The Heterogeneity of Capital: A Survey

By Lucas M. Engelhardt
For E893 Summer 06
The Ohio State University
Under advisement from Professors Paul Evans and Hajime Miyazaki

I. Introduction

Homogeneous capital is a long-held simplifying assumption of modern macroeconomics. However, for some time, there has been great criticism of this assumption. For some the question was one of intuition and simple observation (after all, no one can seriously claim that capital is homogeneous in reality), and how they should hold a place in economic theory. For others, it was primarily an empirical question. Either way, there has long been a train of researchers who were willing to question the place of the homogeneity assumption in capital theory, and in macroeconomics in general. This paper seeks to examine some of the criticisms of and alternatives to the homogeneity assumption.

Section 2 examines the article which sparked the so-called "Cambridge Capital Controversies", an article by Joan Robinson which questions the concept of the "quantity of capital" and attempts to offer an alternative to the standard production function which can save many of the positive aspects of it. Section 3 examines Robert Solow's article which is, in part, a reponse to Joan Robinson's. In this article, he examines the conditions under which a sensible index of capital (and therefore a "quantity of capital") may exist. Section 4 discusses an article by Cohen and Harcourt which seeks to summarize the major points of the Capital Controversies.

In the later sections, an alternative to homogeneous capital are considered. Section 5 offers an overview of the idea of vintage capital in both a Solow "putty-putty" framework, and an ex post fixed factor proportions ("putty-clay") framework. Section 6 examines Bardhan and Priale's work on integrating vintage capital with fixed factor proportions ex ante and ex post ("clay-clay") into endogenous growth theory. Finally, section 7 summarizes an article by Gilchrist and Williams which examines business cycle properties in a putty-clay vintage capital framework. Section 8 concludes.

II. The Production Function and the Theory of Capital: Joan Robinson

Joan Robinson of Cambridge University had the distinction of beginning what became known as the "Capital Controversies" regarding capital. A summary of this debate is contained in Cohen and Harcourt (2003), which has been left for a later section. This section will be devoted to examining the contribution to the debate made by Robinson (1953).

Robinson's initiating article may best be thought of as a list of complaints regarding the use of the neoclassical production function, seeing as it ignores many of the thorny issues in the theory of capital. As she states,

> The student of economic theory if taught to write O = f(L, C) where L is a quantity of labour, C is a quantity of capital and O is a rate of output of commodities. He is instructed to assume all workers alike, and to measure L in man-hours of labour; he is told something about the index-number problem involved in choosing a unit of output; and then he is hurried on to the next question, in the hope that he will forget to ask in what units C is measured. (81)

In short, Robinson is disturbed by the fact that no express assumptions regarding capital homogeneity are given, most likely because such an assumption is far from intuitive, and even further from the student's everyday experience.

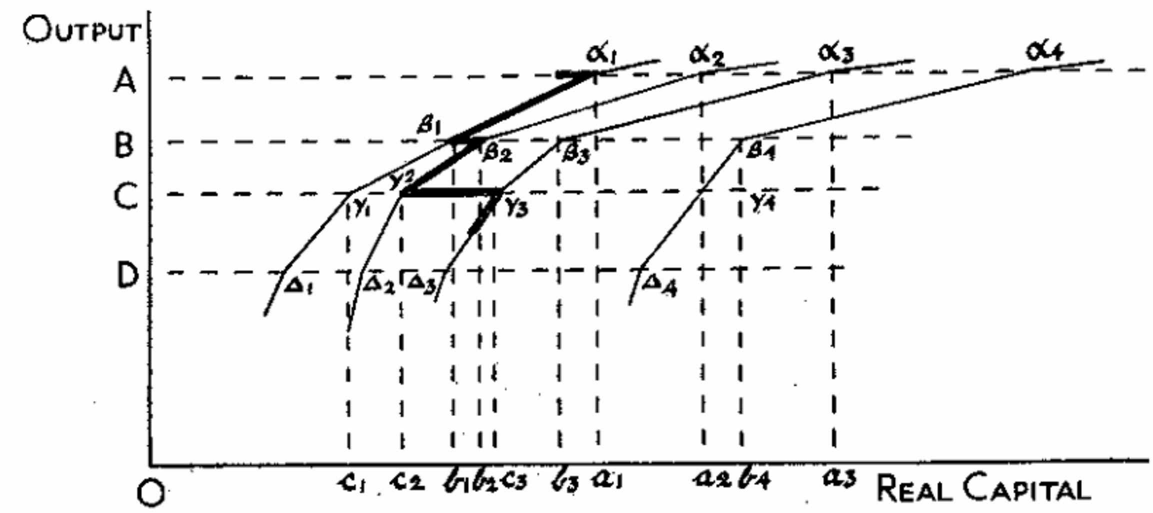
After establishing the problem, Robinson moves to a discussion of possible, sensible solutions in the short-run. For example, capital may be considered as part of the environment in which labour works. In this case, the production function is, essentially a function of labor alone, as capital is held constant. Alternatively, one could view capital as a list of all goods in existence at any moment in time. However, once you leave the very short period, either of these alternatives fails. In the first case, a change in capital, being essentially exogenous, is equivalent to a change in the weather. However, it is clear from simple observation that changes in capital tend to be the result of economic decision making. In the second case, a change in the list of currently existent goods would bring about a change in the method of production, so that the original list is also, in fact changed, by the addition of new goods.

