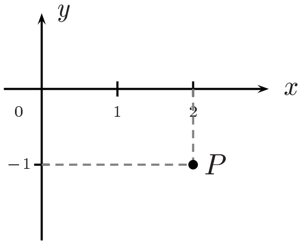
Lista de Exercícios de CVE
Capítulo 5: Vetores no plano e no espaço

Vetores no plano.

Exemplo 1: represente o ponto $P(2,-1)$ no plano cartesiano.

Solução:

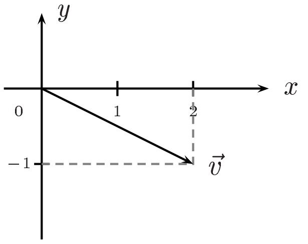


E1) Represente os seguintes pontos no plano cartesiano.

a) $A(2,1)$, b) $B(3,-1)$, c) $C(-2,1)$, d) $D(-3,-2)$.

Exemplo 2: represente o vetor $\vec{v} = 2\hat{\imath} - \hat{\jmath}$ no plano cartesiano.

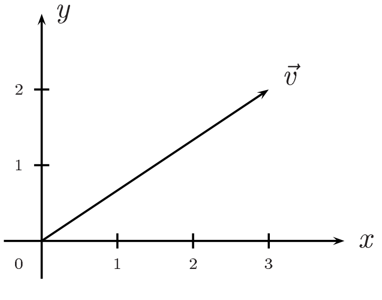
Solução:



E2) Represente os seguintes vetores no plano cartesiano.

a) $\vec{a} = 2\hat{\imath} + 3\hat{\jmath}$, b) $\vec{b} = -2\hat{\imath} + \hat{\jmath}$, c) $\vec{c} = 2\hat{\imath}$, d) $\vec{d} = -\hat{\imath} - 2\hat{\jmath}$.

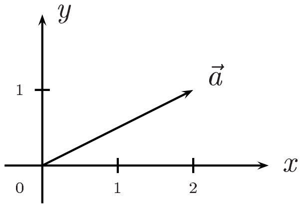
Exemplo 3: escreva o vetor representado abaixo em termos de suas componentes.



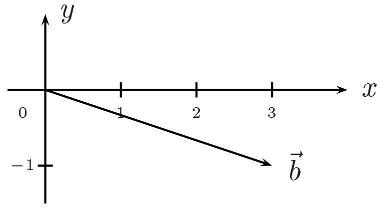
Solução: $\vec{v} = 3\hat{\imath} + 2\hat{\jmath}$.

E3) Escreva os vetores representados abaixo em termos de suas componentes.

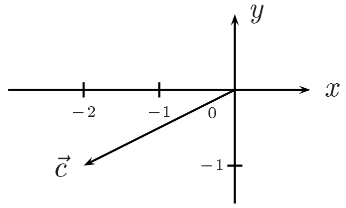
a)



, b)



, c)



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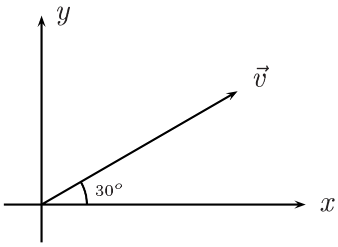
Exemplo 4: calcule o módulo do vetor do exemplo 2.

Solução: $|\vec{v}| = \sqrt{2^2 + (-1)^2} = \sqrt{4+1} = \sqrt{5}$.

E4) Calcule os módulos dos vetores do exercício E2.

Exemplo 5: escreva o vetor representado abaixo em termos de suas componentes sabendo que $|\vec{v}| = 3$.



