APPENDIX A

PSEUDO-CODE FOR VARIANCE CALCULATION PROGRAM

A.1. PDF for two Points in a Square with Side Length of *A*

$$f(x1, y1, r, A) = \int_{x1}^{y1} \frac{4r}{A^2}(1-\frac{r}{A}\cos q)(1-\frac{r}{A}\sin q)dq$$

$$f1 = \{0 < r < A, \quad 0 < q < q1\}$$

$$f2 = \left\{A < r < \sqrt{2}A, \quad a\cos(\frac{A}{r}) < q < a\sin(\frac{A}{r})\right\}$$

A.2. Correlation Coefficient for Two Points in a Rectangle with Side Length *L* and Width *W*

$$a(x2, y2, k, r, L, W, q1) = \frac{\int_{x2}^{y2}(1-\frac{r}{w}\sin q)(1-\frac{r}{l}\cos q)e^{k\cos[2(q)]}dq}{\int_{-q_1}^{q_1} e^{k\cos[2(j)]}dj}$$

$$a1 = \{0 < r < W, \quad 0 < q < q1\}$$

$$a2 = \left\{W < r < L, \quad 0 < q < a\sin(\frac{W}{r})\right\}$$

$$a3 = \left\{L < r < \sqrt{L^2 + W^2}, \quad a\cos(\frac{L}{r}) < q < a\sin(\frac{W}{r})\right\}$$

$$a4 = \{0 < r < L, \quad 0 < q < q1\}$$

$$a5 = \left\{L < r < \frac{W}{\sin q1}, \quad a\cos(\frac{L}{r}) < q < q1\right\}$$

$$a6 = \left\{\frac{W}{\sin q1} < r < \sqrt{L^2 + W^2}, \quad a\cos(\frac{L}{r}) < q < a\sin(\frac{W}{r})\right\}$$

$$a7 = \left\{L < r < \frac{L}{\cos q1}, \quad a\cos(\frac{L}{r}) < q < q1\right\}$$

A.3. Variance Function

$$Variance(W, L, A, Q1) = \int_{R1}^{R2} (f(x1, y1, r, A) \cdot alpha(x2, y2, k, r, L, W, q1) dr$$

A.4. Variance Calculation for the von Mises Distributed Flake Orientation ($k \neq 0$ and $q_1 = \frac{p}{2}$)

Inputs to the program are: flake length (L) and width (W), side length of square zones (A), and concentration parameter (k).

Equations used to implement the Visual Basic program

$$S1(L, W, A, k) := \int_0^A f\left(0, \frac{\pi}{2}, r, A\right) \cdot alpha\left(0, \frac{\pi}{2}, k, r, L, W\right) dr$$

$$S2(L, W, A, k) := \int_A^{\sqrt{2}\cdot A} f\left(acos\left(\frac{A}{r}\right), asin\left(\frac{A}{r}\right), r, A\right) \cdot alpha\left(0, \frac{\pi}{2}, k, r, L, W\right) dr$$

$$var1(L, W, A, k) := S1(L, W, A, k) + S2(L, W, A, k)$$

$$S3(L, W, A, k) := \int_A^W f\left(acos\left(\frac{A}{r}\right), asin\left(\frac{A}{r}\right), r, A\right) \cdot alpha\left(0, \frac{\pi}{2}, k, r, L, W\right) dr$$

$$S4(L, W, A, k) := \int_W^{\sqrt{2}\cdot A} f\left(acos\left(\frac{A}{r}\right), asin\left(\frac{A}{r}\right), r, A\right) \cdot alpha\left(0, asin\left(\frac{W}{r}\right), k, r, L, W\right) dr$$

$$var2(L, W, A, k) := S1(L, W, A, k) + S3(L, W, A, k) + S4(L, W, A, k)$$

$$S5(L, W, A, k) := \int_W^L f\left(acos\left(\frac{A}{r}\right), asin\left(\frac{A}{r}\right), r, A\right) \cdot alpha\left(0, asin\left(\frac{W}{r}\right), k, r, L, W\right) dr$$

$$S6(L, W, A, k) := \int_L^{\sqrt{2}\cdot A} f\left(acos\left(\frac{A}{r}\right), asin\left(\frac{A}{r}\right), r, A\right) \cdot alpha\left(acos\left(\frac{L}{r}\right), asin\left(\frac{W}{r}\right), k, r, L, W\right) dr$$