After a short discussion regarding the valuation of capital, Robinson turns to a more relevant question of how heterogeneous capital could be measured. Here, Robinson describes the difficulty that arises because capital is, at the same time, an output (which should be measured in output units), but also a labor-substitutable input (which should be measured in labor units).

Despite these problems, Robinson ends her introduction by stating that “the problem which the production function professes to analyse, althought it has been too much puffed up by the attention paid to it is a genuine problem…. We cannot abandon the production function without an effort to rescue the element of commonsense that is entangled in it.” (82-83) Throughout the remainder of her article, Robinson seeks to do just that: abandon the production function (as she specified it) and rescue the element of commonsense that that is “entangled” in it.

After some discussion about the difficulty of measuring a quantity of capital, and of the necessity (for Robinson’s purposes) of only comparing long-run equilibria, Robinson discusses techniques of production, and examines how differing levels of mechanization can result in different techniques being used as real wages change (specifically, as wages increase, the degree of mechanization increases). This effect has been called the “Ricardo effect” (by such economists as Hayek). In addition to the Ricardo effect is a Wicksell effect. The Wicksell effect captures the fact that an increase in real wages decreases the overall output, and therefore, given a particular technique of production, capital values will diminish as real wages rise. The combinations of these effects can be seen in this figure from Robinson (93):

FIG. I



In this figure, 0A represents the output from a constant number of men employing technique α, and 0B, 0C, and 0D represent the same quantity for the other techniques. The bold line shows how production progresses as real wages increase. Thus, it can be seen, for example, that as wages increase, the Wicksell effect will cause movement from γ_3 to γ_2. That is, given that technique γ is best for a given range of wage rates, as the wages rise, the quantity of real capital employed will diminish (as will output). Then, there is a range from γ_2 to β_2 where the two techniques are equally profitable, so some mixture of them is likely to be used. Then, as wages increase, β_2 comes to dominate as the Ricardo effect would predict. The process continues with the Ricardo effect dominating as shift from one production technique to another occurs, and the Wicksell effect dominating as shift within a production technique occurs. Using this style of analysis, Robinson reaches the following conclusion:

> The rate of profit on capital will tend to be higher, and real wages lower:
> (1) The more plentiful are the technical opportunities for mechanizing production.
> (2) The slower is the rate of capital accumulation in relation to the growth of population.

(3) The weaker is the force of competition (and the weaker is the bargaining power of the workers, when competition is weak). (103)

As Robinson notes, these results are hardly counterintuitive (though that is perhaps a good sign).

Overall, one may question how well Robinson's method proves what it sets out to prove. That is, it is unclear whether Robinson really captures the most useful results of using a production function (like, for example, how the interest rate relates to the process of capital accumulation). However, Robinson deserves great credit for questioning the usual formulation of the production function, as it does implicitly make assumptions which may prove to be poorly founded from a theoretical perspective. This is the question that Robert Solow decided was worth exploring in his article which shares Robinson's title.

III. The Production Function and the Theory of Capital: Robert Solow

Two years after Robinson's original article, Robert Solow wrote an article of the same title to respond to some of the concerns that Robinson had raised in her article. Specifically, Solow seeks to answer the question "under what conditions can a consistent meaning be given to the quantity of capital?" (1955, 101) That is, under what conditions can we aggregate capital into a meaningful index, which would then allow the production function to be collapsed into a function of labor and this aggregated capital?