Solução: dado o ângulo acima, temos da definição do cosseno que

$$\cos 30^o = \frac{v_x}{v} \Rightarrow v.\cos 30^o = v_x \Rightarrow v_x = v.\cos 30^o \Rightarrow v_x = 3.\frac{\sqrt{3}}{2} \Rightarrow v_x = \frac{3\sqrt{3}}{2}\ .$$

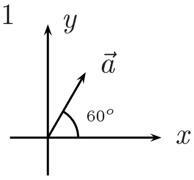
De modo semelhante, da definição do seno, obtemos

$$\text{sen}30^o = \frac{v_y}{v} \Rightarrow v.\text{sen}30^o = v_y \Rightarrow v_y = v.\text{sen}30^o \Rightarrow v_y = 3.\frac{1}{2} \Rightarrow v_y = \frac{3}{2}\ .$$

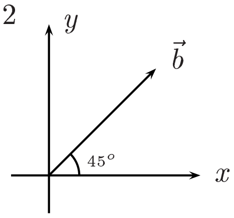
Podemos, então, escrever $\vec{v} = \frac{3\sqrt{3}}{2}\,\hat{i} + \frac{3}{2}\,\hat{j}$.

E5) Escreva os vetores representados abaixo em termos de suas componentes.

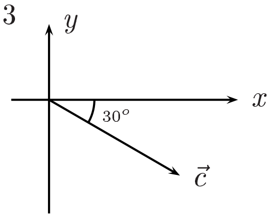
a) $|\vec{a}| = 1$



, b) $|\vec{b}| = 2$



, c) $|\vec{c}| = 3$



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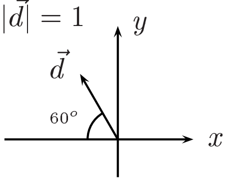
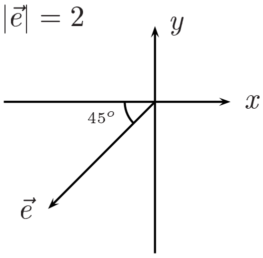
d) $|\vec{d}| = 1$



, e) $|\vec{e}| = 2$



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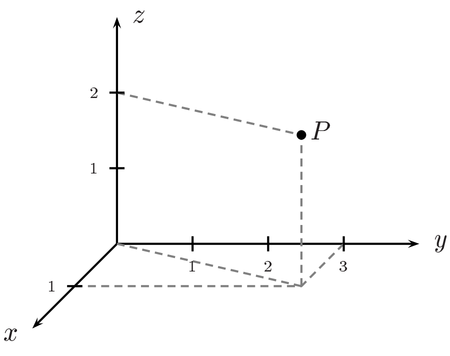
Vetores no espaço.

Exemplo 6: represente o ponto $P(1, 3, 2)$ no espaço.

Solução:

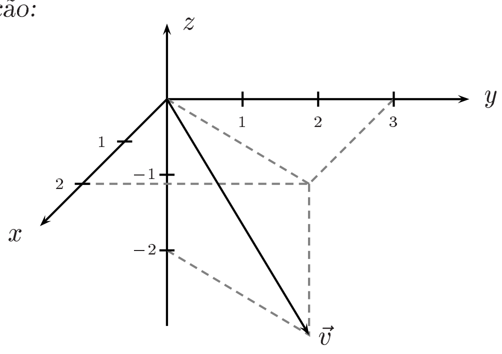


E6) Represente os seguintes pontos no espaço.

a) $A(2,4,3)$, b) $B(3,-1,2)$, c) $C(-2,1,-1)$, d) $D(-3,-2,4)$, e) $E(1,-4,-3)$.

Exemplo 7: represente o vetor $\vec{v} = 2\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$ no espaço.

