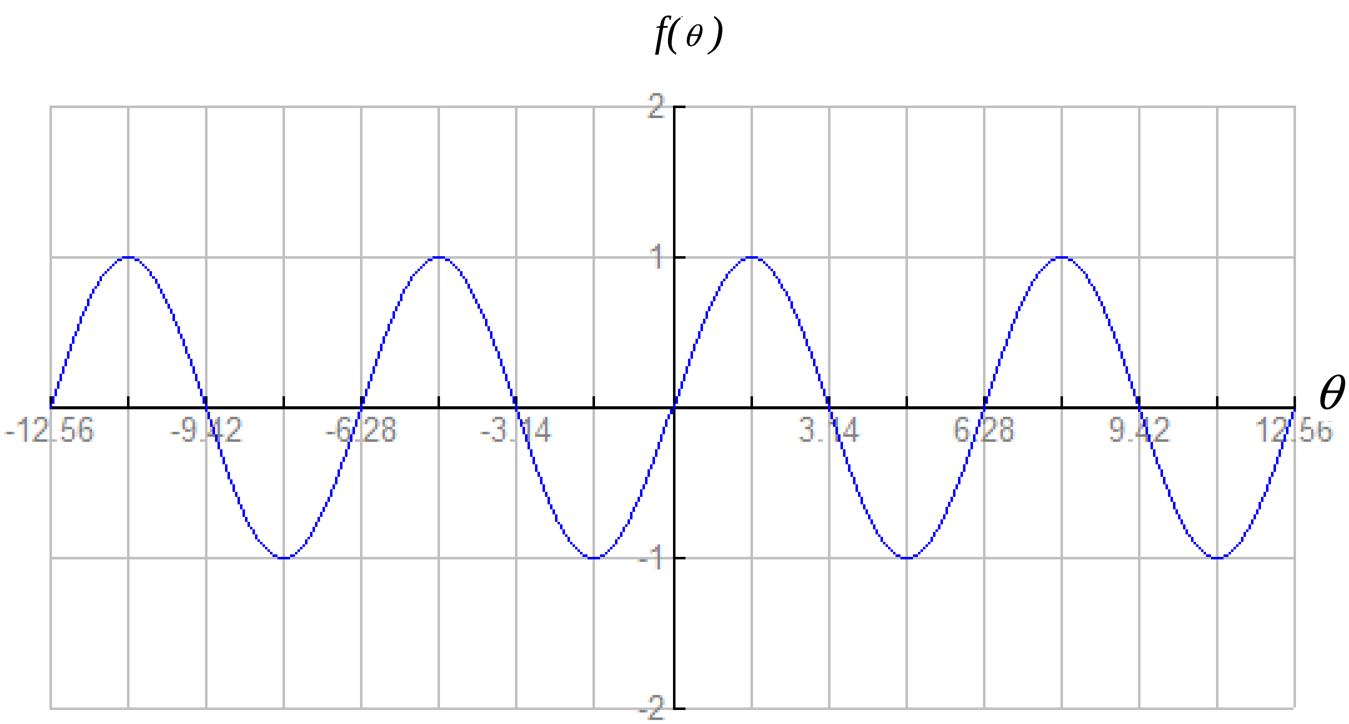
Pre-calculus—Chapter 6

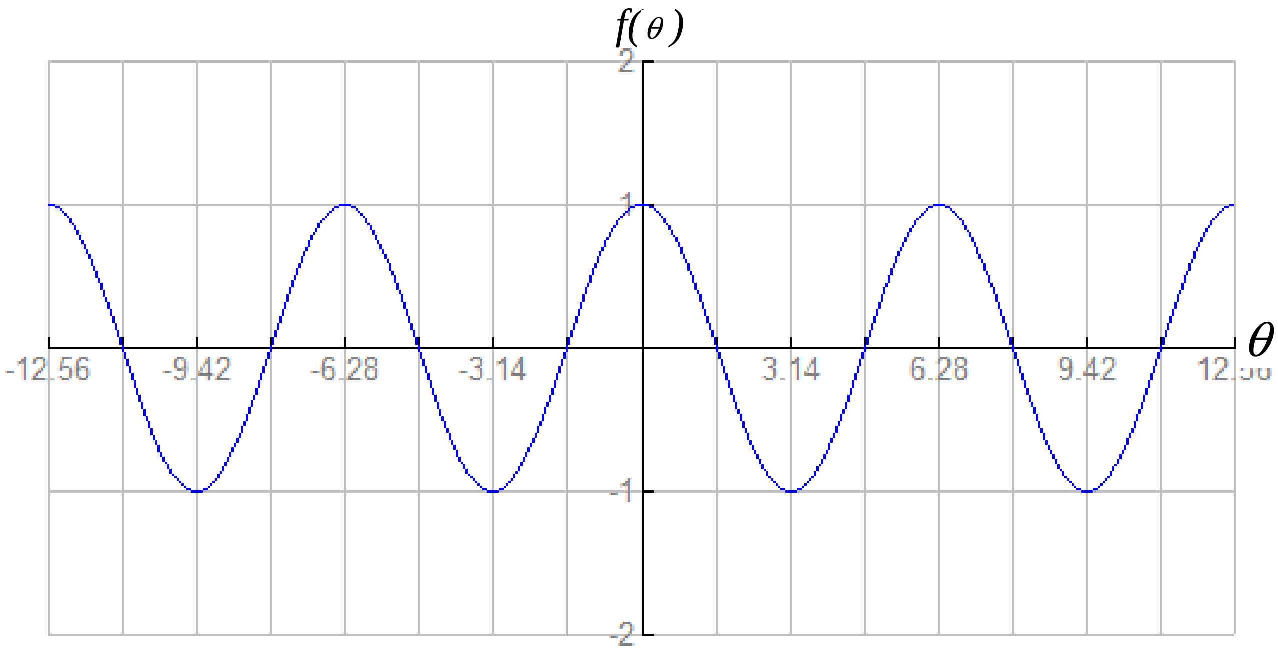