$$var3(L,W,A,k) := S1(L,W,A,k) + S3(L,W,A,k) + S5(L,W,A,k) + S6(L,W,A,k)$$

$$S7(L,W,A,k) := \int_{L}^{\sqrt{L^2+W^2}} f\left(acos\left(\frac{A}{r}\right), asin\left(\frac{A}{r}\right), r, A\right) \cdot alpha\left(acos\left(\frac{L}{r}\right), asin\left(\frac{W}{r}\right), k, r, L, W\right) dr$$

$$var4(L,W,A,k) := S1(L,W,A,k) + S3(L,W,A,k) + S5(L,W,A,k) + S7(L,W,A,k)$$

$$S8(L,W,A,k) := \int_{0}^{W} f\left(0, \frac{\pi}{2}, r, A\right) \cdot alpha\left(0, \frac{\pi}{2}, k, r, L, W\right) dr$$

$$S9(L,W,A,k) := \int_{W}^{A} f\left(0, \frac{\pi}{2}, r, A\right) \cdot alpha\left(0, asin\left(\frac{W}{r}\right), k, r, L, W\right) dr$$

$$S10(L,W,A,k) := \int_{A}^{\sqrt{2}\cdot A} f\left(acos\left(\frac{A}{r}\right), asin\left(\frac{A}{r}\right), r, A\right) \cdot alpha\left(0, asin\left(\frac{W}{r}\right), k, r, L, W\right) dr$$

$$var5(L,W,A,k) := S8(L,W,A,k) + S9(L,W,A,k) + S10(L,W,A,k)$$

$$S11(L,W,A,k) := \int_{A}^{L} f\left(acos\left(\frac{A}{r}\right), asin\left(\frac{A}{r}\right), r, A\right) \cdot alpha\left(0, asin\left(\frac{W}{r}\right), k, r, L, W\right) dr$$

$$var6(L,W,A,k) := S8(L,W,A,k) + S9(L,W,A,k) + S11(L,W,A,k) + S6(L,W,A,k)$$

$$var7(L,W,A,k) := S8(L,W,A,k) + S9(L,W,A,k) + S11(L,W,A,k) + S7(L,W,A,k)$$

$$S12(L,W,A,k) := \int_{W}^{L} f\left(0, \frac{\pi}{2}, r, A\right) \cdot alpha\left(0, asin\left(\frac{W}{r}\right), k, r, L, W\right) dr$$

$$S13(L,W,A,k) := \int_{L}^{A} f\left(0, \frac{\pi}{2}, r, A\right) \cdot alpha\left(acos\left(\frac{L}{r}\right), asin\left(\frac{W}{r}\right), k, r, L, W\right) dr$$

$$S14(L,W,A,k) := \int_{A}^{\sqrt{2}\cdot A} f\left(acos\left(\frac{A}{r}\right), asin\left(\frac{A}{r}\right), r, A\right) \cdot alpha\left(acos\left(\frac{L}{r}\right), asin\left(\frac{W}{r}\right), k, r, L, W\right) dr$$

$$var8(L,W,A,k) := S8(L,W,A,k) + S12(L,W,A,k) + S13(L,W,A,k) + S14(L,W,A,k)$$

$$S15(L,W,A,k) := \int_{A}^{\sqrt{L^2+W^2}} f\left(acos\left(\frac{A}{r}\right), asin\left(\frac{A}{r}\right), r, A\right) \cdot alpha\left(acos\left(\frac{L}{r}\right), asin\left(\frac{W}{r}\right), k, r, L, W\right)\, dr$$

$$var9(L,W,A,k) := S8(L,W,A,k) + S12(L,W,A,k) + S13(L,W,A,k) + S15(L,W,A,k)$$

$$S16(L,W,A,k) := \int_{L}^{\sqrt{L^2+W^2}} f\left(0, \frac{\pi}{2}, r, A\right) \cdot alpha\left(acos\left(\frac{L}{r}\right), asin\left(\frac{W}{r}\right), k, r, L, W\right)\, dr$$

$$var10(L,W,A,k) := S8(L,W,A,k) + S12(L,W,A,k) + S16(L,W,A,k)$$

Visual Basic pseudo-code

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if (A<W)
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    if (√2A<W )
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$$\text{var} = Var1 = \int_0^A f1 \cdot a1\, dr + \int_A^{\sqrt{2}A} f2 \cdot a1\, dr$$

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    elseif (√2A<L )
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$$\text{var} = Var2 = \int_0^A f1 \cdot a1\, dr + \int_A^W f2 \cdot a1\, dr + \int_W^{\sqrt{2}A} f2 \cdot a2\, dr$$

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    elseif (√2A<√(L^2+W^2) )
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$$\text{var} = Var3 = \int_0^A f1 \cdot a1\, dr + \int_A^W f2 \cdot a1\, dr + \int_W^L f2 \cdot a2\, dr + \int_L^{\sqrt{2}A} f2 \cdot a3\, dr$$

