Torque Modelling of a Superconducting Reluctance Machine

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Abstract - This paper discusses the torque characteristics of reluctance machines with and without the inclusion of high temperature superconducting (HTS) elements. The torque equations for the reluctance machines are developed, as well as a method for computing the torque from the finite element flux density model. The models and data are generated through the use of OXFEM, a finite element software package developed at the University of Oxford. The aim of this paper is to both verify the advantages of HTS machines predicted by other research, and to demonstrate the capability of software such as OXFEM in the design of HTS electrical machines.

1. Introduction

Analytical methods for the solution of machine characteristics quickly become prohibitively complex as models move away from simple geometry and uniform magnetic fields, and towards more realistic, useful and innovative designs. For this reason much effort has been put into the development of finite element modelling tools than can make the task only as complex as the initial design, with the analysis being performed automatically. Whereas finite element software has been available to model conventional electrical machines for many years, it is only recently that software has been developed to include the modelling of superconducting materials. OXFEM is such a finite element software package, which was developed at the University of Oxford in 1999.
Heike Kammerlingh Onnes, of Leiden University, discovered superconductivity in 1911, when Mercury was cooled below 4.2 K. However it wasn't until 1987 that superconductivity started to have real potential in engineering applications. It was discovered that with the right compound, the superconducting state could be achieved at temperatures greater than that of liquid nitrogen, a coolant that was both easily available and cheap. The so-called Type-II superconductors usually exist in a mixed state of normal and superconducting regions, called a vortex state where vortices of superconducting currents surrounding cores of normal material. As the critical temperature of the material is approached, the normal cores become more closely packed until they eventually overlap and the superconducting state is lost [1].
The most obvious advantage of superconductors is their ability to sustain current densities an order of magnitude greater than copper. However a second advantage, and the property exploited in their use in reluctance machines, is their ability to repel flux, or to act as flux barriers, due to their diamagnetic properties.
The next section will give an overview of the principal of operation of reluctance machines, focusing on the torque characteristics, and will introduce the advantages of HTS modification. Section four will describe the method used to generate the torque results. Finally, the results and conclusions will be presented in sections five and six respectively.

2. The Reluctance Machine

The reluctance machine shown in figure 1 generates torque due the difference in reluctance along the two perpendicular axes of its rotor [2].

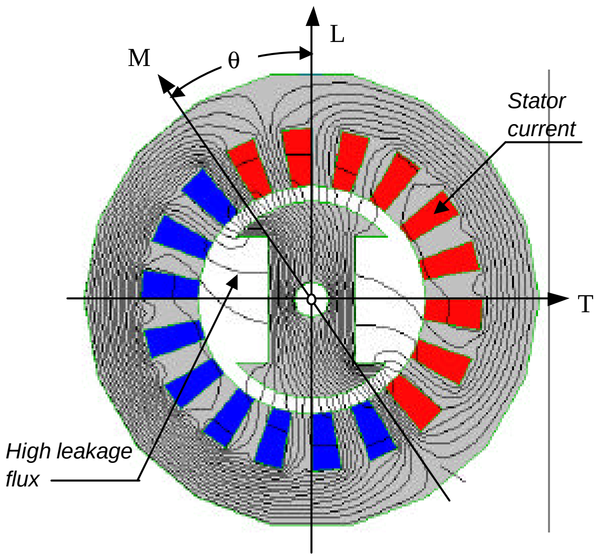

Figure 1 – Plot of Conventional Reluctance Machine

The rotor longitudinal (L) axis represents the path of minimum reluctance, and the rotor transversal (T) axis represents the path of maximum reluctance. If one then considers the stator magnetic (M) axis, one can see that the stator inductance will be at a maximum when the M and L axis are aligned and at a minimum when they are perpendicular, or the M and T axis are aligned. The torque for the three phase machine is given [3] by the following equation

$$T(\theta) = \frac{3}{2} I^2 (L_{\max} - L_{\min}) sin(2\theta) \qquad (1)$$

where $L_{\max}$ and $L_{\min}$ represent the maximum and minimum inductances respectively, and θ is the angle between the M and L axes. Figure 2 shows a plot of this torque versus rotor position θ, generated with MATLAB.

