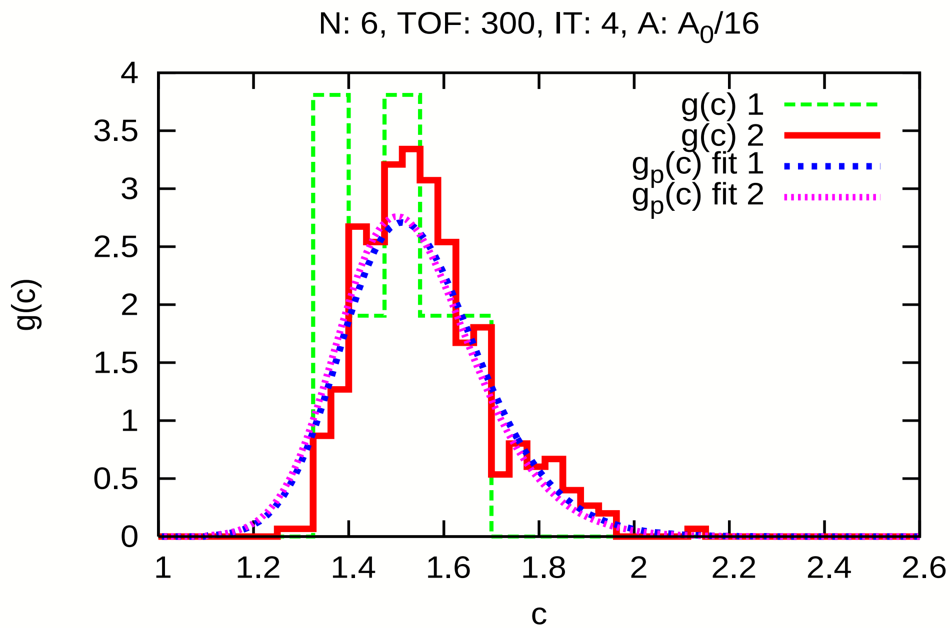


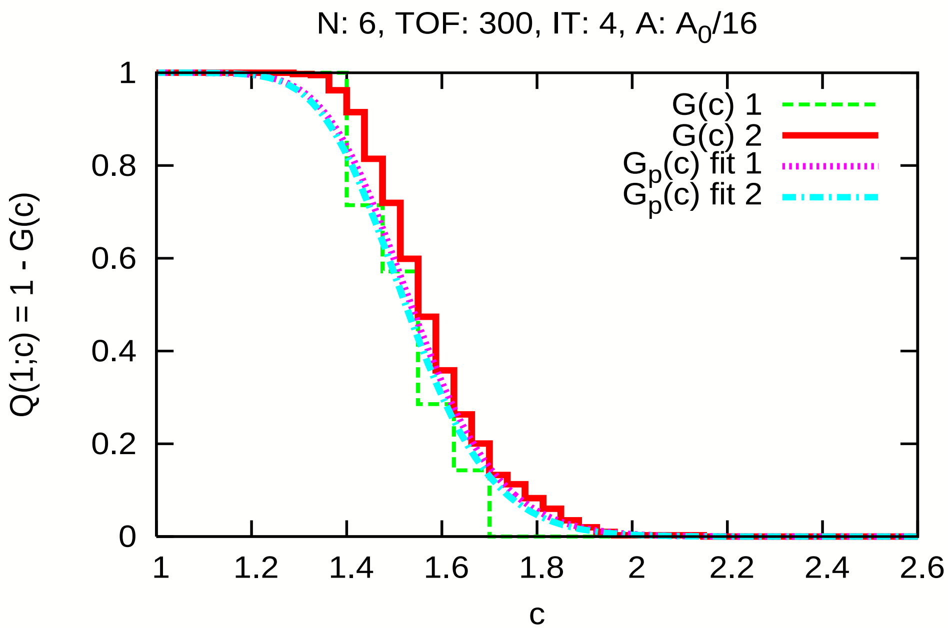
- $p(c)$ obtained for: 8 images (1), 400 images (2).
- Both histograms are well fitted by the tail of a Gaussian (Weibull also fits well).