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    else
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$$\text{var} = Var4 = \int_0^A f1 \cdot a1\, dr + \int_A^W f2 \cdot a1\, dr + \int_W^L f2 \cdot a2\, dr + \int_L^{\sqrt{L^2+W^2}} f2 \cdot a3\, dr$$

```
    endif
elseif (W<A<L )
    if (√2A<L )
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$$\text{var} = Var5 = \int_0^W f1 \cdot a1\, dr + \int_W^A f1 \cdot a2\, dr + \int_A^{\sqrt{2}A} f2 \cdot a2\, dr$$

elseif ($\sqrt{2}A < \sqrt{L^2 + W^2}$)

$$\text{var} = Var6 = \int_0^W f1 \cdot a1\, dr + \int_W^A f1 \cdot a2\, dr + \int_A^L f2 \cdot a2\, dr + \int_L^{\sqrt{2}A} f2 \cdot a3\, dr$$

else

$$\text{var} = Var7 = \int_0^W f1 \cdot a1\, dr + \int_W^A f1 \cdot a2\, dr + \int_A^L f2 \cdot a2\, dr + \int_L^{\sqrt{L^2+W^2}} f2 \cdot a3\, dr$$

endif

elseif ($L < A < \sqrt{L^2 + W^2}$)

if ($\sqrt{2}A < \sqrt{L^2 + W^2}$)

$$\text{var} = Var8 = \int_0^W f1 \cdot a1\, dr + \int_W^L f1 \cdot a2\, dr + \int_L^A f1 \cdot a3\, dr + \int_A^{\sqrt{2}A} f2 \cdot a3\, dr$$

else

$$\text{var} = Var9 = \int_0^W f1 \cdot a1\, dr + \int_W^L f1 \cdot a2\, dr + \int_L^A f1 \cdot a3\, dr + \int_A^{\sqrt{L^2+W^2}} f2 \cdot a3\, dr$$

endif

else

$$\text{var} = Var10 = \int_0^W f1 \cdot a1\, dr + \int_W^L f1 \cdot a2\, dr + \int_L^{\sqrt{L^2+W^2}} f1 \cdot a3\, dr$$

endif

A.5. Variance Calculation for the Uniform Distributed Flake Orientation (k = 0)

Inputs to the program are: flake length (L) and width (W), side length of square zones (A), and range of angles (q_1).

Equations used to implement the Visual Basic program

$$T1(L, W, A, \theta 1) := \int_0^A f\left(0, \frac{\pi}{2}, r, A\right) \cdot \text{alpha}(0, \theta 1, \theta 1, r, L, W)\, dr$$

$$T2(L, W, A, \theta 1) := \int_A^{\sqrt{2}\cdot A} f\left(\text{acos}\left(\frac{A}{r}\right), \text{asin}\left(\frac{A}{r}\right), r, A\right) \cdot \text{alpha}(0, \theta 1, \theta 1, r, L, W)\, dr$$

$$var1(L,W,A,\theta 1) := T1(L,W,A,\theta 1) + T2(L,W,A,\theta 1)$$

$$T3(L,W,A,\theta 1) := \int_{A}^{\frac{W}{\sin(\theta 1)}} f\left(\operatorname{acos}\left(\frac{A}{r}\right), \operatorname{asin}\left(\frac{A}{r}\right), r, A\right) \cdot alpha(0,\theta 1,\theta 1,r,L,W)\, dr$$

$$T4(L,W,A,\theta 1) := \int_{\frac{W}{\sin(\theta 1)}}^{\sqrt{2}\cdot A} f\left(\operatorname{acos}\left(\frac{A}{r}\right), \operatorname{asin}\left(\frac{A}{r}\right), r, A\right) \cdot alpha\left(0, \operatorname{asin}\left(\frac{W}{r}\right), \theta 1, r, L, W\right) dr$$

$$var2(L,W,A,\theta 1) := T1(L,W,A,\theta 1) + T3(L,W,A,\theta 1) + T4(L,W,A,\theta 1)$$

$$T5(L,W,A,\theta 1) := \int_{\frac{W}{\sin(\theta 1)}}^{L} f\left(\operatorname{acos}\left(\frac{A}{r}\right), \operatorname{asin}\left(\frac{A}{r}\right), r, A\right) \cdot alpha\left(0, \operatorname{asin}\left(\frac{W}{r}\right), \theta 1, r, L, W\right) dr$$