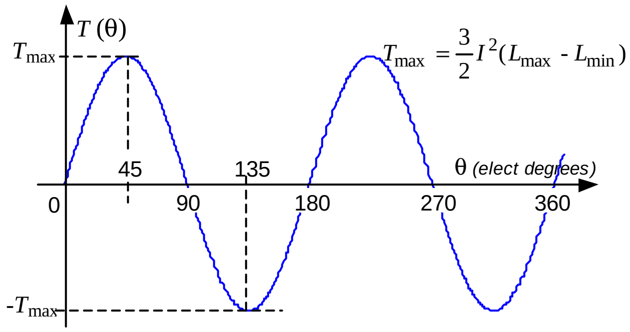

Figure 2 - Plot of Torque versus Rotor Position

With respect to figure 2 it should be noted that the full torque cycle is repeated every 180 elect. degrees, with the maximum and minimum torque appearing at 45^o and 135^o respectively. This is an obvious consequence of the rotor salience, with the same torque being generated when the field is in opposite directions.

3. HTS Reluctance Machine

It has been shown that the use of superconducting elements in the design of reluctance machines can improve the torque output by as much as 60% [4]. By the appropriate incorporation of such material into the rotor, as shown in figure 3, the effective reluctance of the rotor transversal axis can be increased due to the ability of bulk superconducting materials to repel flux from their interior.

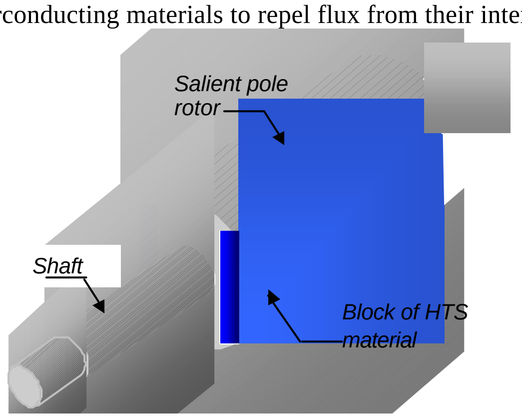

Figure 3 - Reluctance rotor with HTS pieces

This ability, a consequence of their diamagnetic properties, is demonstrated in figure 4 When the rotor incorporates HTS elements the flux leakage decreases for the same applied field, as compared with the conventional rotor shown in figure 1. With reference to equation (1), by increasing the transverse reluctance, the value of $L_{\min}$ is reduced which results in a torque increase.

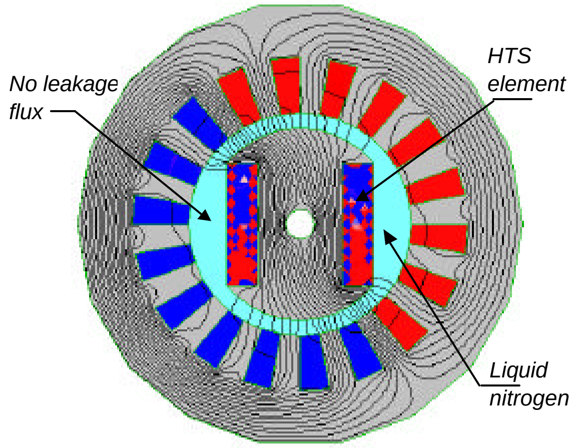

Figure 4 – Plot of HTS Reluctance Machine

The extent to which the superconducting material behaves as a flux barrier is a function of its physical properties, specifically its critical current density. As the critical current density of the material increases so does the extent to which it repels flux [5]. Often, when analysing machines with HTS elements, it is useful to define the shielding parameter s [6], which is given by the following equation

$$s = \frac{\mu_o J_c R}{B_o} , \qquad (2)$$

where R is the radius of the rotor, J_c is the critical current density and B_o is the external field at the centre of the machine. However, for the purposes of this paper it was decided to work directly with J_c as it is a direct representation of the material and is also the value directly defined in OXFEM.

4. Torque Calculation Using FEM

Although the flux plots shown in figure 4 are informative, it is more useful to directly compare the torque characteristics of these motors.

For the purposes of this paper, OXFEM was modified to calculate the torque using an energy method that considered the finite elements that intersected a summation circle drawn around the rotor of the machine. Of course this was not the only possible method, the most common alternative being the use of Maxwell Stress Tensor (MST) theory.

However, the energy method was selected due to its simplicity. The theoretical basis of the method [7] is as follows.