The question can be reformulated as Solow's system (1):

$$\begin{aligned} Q &= F(L, C_1, C_2) \equiv H(L, K) \\ K &\equiv \Phi(C_1, C_2) \end{aligned} \qquad (1)$$

From this set of equations we can derive the marginal rate for substitution between the two types of capital (Solow's equation (2)):

$$\frac{MPP_1}{MPP_2} = \frac{\partial F / \partial C_1}{\partial F / \partial C_2} = \frac{\frac{\partial H}{\partial K}\frac{\partial \Phi}{\partial C_1}}{\frac{\partial H}{\partial K}\frac{\partial \Phi}{\partial C_2}} = \frac{\partial \Phi / \partial C_1}{\partial \Phi / \partial C_2} \qquad (2)$$

It should be noted that (2)'s right hand side is independent of the amount of labor employed. Thus, if a capital index is possible, it must be that the marginal rate of substitution between these the various capital goods is not affected by the quantity of labor employed. This is a necessary condition. In addition, it is also sufficient due to a proposition proven by Leontief which Solow cites.

When considering this condition, it should become immediately obvious that, for most production processes, this condition will not be met. Even the substitution between the use of one-ton and two-ton trucks can easily be impacted by the number of available drivers. The only immediately obvious cases where it does apply are cases where homogeneity is, for all practical purposes, present. (As, to cite Solow's example, when one considers the use of a brick building as opposed to a wooden building.) This suggests that, in nearly all imaginable cases, capital cannot be meaningfully aggregated into a single index.

However, there is a class of situations where this condition does hold. This occurs, as Solow notes, when the two capital goods are used by themselves to manufacture K. In that case, Φ is actually a production function rather than simply an index. Naturally, it is somewhat unusual to try and think of a production process that requires no labor. However, if such a process was found to exist, then it would be a case when heterogeneous capital could be combined into an index because, ultimately, the heterogeneous capital is combined into a single product. Now, seeing that, in this class of cases, the index function is actually a production function, it may be of interest to determine if the function H has the same characteristics as a typical production function. Solow devotes a section of his paper to showing that this is the case, which he states as the following theorem: "Suppose that the underlying production function F exhibits constant returns to scale with respect to L, C_1, and C_2, and obeys the generalized law of diminishing returns to variable proportions, i.e., has property convex equal-output surfaces. Then, exactly the same properties will characterize the index-function Φ and the collapsed production function H" (103). Thus, in the case where Φ is a valid index function, it can also be thought of as a production function.

In summary, Solow's work gives a more mathematically pleasing argument (assuming differentiability) for Robinson's complaint of a year earlier. Given the fact that these two seem to agree on this point (for most cases, at least), it seems strange that any real controversy would exist. To better understand how the controversy occurred, we will turn to a recent article by Cohen and Harcourt which gave an overview of the major issues in the Cambridge Controversies.

IV. Cambridge Capital Theory Controversies: An Overview of Related Issues

In their 2003 article "Retrospectives: Whatever Happened to the Cambridge Capital Theory Controversies?", Cohen and Harcourt revisit the Controversies which, in their estimation, have lain dormant for 40 years. Their article divides the debates into four "rounds", and examines what the results of each round were.

The first round, according to Cohen and Harcourt (C&H) was concerning "meaning and measurement of capital in the scarcity theory of price". This is the round that deals most with the issues that Robinson and Solow's respective "The Production Function and the Theory of Capital" articles. As C&H (201) explain, in the typical simple neoclassical model, one begins with a one-commodity model with a production function of the form that Robinson described, and one makes some usual assumptions regarding the exogeneity of technology, constant returns to scale, diminishing marginal productivity and competitive equilibrium. Using this simple model, one can arrive at three "parables" (to use Samuelson's term):

> 1) The real return on capital (the rate of interest) is determined by the technical properties of the diminishing marginal productivity of capital; 2) a greater quantity of capital leads to a lower marginal product of additional capital and thus to a lower rate of interest, and the same inverse, monotonic relation with the rate of interest also holds for the capital/output ratio and sustainable levels of consumption per head; 3) the distribution of income between laborers and capitalists is explained by relative factor scarcities/supplies and marginal products. The price of capital services (the rate of interest) is determined by the relative scarcity and marginal productivity of aggregate capital, and the price of labor services (the wage rate) is determined by the relative scarcity and marginal productivity of labor (L). (C&H 2003, 201)