$$T6(L,W,A,\theta 1) := \int_{L}^{\sqrt{2}\cdot A} f\left(\operatorname{acos}\left(\frac{A}{r}\right), \operatorname{asin}\left(\frac{A}{r}\right), r, A\right) \cdot alpha\left(\operatorname{acos}\left(\frac{L}{r}\right), \operatorname{asin}\left(\frac{W}{r}\right), \theta 1, r, L, W\right) dr$$

$$var3(L,W,A,\theta 1) := T1(L,W,A,\theta 1) + T3(L,W,A,\theta 1) + T5(L,W,A,\theta 1) + T6(L,W,A,\theta 1)$$

$$T7(L,W,A,\theta 1) := \int_{L}^{\sqrt{L^2+W^2}} f\left(\operatorname{acos}\left(\frac{A}{r}\right), \operatorname{asin}\left(\frac{A}{r}\right), r, A\right) \cdot alpha\left(\operatorname{acos}\left(\frac{L}{r}\right), \operatorname{asin}\left(\frac{W}{r}\right), \theta 1, r, L, W\right) dr$$

$$var4(L,W,A,\theta 1) := T1(L,W,A,\theta 1) + T3(L,W,A,\theta 1) + T5(L,W,A,\theta 1) + T7(L,W,A,\theta 1)$$

$$T8(L,W,A,\theta 1) := \int_{0}^{\frac{W}{\sin(\theta 1)}} f\left(0, \frac{\pi}{2}, r, A\right) \cdot alpha(0,\theta 1,\theta 1,r,L,W)\, dr$$

$$T9(L,W,A,\theta 1) := \int_{\frac{W}{\sin(\theta 1)}}^{A} f\left(0, \frac{\pi}{2}, r, A\right) \cdot alpha\left(0, \operatorname{asin}\left(\frac{W}{r}\right), \theta 1, r, L, W\right) dr$$

$$T10(L,W,A,\theta 1) := \int_{A}^{\sqrt{2}\cdot A} f\left(\operatorname{acos}\left(\frac{A}{r}\right), \operatorname{asin}\left(\frac{A}{r}\right), r, A\right)\cdot \operatorname{alpha}\left(0, \operatorname{asin}\left(\frac{W}{r}\right), \theta 1, r, L, W\right)\, dr$$

$$var5(L,W,A,\theta 1) := T8(L,W,A,\theta 1) + T9(L,W,A,\theta 1) + T10(L,W,A,\theta 1)$$

$$T11(L,W,A,\theta 1) := \int_{A}^{L} f\left(\operatorname{acos}\left(\frac{A}{r}\right), \operatorname{asin}\left(\frac{A}{r}\right), r, A\right)\cdot \operatorname{alpha}\left(0, \operatorname{asin}\left(\frac{W}{r}\right), \theta 1, r, L, W\right)\, dr$$

$$var6(L,W,A,\theta 1) := T8(L,W,A,\theta 1) + T9(L,W,A,\theta 1) + T11(L,W,A,\theta 1) + T6(L,W,A,\theta 1)$$

$$var7(L,W,A,\theta 1) := T8(L,W,A,\theta 1) + T9(L,W,A,\theta 1) + T11(L,W,A,\theta 1) + T7(L,W,A,\theta 1)$$

$$T12(L,W,A,\theta 1) := \int_{\frac{W}{\sin(\theta 1)}}^{L} f\left(0, \frac{\pi}{2}, r, A\right)\cdot \operatorname{alpha}\left(0, \operatorname{asin}\left(\frac{W}{r}\right), \theta 1, r, L, W\right)\, dr$$

$$T13(L,W,A,\theta 1) := \int_{L}^{A} f\left(0, \frac{\pi}{2}, r, A\right)\cdot \operatorname{alpha}\left(\operatorname{acos}\left(\frac{L}{r}\right), \operatorname{asin}\left(\frac{W}{r}\right), \theta 1, r, L, W\right)\, dr$$

$$T14(L,W,A,\theta 1) := \int_{A}^{\sqrt{2}\cdot A} f\left(\operatorname{acos}\left(\frac{A}{r}\right), \operatorname{asin}\left(\frac{A}{r}\right), r, A\right)\cdot \operatorname{alpha}\left(\operatorname{acos}\left(\frac{L}{r}\right), \operatorname{asin}\left(\frac{W}{r}\right), \theta 1, r, L, W\right)\, dr$$

$$var8(L,W,A,\theta 1) := T8(L,W,A,\theta 1) + T12(L,W,A,\theta 1) + T13(L,W,A,\theta 1) + T14(L,W,A,\theta 1)$$