The magnetic energy W_{mag} stored in a magnetic field B is given by:

$$W_{mag} = \frac{1}{2\mu}\int_v |B|^2 dv \qquad (3)$$

It follows that the energy contained within an element in the air-gap of an electrical machine will be given by:

$$\Delta W_{mag} = \frac{B_e^2 A_e}{2\mu_e} \qquad (4)$$

Where the subscript 'e' refers to the values of the individual elements. Hence the energy stored in the air-gap will be given up the sum of the energies of the elements in the air-gap:

$$W_{ag} = \sum \Delta W_{mag} \qquad (5)$$

The torque of the machine can be calculated as:

$$T(\theta) = \frac{\partial W_{ag}}{\partial \theta}\Big|_{i,t=Const} \qquad (6)$$

Hence by combining equations (4), (5) and (6), the torque can be calculated as in equation (7):

$$T(\theta) = \frac{\partial\left\{\sum\left(\frac{B_e^2 A_e}{2\mu_e}\right)\right\}}{\partial\theta}\Bigg|_{i,t=Const} \qquad (7)$$

The summation was implemented in the OXFEM, by modifying the JAVA code [8], and designed to run automatically upon the selection of a suitable summation circle. This circle was consistently chosen to lie within the air-gap of the machine. The location of circle line within the air-gap was not relevant as the finite element mesh was arranged such that the air-gap only had a thickness of one element.
Once OXFEM had calculated the energy distribution around the air-gap, differentiating this distribution with respect to ? generated the torque of the machine. This final stage was executed with MATLAB.

5. Numerical Results

The torque curve for the reluctance machine with HTS elements was generated for three different values of the critical current density of the superconducting material: J_c = 1.0x10^7 A/m^2, 5.0x10^7 A/m^2 and 1.0x10^8 A/m^2. The machine design was the same as that shown in figure 4. These curves, together with the curve generated for the conventional reluctance machine are presented in figure 5. The results show that the use of HTS elements with critical current densities of 1.0x10^7 A/m^2 will offer an insignificant increase in torque. However, the use of HTS elements with critical current densities of 5.0x10^7 A/m^2 and 1.0x10^8 A/m^2 will give a torque increase of approximately 35% and 50% respectively.
It should be noted that peaks of the curves to not all lie at 45^o or 135^o, although they are close. The errors in the results could be accounted for by the fact that the computational process included the approximate differentiation of sampled discrete and noisy data, a process notorious for generating errors. For this reason, the results should be considered a qualitative, not quantitative, indication of the relative torque improvements.

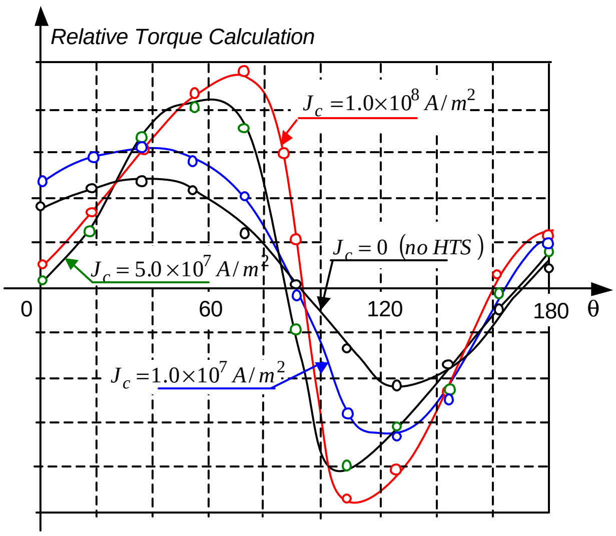

Figure 5 – Plot of Torque versus Rotor Position for different values of current density J_c

6. Conclusions

The primary aim of this paper was to demonstrate, though the use of finite element modelling, the increased torque production of a reluctance machine with HTS elements when compared with a conventional reluctance machine. This was achieved by the generation of torque curves for the both the non-superconducting case and for three superconducting cases each with different values of critical current density.
It was shown that the increase in torque due to the inclusion of bulk HTS material on the rotor could be as great as 50%. This is in agreement with previous research.
A method for the torque calculation was developed based upon the analysis of the stored energy in the air-gap. This method was implemented within OXFEM, a modelling program based on finite elements, and used to generate the torque results.
It is clear that, in the design and analysis of electrical machines with HTS elements, the use of a modelling program such as OXFEM is invaluable. The optimisation of these programs is essential for the rapid integration of

superconducting machine theory into mainstream engineering solutions.

Acknowledgements

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