These parables are, to most modern economists, a basic extension of common sense. However, they rest on a meaningful, measurable quantity of physical capital (and labor) for the results to follow properly. But, when heterogeneity of capital is introduced capital cannot be meaningfully aggregated except in the unusual cases described by Solow (1955-56). Rather, capital valuation must be used. However, capital can be valued either by present values of future output (which is the typical modern way) or according to its cost of production (which was more common among classical economists). But, in either case, the rate of interest is required for valuation. However, according to parable 1, the rate of interest is determined by the quantity of capital. Thus, a circular relationship develops between capital valuation and the rate of interest. This gives rise to various so-called Wicksell effects (which are, in certain ways, similar to those described by Robinson). In the Cambridge controversies, the two effects that came to the forefront were reswitching and capital reversing. "Reswitching occurs when the same technique is preferred at two or more rates of interest while other techniques are preferred at intermediate rates" (C&H 2003, 202). For example, suppose there are two production processes. Production process A requires a small investment on day 1 and a moderate investment on day 3 to produce a product on day 4. Production process B requires a large amount of investment (slightly smaller than the undiscounted sums from production process A) on day 2 to produce the same product on day 4. It is possible that such a choice between production processes would mean that B is preferred at very low and very high rates of interest. It is preferred at very low rates because the costs are not discounted very much, so the lower total cost dominates the calculation. It is preferred at very high rates of interest because costs are discounted greatly, so having a large investment on day 2 is preferred to a slightly smaller moderate investment on day 3. This reswitching violates parables 1 and 2.

Capital reversal occurs when a lower capital/labor ratio is associated with a lower interest rate. That is, capital that is “more scarce” is, in fact, cheaper. This phenomenon violates parables 2 and 3, and would also occur in the example above. The difficulty arises from the fact that, with heterogeneous capital, endogenously determined prices (like the interest rate) change which of a variety of potential technical properties are chosen. This endogeneity can give rise to multiple equilibria, which complicates the parables that the simple neoclassical model hoped to establish. Thus, we are left with a more abiding question. Even if we could sensibly calculate a quantity of capital, what use would that serve if it cannot establish the rate of interest?

Round 2 was concerned with issues of equilibrium and time. Naturally, capital is intertwined with time, as it is a stock (homogeneous or heterogeneous) that is built up or depleted over time. Any dynamic model involving capital in some way captures the fact that capital ties the past, present, and future together as the quantity of capital is the result of decisions which must take all of these into account in some way. This round, however, was only loosely related to capital issues. Mostly, it was composed of complaints by Robinson against the use of comparative statics to describe economic outcomes when, in reality, change is required to move from one equilibrium to another rather than just some difference in parameters arising.

However, recent work in macroeconomics does a far better job of examining changes rather than just differences, so Robinson's critique may be considered out of date.

The third round occurred between 1956 and 1966 and was composed of the American neoclassicals rising to defend the aggregate production function on one ground or another. The theoretical response was three-fold. First, Swan attempted to establish the metaphor of "putty capital", that is, capital which can be continuously remolded as necessary. However, in this theoretical construction, capital is effectively homogeneous putty which, in each period, happens to be in some concrete form before it transmute back into putty. This setup only avoided the problem of heterogeneity by adding a step to collapse heterogeneity back into the one-commodity problem. Solow attempted to sidestep the problem of capital by establishing the rate of interest in terms of the rate of return on investment rather than the rate of return on capital. However, this avoids many of the other real problems that occur, and in itself may not be a valid approach. The third attempt was one by Samuelson to create a "surrogate production function". However, in his setup he assumed equal factor proportions in all industries, effectively collapsing capital into one commodity (as he himself later realized). So, on the theoretical front, round three was a disappointing time for the neoclassicals. However, Solow did make some headway in his work showing that the one commodity model provided a good empirical approximation. But, from a theoretical standpoint, aggregate production functions were a loss, and as such were out of favor in the 1970s and early 1980s until endogenous growth and real business cycle theories revived them (C&H, 206).

The final round primarily centered around the concept of general equilibrium and whether it was capable of saving the parables derived from the homogeneous good case. However, even the neoclassical side of the debate (primarily represented by Hahn and Bliss) came to the conclusion that general equilibrium theory does not imply a monotonically decreasing relationship between the quantity of capital and the interest rate. Rather, it simply implies that factor returns are equal to their disaggregated marginal productivities. On the whole, this round spoke little to the topic of capital heterogeneity. However, it does revive Robinson's concerns about the concept of equilibrium (which were spoken of in Round 2). Here, due to results concerning the stability of equilibria in general equilibrium models, Hahn was forced to admit: "The Arrow-Debreu construction…must relinquish the claim of providing necessary descriptions of terminal states of economic processes" (1984, 53).