$$T15(L,W,A,\theta 1) := \int_{A}^{\sqrt{L^2+W^2}} f\left(\operatorname{acos}\left(\frac{A}{r}\right), \operatorname{asin}\left(\frac{A}{r}\right), r, A\right)\cdot \operatorname{alpha}\left(\operatorname{acos}\left(\frac{L}{r}\right), \operatorname{asin}\left(\frac{W}{r}\right), \theta 1, r, L, W\right)\, dr$$

$$var9(L,W,A,\theta 1) := T8(L,W,A,\theta 1) + T12(L,W,A,\theta 1) + T13(L,W,A,\theta 1) + T15(L,W,A,\theta 1)$$

$$T16(L,W,A,\theta 1) := \int_{L}^{\sqrt{L^2+W^2}} f\left(0, \frac{\pi}{2}, r, A\right)\cdot \operatorname{alpha}\left(\operatorname{acos}\left(\frac{L}{r}\right), \operatorname{asin}\left(\frac{W}{r}\right), \theta 1, r, L, W\right)\, dr$$

$$var10(L,W,A,\theta 1) := T8(L,W,A,\theta 1) + T12(L,W,A,\theta 1) + T16(L,W,A,\theta 1)$$

$$var11(L,W,A,\theta1) := var1(L,W,A,\theta1)$$

$$T17(L,W,A,\theta1) := \int_{A}^{L} f\left(acos\left(\frac{A}{r}\right), asin\left(\frac{A}{r}\right), r, A\right) \cdot alpha(0,\theta1,\theta1,r,L,W)\, dr$$

$$T18(L,W,A,\theta1) := \int_{L}^{\sqrt{2}\cdot A} f\left(acos\left(\frac{A}{r}\right), asin\left(\frac{A}{r}\right), r, A\right) \cdot alpha\left(acos\left(\frac{L}{r}\right),\theta1,\theta1,r,L,W\right) dr$$

$$var12(L,W,A,\theta1) := T1(L,W,A,\theta1) + T17(L,W,A,\theta1) + T18(L,W,A,\theta1)$$

$$T19(L,W,A,\theta1) := \int_{L}^{\frac{W}{\sin(\theta1)}} f\left(acos\left(\frac{A}{r}\right),\theta1,r,A\right) \cdot alpha\left(acos\left(\frac{L}{r}\right),\theta1,\theta1,r,L,W\right) dr$$

$$T20(L,W,A,\theta1) := \int_{\frac{W}{\sin(\theta1)}}^{\sqrt{2}\cdot A} f\left(acos\left(\frac{A}{r}\right), asin\left(\frac{A}{r}\right), r, A\right) \cdot alpha\left(acos\left(\frac{L}{r}\right), asin\left(\frac{W}{r}\right),\theta1,r,L,W\right) dr$$

$$var13(L,W,A,\theta1) := T1(L,W,A,\theta1) + T17(L,W,A,\theta1) + T19(L,W,A,\theta1) + T20(L,W,A,\theta1)$$

$$T21(L,W,A,\theta1) := \int_{\frac{W}{\sin(\theta1)}}^{\sqrt{L^2+W^2}} f\left(acos\left(\frac{A}{r}\right), asin\left(\frac{A}{r}\right), r, A\right) \cdot alpha\left(acos\left(\frac{L}{r}\right), asin\left(\frac{W}{r}\right),\theta1,r,L,W\right) dr$$

$$var14(L,W,A,\theta1) := T1(L,W,A,\theta1) + T17(L,W,A,\theta1) + T19(L,W,A,\theta1) + T21(L,W,A,\theta1)$$

$$T22(L,W,A,\theta1) := \int_{0}^{L} f\left(0,\frac{\pi}{2},r,A\right) \cdot alpha(0,\theta1,\theta1,r,L,W)\, dr$$

$$T23(L,W,A,\theta1) := \int_{L}^{A} f\left(0,\frac{\pi}{2},r,A\right) \cdot alpha\left(acos\left(\frac{L}{r}\right),\theta1,\theta1,r,L,W\right) dr$$

$$T24(L,W,A,\theta 1) := \int_{A}^{\sqrt{2}\cdot A} f\left(\operatorname{acos}\left(\frac{A}{r}\right), \operatorname{asin}\left(\frac{A}{r}\right), r, A\right) \cdot alpha\left(\operatorname{acos}\left(\frac{L}{r}\right), \theta 1, \theta 1, r, L, W\right) dr$$

$$var15(L,W,A,\theta 1) := T22(L,W,A,\theta 1) + T23(L,W,A,\theta 1) + T24(L,W,A,\theta 1)$$

$$T25(L,W,A,\theta 1) := \int_{A}^{\frac{W}{\sin(\theta 1)}} f\left(\operatorname{acos}\left(\frac{A}{r}\right), \operatorname{asin}\left(\frac{A}{r}\right), r, A\right) \cdot alpha\left(\operatorname{acos}\left(\frac{L}{r}\right), \theta 1, \theta 1, r, L, W\right) dr$$