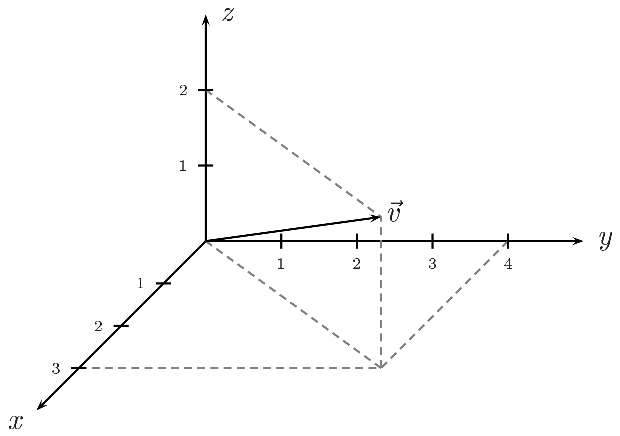
Solução:



E7) Represente os seguintes vetores no espaço.

a) $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$, b) $\vec{b} = -2\hat{\imath} + \hat{\jmath} - 2\hat{k}$, c) $\vec{c} = 2\hat{\imath} + 4\hat{k}$, d) $\vec{d} = 3\hat{\imath} - 2\hat{\jmath}$.

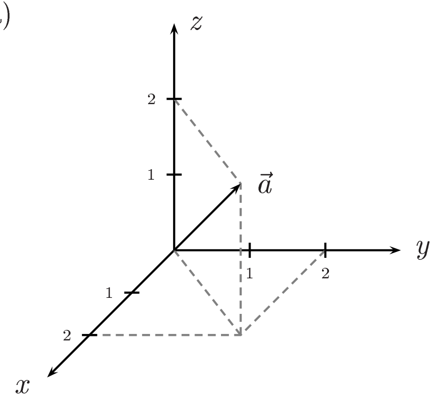
Exemplo 8: escreva o vetor representado abaixo em termos de suas componentes.



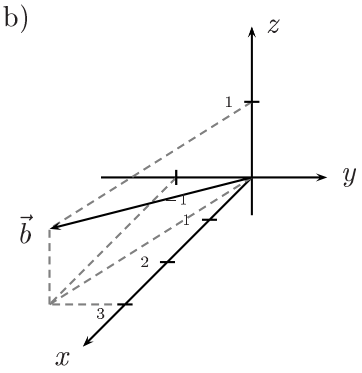
Solução: $\vec{v} = 3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$.

E8) Escreva os vetores representados abaixo em termos de suas componentes.

a)



, b)



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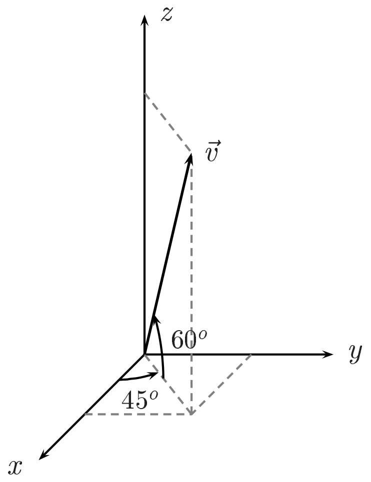
Exemplo 9: calcule o módulo do vetor do exemplo 7.

Solução: $|\vec{v}| = \sqrt{2^2 + 3^2 + (-2)^2} = \sqrt{4+9+4} = \sqrt{17}$.

E9) Calcule os módulos dos vetores do exercício E7.

Exemplo 10: escreva o vetor representado abaixo em termos de suas componentes sabendo que $|\vec{v}| = 4$.



Solução: usamos o ângulo de inclinação (60^o) para calcular os módulos do vetor $\vec{v}_z$ e do vetor resultante $\vec{r} = \vec{v}_x + \vec{v}_y$.

$$\cos 60^o = \frac{r}{v} \Rightarrow v.\cos 60^o = r \Rightarrow r = v.\cos 60^o \Rightarrow r = 4 \cdot \frac{1}{2} \Rightarrow r = 2 \; ,$$
$$\text{sen}60^o = \frac{v_z}{v} \Rightarrow v.\text{sen}60^o = v_z \Rightarrow v_z = v.\text{sen}60^o \Rightarrow v_z = 4 \cdot \frac{\sqrt{3}}{2} \Rightarrow v_z = 2\sqrt{3} \; .$$