On the whole, one may be tempted to say that Robinson and her allies "won" the controversies. However, even a cursory glance at many recent papers in macroeconomics suggest that the American contingent "won" instead. Ultimately, as C&H conclude, determining a winner in this debate is impossible, as the issues where never fully resolved because there was never any agreement about the significance of the various results. On the one hand, there appears to be wide agreement that, from a theoretical standpoint, the assumption of capital homogeneity is unsatisfactory except in extremely unusual cases. However, on the other hand, there seems to be a consensus that one can set this aside because, empirically, the homogeneity assumption appears to give a reasonable approximation of relevant data.

As continuing evidence of the disagreement as to the significance of these results, Cohen and Harcourt's article received several comments, which were compiled in the comments section under the title "Cambridge Capital Controversies". Among the most notable comments, Franklin Fisher notes that his own work suggests: "Aggregate production functions can exist, but such existence requires very stringent and unlikely conditions. Further, one cannot reasonably suppose that aggregate production function can be used as good approximations in most circumstances" (Pasinetti et al., 228-229). Also, empirical findings that agree with an aggregate Cobb-Douglas production function cannot be used to justify the existence of aggregate production functions. To strengthen Fisher's case, Felipe and McCombie, in the same comment section, suggest that the Cobb-Douglas production function is simply a transformation of an income identity under certain assumptions (230). Thus, the Cobb-Douglas production function providing a good match for empirical data could just as easily be explained by this income identity. For all of these reasons, there is at the very least a theoretical need for accounting for the heterogeneity of capital. However, doing so in a way that is simultaneously theoretically satisfying, empirically verifiable, and relatively simple proves to be a great challenge which capital theorists are still struggling with.

With this fact in hand, we turn to two methods of accounting for capital heterogeneity, which are often used in conjunction with one another. First is the concept of vintage capital in which technology is embodied in capital goods, and thus capital goods from different times have different productivities due to differing production technologies. Second is the concept of putty-clay capital in which capital, prior to the actual contruction of a capital good can take a variety of forms (that is, can be designed to require any particular labor/capital ratio to operate), but after construction is inflexible in factor proportions.

V. The Theory of Vintage Capital: An Overview[1]

In standard neoclassical growth theory capital is assumed to be homogeneous and technical progress is assumed to be disembodied. Traditionally, this is seen in the typical neoclassical Cobb-Douglas production function:

$$Y_t = A_t^{\alpha} L_t^{\beta} K_t^{\gamma}$$

[1] Much of this section is from Boucekkine, et al. (2006). However, due to the general nature of the model presented, no specific citations are listed.

Thus, any increase in technology (A) will increase production for all capital. Intuitively, this type of model of technology captures cases where technological change is such that combining the same capital and labor in a different way can increase output. However, in experience, technological change generally occurs in a different way. Rather than being disembodied, technological change is embodied in capital. That is, the current state of technological knowledge is used when capital goods are built. Thus, the capital embodies the technology of the time that it is built. Here, the production function takes a different form:

$$Y(v,t) = F(I(v), L(v,t), e^{\gamma v})$$

Y(v,t) indicates the production at time t from capital of vintage v. I(v) is the amount that was invested in vintage v capital (here we assume no physical depreciation). L(v,t) is the amount of labor devoted to vintage v at time t. Finally, γ is the rate of technological progress, so that the last term captures the state of technology for vintage v. Given this production function for each vintage, to obtain aggregate production one must integrate across all operational vintages (in some variations of this model, it is unprofitable to operate some vintages).