$$var16(L,W,A,\theta 1) := T22(L,W,A,\theta 1) + T23(L,W,A,\theta 1) + T25(L,W,A,\theta 1) + T20(L,W,A,\theta 1)$$

$$var17(L,W,A,\theta 1) := T22(L,W,A,\theta 1) + T23(L,W,A,\theta 1) + T25(L,W,A,\theta 1) + T21(L,W,A,\theta 1)$$

$$T26(L,W,A,\theta 1) := \int_{L}^{\frac{W}{\sin(\theta 1)}} f\left(0, \frac{\pi}{2}, r, A\right) \cdot alpha\left(\operatorname{acos}\left(\frac{L}{r}\right), \theta 1, \theta 1, r, L, W\right) dr$$

$$T27(L,W,A,\theta 1) := \int_{\frac{W}{\sin(\theta 1)}}^{A} f\left(0, \frac{\pi}{2}, r, A\right) \cdot alpha\left(\operatorname{acos}\left(\frac{L}{r}\right), \operatorname{asin}\left(\frac{W}{r}\right), \theta 1, r, L, W\right) dr$$

$$var18(L,W,A,\theta 1) := T22(L,W,A,\theta 1) + T26(L,W,A,\theta 1) + T27(L,W,A,\theta 1) + T14(L,W,A,\theta 1)$$

$$var19(L,W,A,\theta 1) := T22(L,W,A,\theta 1) + T26(L,W,A,\theta 1) + T27(L,W,A,\theta 1) + T15(L,W,A,\theta 1)$$

$$T28(L,W,A,\theta 1) := \int_{\frac{W}{\sin(\theta 1)}}^{\sqrt{L^2+W^2}} f\left(0, \frac{\pi}{2}, r, A\right) \cdot alpha\left(\operatorname{acos}\left(\frac{L}{r}\right), \operatorname{asin}\left(\frac{W}{r}\right), \theta 1, r, L, W\right) dr$$

$$var20(L,W,A,\theta 1) := T22(L,W,A,\theta 1) + T26(L,W,A,\theta 1) + T28(L,W,A,\theta 1)$$

$$var21(L,W,A,\theta 1) := var11(L,W,A,\theta 1)$$

$$var22(L,W,A,\theta 1) := var12(L,W,A,\theta 1)$$

$$T29(L,W,A,\theta 1) := \int_{L}^{\frac{L}{\cos(\theta 1)}} f\left(\operatorname{acos}\left(\frac{A}{r}\right), \operatorname{asin}\left(\frac{A}{r}\right), r, A\right) \cdot alpha\left(\operatorname{acos}\left(\frac{L}{r}\right), \theta 1, \theta 1, r, L, W\right) dr$$

$$var23(L,W,A,\theta 1) := T1(L,W,A,\theta 1) + T17(L,W,A,\theta 1) + T29(L,W,A,\theta 1)$$

$$T30(L,W,A,\theta 1) := \int_{L}^{A} f\left(0, \frac{\pi}{2}, r, A\right) \cdot alpha\left(\operatorname{acos}\left(\frac{L}{r}\right), \theta 1, \theta 1, r, L, W\right) dr$$

$$T31(L,W,A,\theta 1) := \int_{A}^{\sqrt{2}\cdot A} f\left(\operatorname{acos}\left(\frac{A}{r}\right), \operatorname{asin}\left(\frac{A}{r}\right), r, A\right) \cdot alpha\left(\operatorname{acos}\left(\frac{L}{r}\right), \theta 1, \theta 1, r, L, W\right) dr$$

$$var24(L,W,A,\theta 1) := T22(L,W,A,\theta 1) + T30(L,W,A,\theta 1) + T31(L,W,A,\theta 1)$$

$$T32(L,W,A,\theta 1) := \int_{A}^{\frac{L}{\cos(\theta 1)}} f\left(\operatorname{acos}\left(\frac{A}{r}\right), \operatorname{asin}\left(\frac{A}{r}\right), r, A\right) \cdot alpha\left(\operatorname{acos}\left(\frac{L}{r}\right), \theta 1, \theta 1, r, L, W\right) dr$$

$$var25(L,W,A,\theta 1) := T22(L,W,A,\theta 1) + T30(L,W,A,\theta 1) + T32(L,W,A,\theta 1)$$

$$T33(L,W,A,\theta 1) := \int_{L}^{\frac{L}{\cos(\theta 1)}} f\left(0, \frac{\pi}{2}, r, A\right) \cdot alpha\left(\operatorname{acos}\left(\frac{L}{r}\right), \theta 1, \theta 1, r, L, W\right) dr$$

$$var26(L,W,A,\theta 1) := T22(L,W,A,\theta 1) + T33(L,W,A,\theta 1)$$

Visual Basic pseudo-code