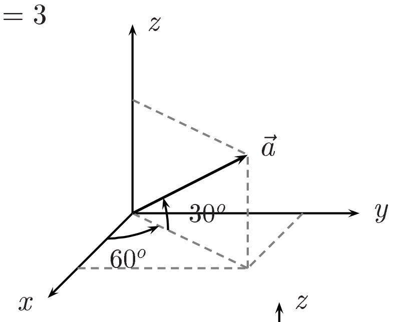
Agora, usamos o ângulo de azimute (45^o) para calcular os módulos dos vetores $\vec{v}_x$ e $\vec{v}_y$.

$$\cos 45^o = \frac{v_x}{r} \Rightarrow r.\cos 45^o = v_x \Rightarrow v_x = r.\cos 45^o \Rightarrow v_x = 2.\frac{1}{\sqrt{2}} \Rightarrow v_x = \frac{2}{\sqrt{2}} \Rightarrow v_x = \frac{2\sqrt{2}}{\sqrt{2}.\sqrt{2}} \Rightarrow v_x = \frac{2\sqrt{2}}{2} \Rightarrow$$
$$\Rightarrow v_x = \sqrt{2} \; ,$$
$$\text{sen}45^o = \frac{v_y}{r} \Rightarrow r.\text{sen}45^o = v_y \Rightarrow v_y = r.\text{sen}45^o \Rightarrow v_y = 2.\frac{1}{\sqrt{2}} \Rightarrow v_y = \sqrt{2} \; .$$

Temos, então, que $\vec{v} = \sqrt{2}\,\hat{i} + \sqrt{2}\,\hat{j} + 2\sqrt{3}\,\hat{k}$.

E10) Escreva os vetores representados abaixo em termos de suas componentes.

a) $|\vec{a}| = 3$



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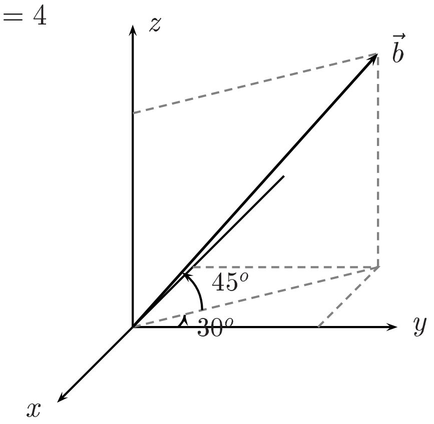
b) $|\vec{b}| = 4$



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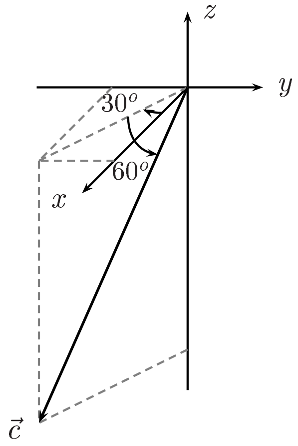
c) $|\vec{c}| = 4$



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Soma de vetores.

Exemplo 11: dados os vetores $\vec{u} = 3\hat{\imath} + 2\hat{\jmath} - \hat{k}$ e $\hat{v} = -\hat{\imath} + 2\hat{\jmath}$, calcule $\vec{u} + \vec{v}$.

Solução: $\vec{u} + \vec{v} = (3\hat{\imath} + 2\hat{\jmath} - \hat{k}) + (-\hat{\imath} + 2\hat{\jmath}) = (3-1)\hat{\imath} + (2+2)\hat{\jmath} + (-1+0)\hat{k} = 2\hat{\imath} + 4\hat{\jmath} - \hat{k}$.

E11) Dados os vetores $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$, $\vec{b} = -2\hat{\imath} + \hat{\jmath}$ e $\vec{c} = \hat{\imath} + \hat{\jmath} - \hat{k}$, calcule:

a) $\vec{a} + \vec{b}$, b) $\vec{a} + \vec{c}$, c) $\vec{b} + \vec{c}$, d) $\vec{a} + \vec{b} + \vec{c}$.