Solow (1960) gives a definite form to the Y(v,t) equation by making it Cobb-Douglas:

$$Y(v,t) = [e^{\gamma v} I(v)]^{1-\alpha} L(v,t)^{\alpha}$$

In this model, the continuously variability of the labor/capital ratio allows for all vintages to remain operational. However, over time the quantity of labor devoted to a particular vintage would decrease given technological improvement over time. One interesting outcome of Solow's model is that it allows for the aggregation of capital.[2] Simply define:

[2] This aggregation is valid, as can be easily seen from the result in Solow (1955) in an example case of 2 vintages where the total output is simply figured by the sum. When one finds the ratio of marginal physical products between the two vintages, it is easily seen that the total labor supply does not matter, assuming that the allocation of labor is chosen to maximize output.

$$K(t) = \int_{-\infty}^{t} e^{\gamma v} I(v) dv$$

Since, given maximization across vintages, labor productivity will equalize across vintages, aggregate output becomes

$$Y(t) = K(t)^{1-\alpha} L(t)^{\alpha}$$

This equation for Y(t) is clearly a typical neoclassical Cobb-Douglas production function without technology (which is sensible as technology is captured by the vintage-adjusted capital aggregate). However, despite the fact that the production function collapses into a typical Cobb-Douglas there is still an interesting result when one examines the time derivative of the aggregated capital stock:

$$\dot{K} = e^{\gamma t} I(t)$$

This equation shows that technological change has an impact on the growth of the vintage adjusted aggregate capital stock. Thus, over time, an equivalent level of investment will yield a larger growth in vintage-adjusted capital stock due to the better technology embodied in new capital.

An alternative formulation using vintage capital is in the spirit of Johansen (1950), and combines the vintage capital concept with the concept of putty-clay technology. That is, capital-labor substitution is permitted ex-ante, but not after capital is installed. Thus, because capital/labor ratios are fixed ex-post one can rewrite the Y(v,t) equation in the following way:

$$Y(v,t) = g(\lambda(v)) I(v)$$

where $\lambda(v)$ is the labor-capital ratio for vintage v. Thus, employment on vintage v will be: $L(v,t) = \lambda(v)I(v)$. Then, quasi-rents of vintage v at date t are proportional to the difference: $g(\lambda(v))- \lambda(v)w(t)$, were w(t) is the equilibrium wage. Thus, if wages are increasing over time, then quasi-rents will become negative and capital of a particular vintage will be abandoned. If, for example, we assume that technology is labor-saving (as Johansen did), then technological progress will imply wage growth, and therefore, endogenous obsolescence. Thus, the vintage capital model is capable, when combined with putty-clay production technology, of producing capital obsolescence. This is an important contribution, as capital obsolescence is a phenomenon which is empirically obvious (simply driving through "Rust Belt" cities provides ample evidence), but which is poorly accounted for in standard neoclassical models.

VI. Bardhan and Priale: Endogenous Growth Theory in a Vintage Capital Model

Bardhan and Priale (1996) present a model using a variant on vintage capital. In their model, labor and vintage capital enter into the production function with fixed proportions (this is sometimes called the "clay-clay" model). The embodied technology is accumulated by a process of research and development. Thus, vintages are important due to the difference in embodied technology, and the fixed proportion between labor and capital ex post implies that sufficiently old capital goods may be scrapped. Using this setup, they explore how policies may affect the growth rate through changing the duration of the productive use of a particular vintage of capital equipment.

Essentially their result can be stated as follows: Growth rates vary inversely with the profitable duration of vintage capital. Essentially, an economy where it remains profitable to operate machines for a very long time will grow more slowly, ceteris paribus, than one where machines can only be profitably operated for a shorter period of time. There are several variables which may change the length of time that machines can be operated profitably. First, if the labor force increases, then the economic life of machines is smaller (due, at least in part, to the faster rate of development in embodied technology and also to the increased ability to replace machines as more workers can produce new capital goods to replace older vintages). An increase in the savings rate (which, in Bardhan and Priale, is exogenously determined) will similarly decrease the economic life of machines, causing an increase in the growth rate. This phenomenon can be explained intuitively as higher savings rates increase the rate at which new capital is accumulated, and therefore decreases the number of old machines that can be profitably operated.

After setting up and examining some of the properties of their basic model, Bardhan and Priale move on to examine a potential extension of their model. They examine a model in which there are two sectors producing final goods. One sector has a labor-intensive mode of production (which utilizes no capital), and the other sector has a capital-intensive mode of production which follows the basic model presented in the paper. This extension gives many of the same results as the basic model. Like in the basic model, larger labor forces and higher savings rates will decrease the economic life of machines and, consequently, increase the growth rate. However, there is one additional result which could not be derived simply by using the basic model. If the relative price of the labor-intensive good falls, then the economic life of machines decrease and the growth rate increases. Such a change may, for example, be the result of policies which are designed to protect the capital-intensive industry.