```
if ( q1 > a sin( W / L ) )
    if ( A < W / sin q1 )
        if ( √2 A < W / sin q1 )
```

$$Var1 = \int_0^A f1 \cdot a1 \, dr + \int_A^{\sqrt{2}A} f2 \cdot a1 \, dr$$

elseif ($\sqrt{2}A < L$)

$$Var2 = \int_0^A f1 \cdot a1 \, dr + \int_A^{\frac{W}{\sin q1}} f2 \cdot a1 \, dr + \int_{\frac{W}{\sin q1}}^{\sqrt{2}A} f2 \cdot a2 \, dr$$

elseif ($\sqrt{2}A < \sqrt{L^2 + W^2}$)

$$Var3 = \int_0^A f1 \cdot a1 \, dr + \int_A^{\frac{W}{\sin q1}} f2 \cdot a1 \, dr + \int_{\frac{W}{\sin q1}}^{L} f2 \cdot a2 \, dr + \int_L^{\sqrt{2}A} f2 \cdot a3 \, dr$$

else

$$Var4 = \int_0^A f1 \cdot a1 \, dr + \int_A^{\frac{W}{\sin q1}} f2 \cdot a1 \, dr + \int_{\frac{W}{\sin q1}}^{L} f2 \cdot a2 \, dr + \int_L^{\sqrt{L^2+W^2}} f2 \cdot a3 \, dr$$

endif

elseif ($\frac{W}{\sin q1} < A < L$)

if ($\sqrt{2}A < L$)

$$Var5 = \int_0^{\frac{W}{\sin q1}} f1 \cdot a1 \, dr + \int_{\frac{W}{\sin q1}}^{A} f1 \cdot a2 \, dr + \int_A^{\sqrt{2}A} f2 \cdot a2 \, dr$$

elseif ($\sqrt{2}A < \sqrt{L^2 + W^2}$)

$$Var6 = \int_0^{\frac{W}{\sin q1}} f1 \cdot a1 \, dr + \int_{\frac{W}{\sin q1}}^{A} f1 \cdot a2 \, dr + \int_A^{L} f2 \cdot a2 \, dr + \int_L^{\sqrt{2}A} f2 \cdot a3 \, dr$$

else

$$Var7 = \int_0^{\frac{W}{\sin q1}} f1 \cdot a1 \, dr + \int_{\frac{W}{\sin q1}}^{A} f1 \cdot a2 \, dr + \int_A^{L} f2 \cdot a2 \, dr + \int_L^{\sqrt{L^2+W^2}} f2 \cdot a3 \, dr$$

endif

elseif ($L < A < \sqrt{L^2 + W^2}$)

if ($\sqrt{2}A < \sqrt{L^2 + W^2}$)

$$Var8 = \int_0^{\frac{W}{\sin q1}} f1 \cdot a1 \, dr + \int_{\frac{W}{\sin q1}}^{L} f1 \cdot a2 \, dr + \int_L^{A} f1 \cdot a3 \, dr + \int_A^{\sqrt{2}A} f2 \cdot a3 \, dr$$

else

$$Var9 = \int_0^{\frac{W}{\sin q1}} f1 \cdot a1\, dr + \int_{\frac{W}{\sin q1}}^{L} f1 \cdot a2\, dr + \int_L^A f1 \cdot a3\, dr + \int_A^{\sqrt{L^2+W^2}} f2 \cdot a3\, dr$$

endif

else

$$Var10 = \int_0^{\frac{W}{\sin q1}} f1 \cdot a1\, dr + \int_{\frac{W}{\sin q1}}^{L} f1 \cdot a2\, dr + \int_L^{\sqrt{L^2+W^2}} f1 \cdot a3\, dr$$

endif

elseif $(q_1 > a\sin(\frac{W}{\sqrt{L^2+W^2}})$

if $(A < L)$

if $(\sqrt{2}A < L)$

$$Var11 = \int_0^A f1 \cdot a4\, dr + \int_A^{\sqrt{2}A} f2 \cdot a4\, dr$$

elseif $(\sqrt{2}A < \frac{W}{\sin q1})$

$$Var12 = \int_0^A f1 \cdot a4\, dr + \int_A^L f2 \cdot a4\, dr + \int_L^{\sqrt{2}A} f2 \cdot a5\, dr$$

elseif $(\sqrt{2}A < \sqrt{L^2+W^2})$