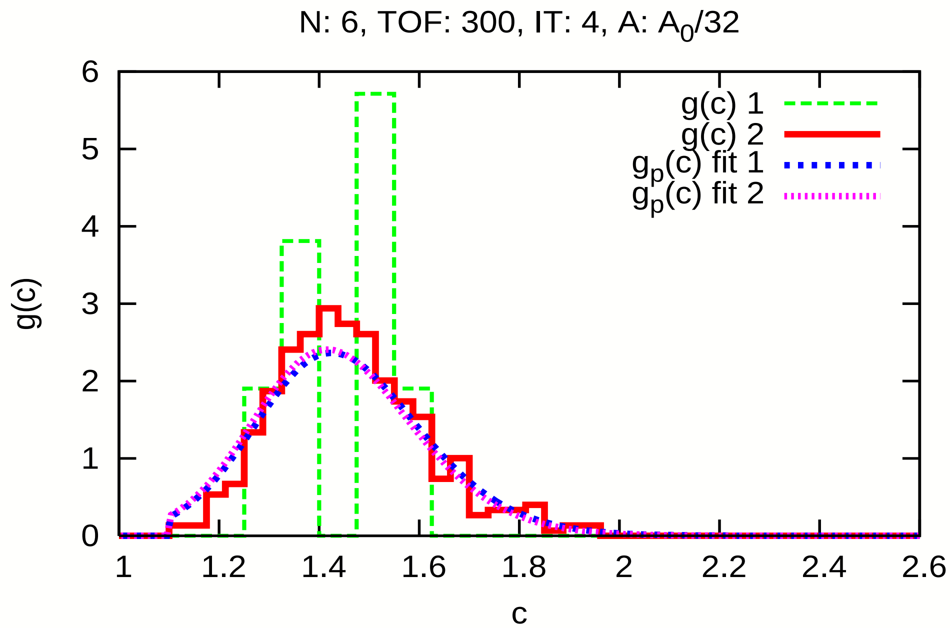
Max-scan $g(c)$ vs. estimated max-scan from $p(c)$ *I*

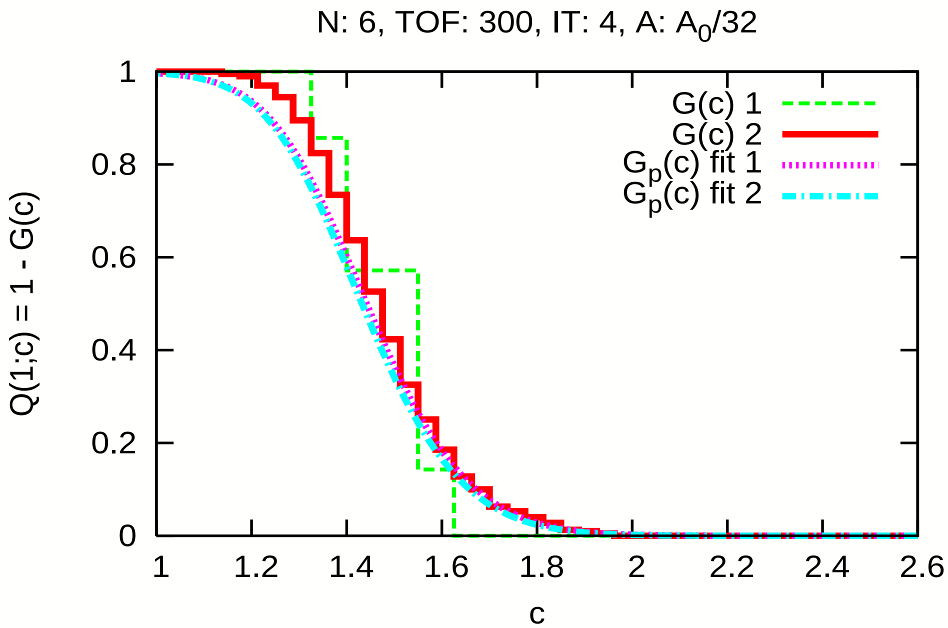


Max-scan $g(c)$ vs. estimated max-scan from $p(c)$ *II*









Conclusion

- Compared to fixed position evaluations the scan-statistic image evaluations are **more sensitive** to TOF resolution and the number of data counts.
- The search **area size is a factor that should be carefully considered** when the image evaluations are meant to produce results proportional to the real performance of the evaluated methods, as is required in cost-benefit analysis.

Conclusion

- The **generalized scan-statistic** approach has several advantages compared to the direct determination of the max-scan distribution $g(c)$.
  - Greater **efficiency**. One scan obtains multiple values for the determination of $p(c)$.

    It can be applied with only a **few image samples**.
  - Greater **flexibility**. The $p(c)$ distribution can be determined by searching areas of arbitrary size, and then the theory applied to a certain reference area size.
- Provides a **framework for treating the multiple features per image** case (Popescu & Lewitt, *Phys. Med. Biol.*, in print).