Bardhan and Priale are fast to admit that the primary purpose of their model is not to provide any kind of conclusive answers, but rather to illustrate a number of interesting questions that arise when vintage capital concepts are applied to theories of endogenous growth. In particular, Bardhan and Priale mention several specific areas where their model (or a model that has similar characteristics) may be extended:

1) Their model only examined steady states. Examining transitional dynamics off of steady states needs to be explored.

2) The model could be modified to dispose of the full-employment assumption or to account for vintage human capital.

3) The model could be modified to fit into a putty-clay technology rather than a clay-clay technology.

4) In the two sector model, Bardhan and Priale only allow for tradeability of final goods, not for capital goods. They suggest that the tradeability of capital goods may have some relation to the growth of East Asia where trade allowing for imports of high technology machines is supposed to have played an important role.

On the whole, Bardhan and Priale present an interesting paper[3] which offers great potential for future research possibilities.

VII. Gilchrist and Williams: Putty Clay and Investment: A Business Cycle Analysis

In 2000, Gilchrist and Williams (G&W) examined the use of putty-clay production technology in a general equilibrium model to examine business cycle effects of such technology. They find that the inclusion of at least some putty-clay production into the economy significantly improves the fit between model predicted values and data over standard business cycle models. Specifically, they find that "the distance between model and data moments is minimized for an estimated putty-clay share of total output that is on the order of 50-70 percent. The data overwhelmingly reject the restriction of no role for putty-clay capital." (929) This result

[3] One great weakness of their paper, however, is formatting. The fifty-six equations were not numbered, and nearly all minus signs were omitted. Why these formatting errors occurred is unknown.

suggests that models which ignore the putty-clay aspect of capital will likely have less than optimal fits to the data.

The G&W model is designed to be putty-clay (as opposed to the B&P model which was clay-clay). That is, ex ante, there is substitutability between labor and capital, and ex post labor/capital proportions are fixed. In this model, the ex ante choice of capital intensity is based on a standard Cobb-Douglas production function. Final-goods production at time t by a machine built in period t-j with embodied technology $\mu_{t-j}\theta_{t-j}$ (where μ captures an idiosyncratic shock, and θ is the economy-wide level of vintage technology) and capital k_{t-j} is given by:

$$Y_t(\mu_{t-j}\theta_{t-j}k_{t-j}^{\alpha}) = 1\{L_t(\mu_{t-j}\theta_{t-j}k_{t-j}^{\alpha}) = 1\}\mu_{t-j}\theta_{t-j}k_{t-j}^{\alpha}$$

Where 1{} is a zero-one indicator function that shows if the machine is being used at time t. Total output can be found by multiplying the output from one machine of a particular labor productivity by the number of machines with that labor productivity, and then integrating across all the different levels of labor productivity.

The utilization decision on a particular machine is intuitively simple. If expending a unit of labor on a machine will lead to an increase in output that is at least as high as the wage rate, then the machine will be utilized. The investment decision should also then be clear. New machines are put into place until the value of a new machine (the present discounted value of net income) is equal to the cost of a machine. Finally, household preferences depend on consumption per labor endowment and labor per labor endowment in the representative household, where the labor endowment grows at a constant rate n. These preferences are maximized given a standard intertemporal budget constraint, and shares in machines are traded. (In equilibrium, this results in representative households holding diversified portfolios of all such claims.)

The model offers several interesting results. First, the short-run marginal cost curve is non-linear in logs (as opposed to the Solow vintage capital model). This results comes from the fact that, at small levels of output, there are a number of machines which are highly efficient which may be brought into operation at low cost. However, at high levels of output, the efficiency level of the marginal machine drops sharply, so increasing output becomes quite costly. The degree of this curvature depends on the degree of idiosyncratic uncertainty. If there is a low level of idiosyncratic uncertainty, then the short-run marginal cost curve gets steeper faster. (As the low level of uncertainty does not result in a wide distribution of machine productivities.) But, as the level of idiosyncratic uncertainty becomes large, the short-run marginal cost curve approximates the log-linear form as in the Solow vintage capital model.

The second result comes from a temporary, but persistent decrease in the cost of capital (which is equivalent to a similarly described increase in the economy-wide level of vintage technology). The response to such a shock is quite different from in the Solow vintage model. In the Solow model, the largest response in both labor hours and output is immediate at the time of the shock. In the G&W model, the maximum response in labor takes about 2 years, and in output, about 4 years. This delay occurs because the ex post fixed proportions (and L shaped short-run marginal cost curve) greatly dampen immediate effects until new capital goods can be built, and new workers hired. Eventually, as the cost of capital rises back to its original level, output and labor fall back to the steady state.