Produto de um vetor por um escalar.

Exemplo 12: dado o vetor $\vec{v} = 3\hat{\imath} + 2\hat{\jmath} - \hat{k}$, calcule $2\vec{v}$.

Solução: $2\vec{v} = 2(3\hat{\imath} + 2\hat{\jmath} - \hat{k}) = 2.3\hat{\imath} + 2.2\hat{\jmath} + 2(-1)\hat{k} = 6\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$.

E12) Dados os vetores $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ e $\vec{b} = -2\hat{\imath} + \hat{\jmath}$, calcule:

a) $3\vec{a}$, b) $-\vec{b}$, c) $\frac{1}{2}\vec{a}$, d) $-\frac{1}{3}\vec{b}$.

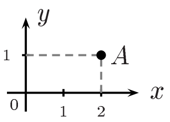
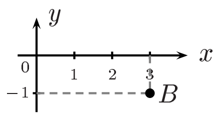
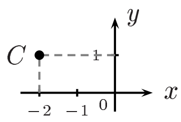
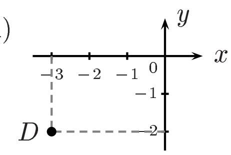
Exemplo 13: dados os vetores $\vec{u} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$ e $\vec{v} = \hat{\imath} - 3\hat{k}$, calcule $2\vec{u} - \vec{v}$.

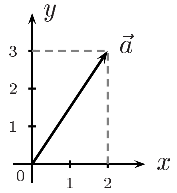
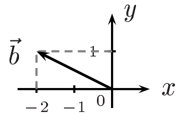
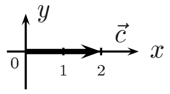
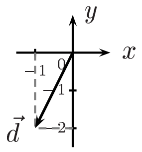
Solução: $2\vec{u} - \vec{v} = 2(3\hat{\imath} - \hat{\jmath} + 2\hat{k}) - (\hat{\imath} - 3\hat{k}) = 6\hat{\imath} - 2\hat{\jmath} + 4\hat{k} - \hat{\imath} + 3\hat{k} = (6-1)\hat{\imath} + (-2+0)\hat{\jmath} + (4+3)\hat{k} = 5\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$.

E13) Dados os vetores $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$, $\vec{b} = -2\hat{\imath} + \hat{\jmath}$ e $\vec{c} = \hat{\imath} + \hat{\jmath} - 3\hat{k}$, calcule:

a) $2\vec{a} - \vec{b}$, b) $\vec{a} + 3\vec{b}$, c) $\frac{1}{2}\vec{a} - 2\vec{c}$, d) $0,5\vec{a} - 0,1\vec{b} + 0,4\vec{c}$.

Respostas

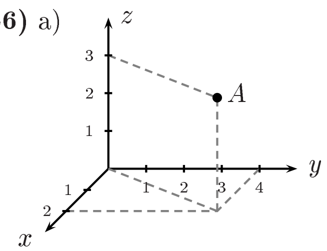
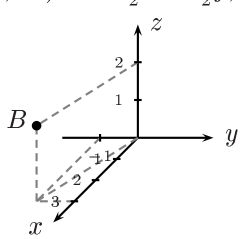
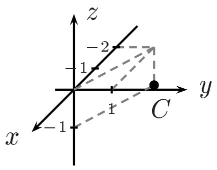
E1) a)  , b)  , c)  , d)  .

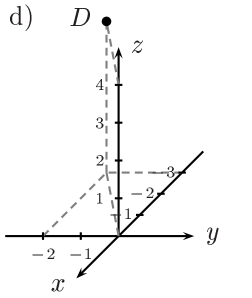
E2) a)  , b)  , c)  , d)  .

