

## Pre-Calculus—Chapter 7

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# Pre-Calculus—Chapter 7-1

## Basic Trig Identities

### Identity →

A statement of equality (equation) between two expressions that is true for all values of the variable(s) is called an *Identity*.

Some identities you already know-----

### Reciprocal Identities:

$$\sin \theta = 1/\csc \theta \quad \text{etc.}$$

Ex: If  $\tan \theta = 0.8$ , find  $\cot \theta \dots$

$$\cot \theta = 1/\tan \theta$$

$$\cot \theta = 1/0.8 = 1.25$$

## Quotient Identities:

$$\frac{\sin \theta}{\cos \theta} = \tan \theta \quad \rightarrow \quad \sin \theta = \tan \theta \cos \theta$$

$$\frac{\cos \theta}{\sin \theta} = \cot \theta \quad \rightarrow \quad \cos \theta = \sin \theta \cot \theta$$

## Pythagorean Identities:

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

\*\*\*Remember that all trig identities hold for all values except those for which the function is undefined.

## Symmetry Identities:

Use the reference angle and the quadrant in which the angle lies to find the Symmetric identity.

$$\tan 315^\circ$$

Ex:  $315^\circ = 360^\circ - 45^\circ$  Reference angle = 45

Now what quadrant does 315 lie in? 3<sup>rd</sup>-----

Thus tangent is negative----Answer.....

$$\tan 315^\circ = -\tan 45^\circ$$

Simplifying Ex: Simplify:

$$\sin^2 \theta + \sin^2 \theta \tan^2 \theta$$

$$\sin^2 \theta (1 + \tan^2 \theta)$$

$$\sin^2 \theta (\sec^2 \theta)$$

$$\sin^2 \theta \left( \frac{1}{\cos^2 \theta} \right)$$

$$\tan^2 \theta$$

Note that we moved from a stated expression to a more simplified one using *identities*.



# Pre-Calculus—Chapter 7-2

## Verifying Trig Identities

Verifying trig identities involves transforming **ONE** side of the equation into the same form as the other side.

### Hints or Steps-----

- Choose the more complicated looking side to transform.
- Substitute the basic trig identities to simplify.
- Try factoring or multiplying to simplify the expression.
- Try multiplying both the numerator and denominator by the same trig expression.

Any time you are to “Verify”, you know that the identity is true—you are just verifying.



# Pre-Calculus—Chapter 7-2B

## Using the Graphing Calculator to verify trig Identities

Steps:

- Open the graphing editor--  $\langle y = \rangle$
- Enter the expression on one side of the identity into  $y =$
- Then enter the expression on the other side into the next  $y =$
- *ZoomTrig* Sometime you will have to adjust the window so that you may see the graph.

WOW!! If the graphs coincide then the equation is a trig identity.

Important to note where you put the parentheses.



# Pre-Calculus—Chapter 7-5

## Solving Trig Equations

Trig Identities  $\rightarrow$  true for all values of the variable for which the expression is defined.

Trig Equations  $\rightarrow$  true for some but not all values of the variable—do not have *unique solutions*....

WHY NOT???

Solve:  $2 \cos^2 x - 5 \cos x + 2 = 0$

$$(\cos x - 2)(2 \cos x - 1) = 0$$

$$\cos x = 2 \qquad \cos x = \frac{1}{2}$$

Wait!! Must always check solutions.

Solve:  $2 \tan x \sin x + 2 \sin x = \tan x + 1$

$$2 \sin x (\cancel{\tan x + 1}) = \cancel{\tan x + 1}$$

$$2 \sin x = 1$$

$$\sin x = \frac{1}{2}$$

**STOP!!!** When you divide through with an expression or term that contains a variable, you could be eliminating a solution(s).

Try again: for values of  $0^\circ \leq x \leq 360^\circ$

**Check Solutions**

$$2 \tan x \sin x + 2 \sin x = \tan x + 1$$

$$2 \sin x (\tan x + 1) = \tan x + 1$$

$$2 \sin x (\tan x + 1) - (\tan x + 1) = 0$$

$$(\tan x + 1)(2 \sin x - 1) = 0$$

$$\sin x = \frac{1}{2} \qquad \tan x = -1$$

Solutions-----Note: solution(s) must be within the domain.

$$x = 30^\circ, 150^\circ$$

$$x = 135^\circ, 315^\circ$$

Sometimes you solve for all values of  $x$ .

Steps:

1. Simplify until you are able to determine values for the angle using factoring, collecting like terms, or using identities.
2. Check the solution(s) to see if they are possible.
3. Check the interval given to determine which quadrant solution lies in.
4. If solution is for all possible values then add  $180k$  or  $360k$ .
5. Finally, check solution.....some algebraic operations may introduce extraneous solution(s) or eliminate solution(s).



# Pre-Calculus—Chapter 7-7

## Distance from a Point to a Line

### Formula

$$d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$$

The sign of the radical is chosen opposite the sign of  $C$ .

can be used to find the distance from a point  $(x_1, y_1)$  to a line with equation  $Ax_1 + By_1 + C = 0$ .

Example: Find the distance between  $P(3, -1)$  and the line with equation  $2x + 5y = 2$ .

Rewrite equation:  $2x + 5y - 2 = 0$

$$d = \frac{2(3) + 5(-1) + (-2)}{+\sqrt{2^2 + 5^2}} = \frac{-1}{\sqrt{29}} \quad \text{OH! NO!!!}$$

**Twilight Zone!!!!** In this situation make the distance positive.

Now suppose want to find the distance between the lines with equations  $3x + 2y = 10$  and

$$y = \frac{3}{2}x + 7 \text{ ???}$$

What do you notice about the position of these two lines? (Parallel)

Okay—If I find the distance between any point on one line to the other line then that will be the distance between the lines.

What would be an easy point to find on one line?? (y-int) Now use point  $(0, 7)$  and equation  $3x + 2y = 10$ .

Homework: Practice: 1-8  
Pg 402 7, 9, 15-18