The third feature worth note is the asymmetric response to positive versus negative shocks, especially for large shocks. In brief, large negative shocks result in more pronounced responses than large positive shocks. The reason for this asymmetry should be apparent. A large negative shock is responded to by shutting down the marginal machines. A large positive shock will be responded to by building new machines, which takes more time. (The example G&W cited to demonstrate this is a shock to labor costs, specifically, the introduction of a payroll tax.)

Finally, the G&W model provides support for business cycles based on permanent technology shocks. Previous work had been unable to approach the empirical business cycle data simply by allowing for permanent technology shocks. However, the introduction of embodied putty-clay technology "generates dynamic responses to permanent technology shocks that accord well with key properties of the data." (948) Thus, G&W suggest that there is still hope for the hypothesis that permanent technology shocks are a major determinant of the business cycle. (Though certain aspects of their model still do not match the data well.)

On the whole, G&W do well demonstrating the importance of putty-clay technology in replicating certain business cycle features that many standard models are incapable of reproducing. This result suggests that putty-clay technologies should have a greater place in macroeconomic modeling, especially in those models designed to study business cycle features.

VIII. Conclusion

In this paper, several important results may be seen. First, capital homogeneity, when it is assumed, must, in nearly all cases, be considered a simplifying assumption that does differ from reality. Solow (1955) gives the stringent mathematical conditions for this assumption to actually be a somewhat accurate reflection of reality. However, these conditions are almost never met in the real world, and certainly not in the macroeconomy. This is the most important result that can be drawn from Sections 2-4. Sections 5-7 can best be summarized as follows: vintage capital models offer significant improvements over standard homogeneous capital, disembodied technology models. These improvements help to explain every day economic phenomena (scrapping of capital) which does not occur in homogeneous capital models, and also (as in the case of the G&W model) helps to match empirical data better than homogeneous capital models.

If nothing else, it should be obvious that the manner in which technology affects production has significant dynamic consequences for macroeconomic models. This suggests that more examination of this field is warranted if we are to build a better understanding, and better predictions, of the macroeconomy.

References:

Bardhan, Pranab and Rodrigo Priale, "Endogenous Growth Theory in a Vintage Capital Model" (June 7, 1996). *Center for International and Development Economics Research.* Paper C96-069. http://repositories.cdlib.org/iber/cider/C96-069

Boucekkine, Raouf, David de la Croix, and Omar Licandro, "Vintage Capital", http://ideas.repec.org/p/eui/euiwps/eco2006-8.html (2006).

Cohen, Avi J. and G. C. Harcourt, "Cambridge Capital Controversies: Response from Avi J. Cohen and G. C. Harcourt", *Journal of Economic Perspectives*, Vol. 17, No. 4 (Autumn 2003), pp. 232-233.

Cohen, Avi J. and G. C. Harcourt, "Retrospectives: Whatever Happened to the Cambridge Capital Theory Controversies?", *Journal of Economic Perspectives*, Vol. 17, No. 1 (Winter 2003), pp. 199-214.

Gilchrist, Simon, and John C. Williams, "Putty-Clay and Investment: A Business Cycle Analysis", *The Journal of Political Economy*, Vol. 108, No. 5. (Oct., 2000), pp. 928-960.

Hahn, Frank H. *Equilibrium and Macroeconomics*. Oxford: Basil Blackwell.

Pasinetti, Luigi L., Franklin M. Fisher, Jesus Felipe, J.S.L. McCombie, Robert L. Greenfield, "Cambridge Capital Controversies", *Journal of Economic Perspectives*, Vol. 17, No. 4 (Autumn 2003), pp. 227-232.

Robinson, Joan, "The Production Function and the Theory of Capital", *The Review of Economic Studies,* Vol. 21, No. 2. (1953-1954), pp. 81-106.

Solow, Robert M. "The Production Function and the Theory of Capital", *The Review of Economic Studies,* Vol. 23, No. 2. (1955-1956), pp. 101-108

Solow, Robert M. "Investment and technological progress." In K. Arrow, S. Karlin, and P. Suppes (eds.), *Mathematical Methods in Social Sciences 1959*, 89-104. Stanford University Press.