$$Var13 = \int_0^A f1 \cdot a4\, dr + \int_A^L f2 \cdot a4\, dr + \int_L^{\frac{W}{\sin q1}} f2 \cdot a5\, dr + \int_{\frac{W}{\sin q1}}^{\sqrt{2}A} f2 \cdot a6\, dr$$

else

$$Var14 = \int_0^A f1 \cdot a4\, dr + \int_A^L f2 \cdot a4\, dr + \int_L^{\frac{W}{\sin q1}} f2 \cdot a5\, dr + \int_{\frac{W}{\sin q1}}^{\sqrt{L^2+W^2}} f2 \cdot a6\, dr$$

endif

elseif $(L < A < \frac{W}{\sin q1})$

if $(\sqrt{2}A < \frac{W}{\sin q1})$

$$Var15 = \int_0^L f1 \cdot a4\, dr + \int_L^A f1 \cdot a5\, dr + \int_A^{\sqrt{2}A} f2 \cdot a5\, dr$$

elseif $(\sqrt{2}A < \sqrt{L^2+W^2})$

$$Var16 = \int_0^L f1 \cdot a4\, dr + \int_L^A f1 \cdot a5\, dr + \int_A^{\frac{W}{\sin q1}} f2 \cdot a5\, dr + \int_{\frac{W}{\sin q1}}^{\sqrt{2}A} f2 \cdot a6\, dr$$

else

$$Var17 = \int_0^L f1 \cdot a4\, dr + \int_L^A f1 \cdot a5\, dr + \int_A^{\frac{W}{\sin q1}} f2 \cdot a5\, dr + \int_{\frac{W}{\sin q1}}^{\sqrt{L^2+W^2}} f2 \cdot a6\, dr$$

endif

elseif ($\frac{W}{\sin q1} < A < \sqrt{L^2+W^2}$)

if ($\sqrt{2}A < \sqrt{L^2+W^2}$)

$$Var18 = \int_0^L f1 \cdot a4\, dr + \int_L^{\frac{W}{\sin q1}} f1 \cdot a5\, dr + \int_{\frac{W}{\sin q1}}^{A} f1 \cdot a6\, dr + \int_A^{\sqrt{2}A} f2 \cdot a6\, dr$$

else

$$Var19 = \int_0^L f1 \cdot a4\, dr + \int_L^{\frac{W}{\sin q1}} f1 \cdot a5\, dr + \int_{\frac{W}{\sin q1}}^{A} f1 \cdot a6\, dr + \int_A^{\sqrt{L^2+W^2}} f2 \cdot a6\, dr$$

endif

else

$$Var20 = \int_0^L f1 \cdot a4\, dr + \int_L^{\frac{W}{\sin q1}} f1 \cdot a5\, dr + \int_{\frac{W}{\sin q1}}^{\sqrt{L^2+W^2}} f1 \cdot a6\, dr$$

endif

else

if ($A < L$)

if ($\sqrt{2}A < L$)

$$Var21 = \int_0^A f1 \cdot a4\, dr + \int_A^{\sqrt{2}A} f2 \cdot a4\, dr$$

elseif ($\sqrt{2}A < \frac{L}{\cos q1}$)

$$Var22 = \int_0^A f1 \cdot a4\, dr + \int_A^L f2 \cdot a4\, dr + \int_L^{\sqrt{2}A} f2 \cdot a7\, dr$$

else

$$Var23 = \int_0^A f1 \cdot a4\, dr + \int_A^L f2 \cdot a4\, dr + \int_L^{\frac{L}{\cos q1}} f2 \cdot a7\, dr$$

```
        endif
    elseif ( L < A < L / cos(q1) )
        if ( sqrt(2)A < L / cos(q1) )
```

$$Var24 = \int_0^L f1 \cdot a4\, dr + \int_L^A f1 \cdot a7\, dr + \int_A^{\sqrt{2}A} f2 \cdot a7\, dr$$

```
        else
```

$$Var25 = \int_0^L f1 \cdot a4\, dr + \int_L^A f1 \cdot a7\, dr + \int_A^{\frac{L}{\cos q1}} f2 \cdot a7\, dr$$

```
        endif
    else
```

$$Var26 = \int_0^L f1 \cdot a4\, dr + \int_L^{\frac{L}{\cos q1}} f1 \cdot a7\, dr$$

```
    endif
endif
```