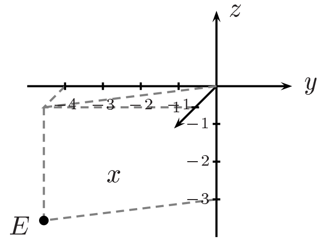
E3) a) $\vec{a} = 2\hat{\imath} + \hat{\jmath}$, b) $\vec{b} = 3\hat{\imath} - \hat{\jmath}$, c) $\vec{c} = -2\hat{\imath} - \hat{\jmath}$.

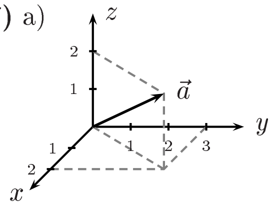
E4) a) $\sqrt{13}$, b) $\sqrt{5}$, c) 2, d) $\sqrt{5}$.

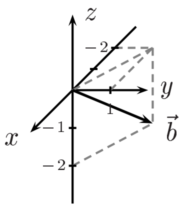
E5) a) $\vec{a} = \frac{1}{2}\hat{\imath} + \frac{\sqrt{3}}{2}\hat{\jmath}$, b) $\vec{b} = \sqrt{2}\,\hat{\imath} + \sqrt{2}\,\hat{\jmath}$, c) $\vec{c} = \frac{3\sqrt{3}}{2}\hat{\imath} - \frac{1}{2}\hat{\jmath}$, d) $\vec{d} = -\frac{1}{2}\hat{\imath} + \frac{\sqrt{3}}{2}\hat{\jmath}$, d) $\vec{d} = -\sqrt{2}\,\hat{\imath} - \sqrt{2}\,\hat{\jmath}$.

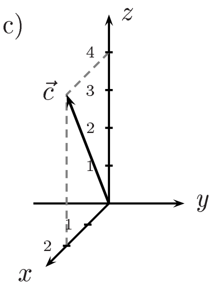
E6) a)  , b)  , c)  ,

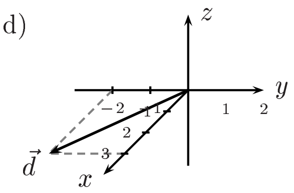
d) 

, e) 

E7) a) 

, b) 

, c) 

, d) 

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E8) a) $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$, b) $\vec{b} = 3\hat{\imath} - \hat{\jmath} + \hat{k}$.

E9) a) $\sqrt{17}$, b) 3, c) $2\sqrt{5}$, d) $\sqrt{13}$.

E10) a) $\vec{a} = \frac{3\sqrt{3}}{2}\hat{\imath} + \frac{9}{4}\hat{\jmath} + \frac{3}{2}\hat{k}$, b) $\vec{b} = -\sqrt{2}\,\hat{\imath} + \sqrt{6}\,\hat{\jmath} + 2\sqrt{2}\,\hat{k}$, c) $\vec{c} = \sqrt{3}\,\hat{\imath} - \hat{\jmath} + 2\sqrt{3}\,\hat{k}$.

E11) a) $\hat{\imath} - \hat{\jmath} + \hat{k}$, b) $4\hat{\imath} - \hat{\jmath}$, c) $-\hat{\imath} + 2\hat{\jmath} - \hat{k}$, d) $2\hat{\imath}$.

E12) a) $9\hat{\imath} - 6\hat{\jmath} + 3\hat{k}$, b) $2\hat{\imath} - \hat{\jmath}$, c) $\frac{1}{2}\hat{\imath} - \hat{\jmath} + \frac{1}{2}\hat{k}$, d) $\frac{2}{3}\hat{\imath} - \frac{1}{3}\hat{\jmath}$.

E13) a) $8\hat{\imath} - 5\hat{\jmath} + 2\hat{k}$, b) $-3\hat{\imath} + \hat{\jmath} + \hat{k}$, c) $-\frac{1}{2}\hat{\imath} - 3\hat{\jmath} + \frac{13}{2}\hat{k}$, d) $2,1\hat{\imath} - 0,7\hat{\jmath} - 0,7\hat{k}$.