Pre-calculus—Chapter 6-1

Graphs of the 6 Basic Trigonometric Functions

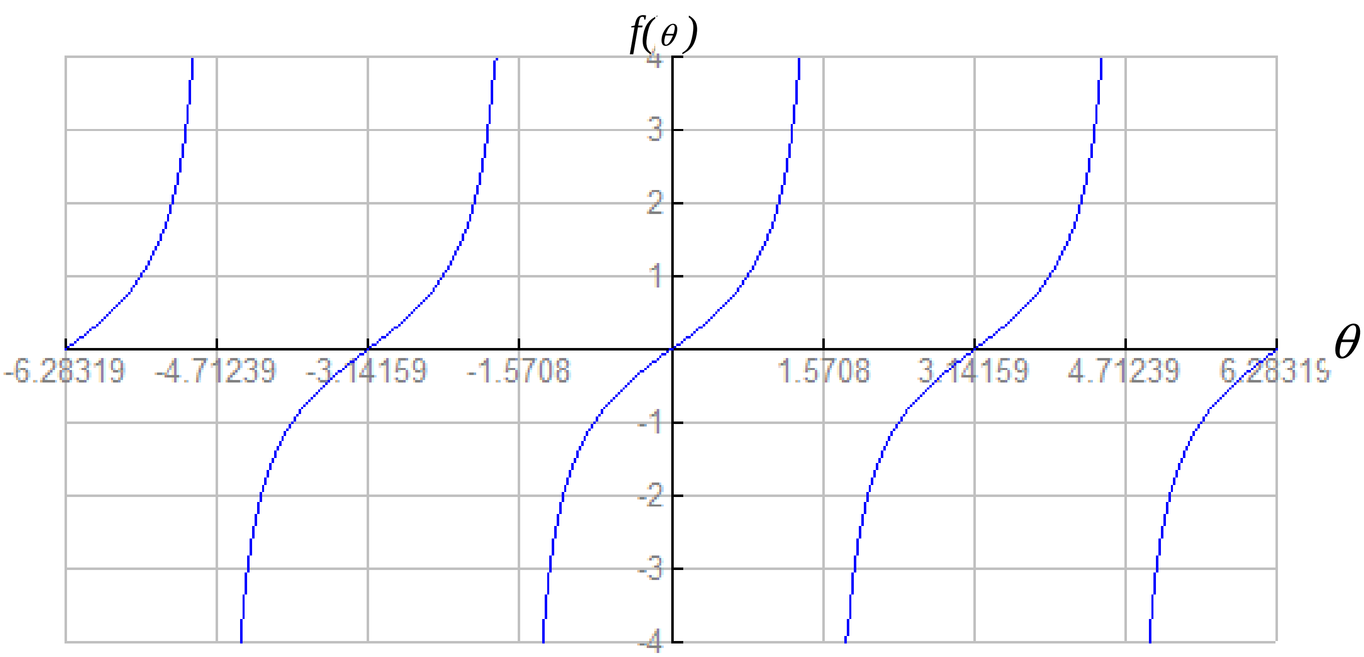
1. Sine Function → $f(\theta) = \sin\theta$



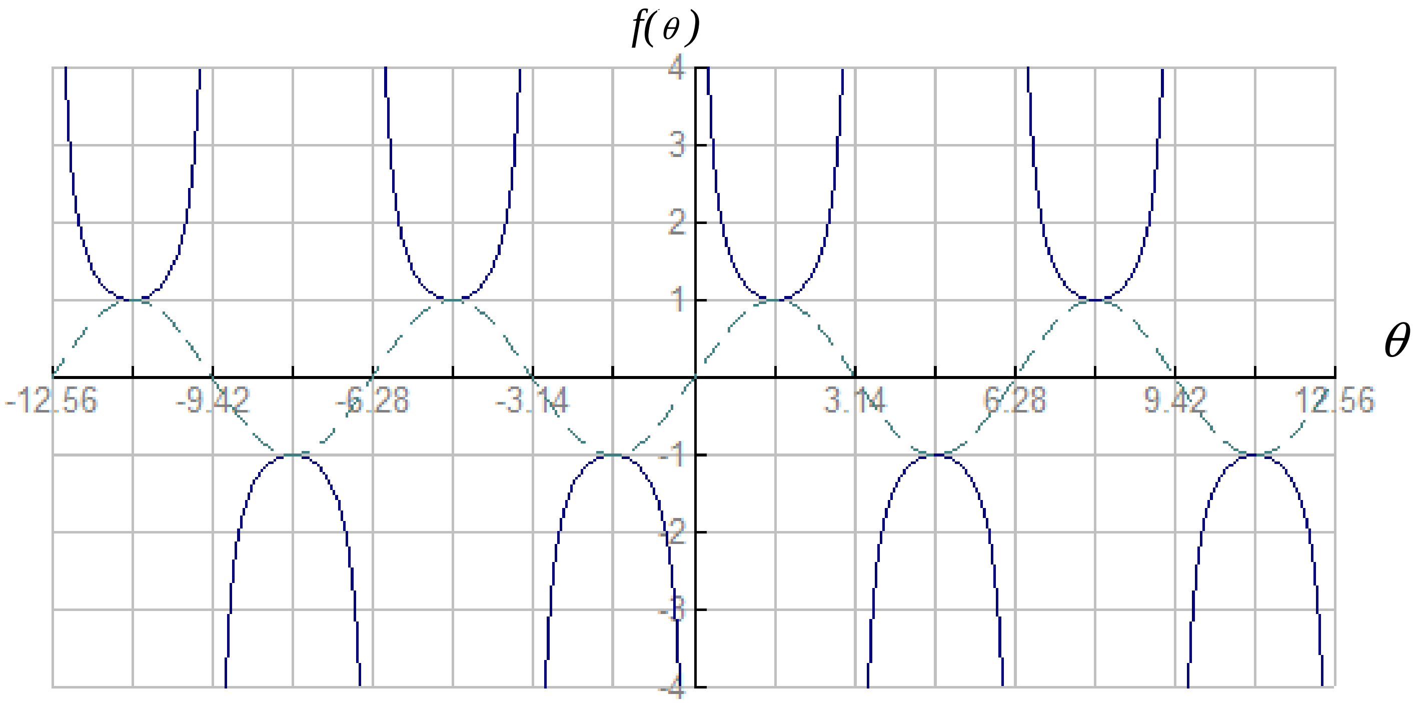
2. Cosine Function → $f(\theta) = \cos\theta$



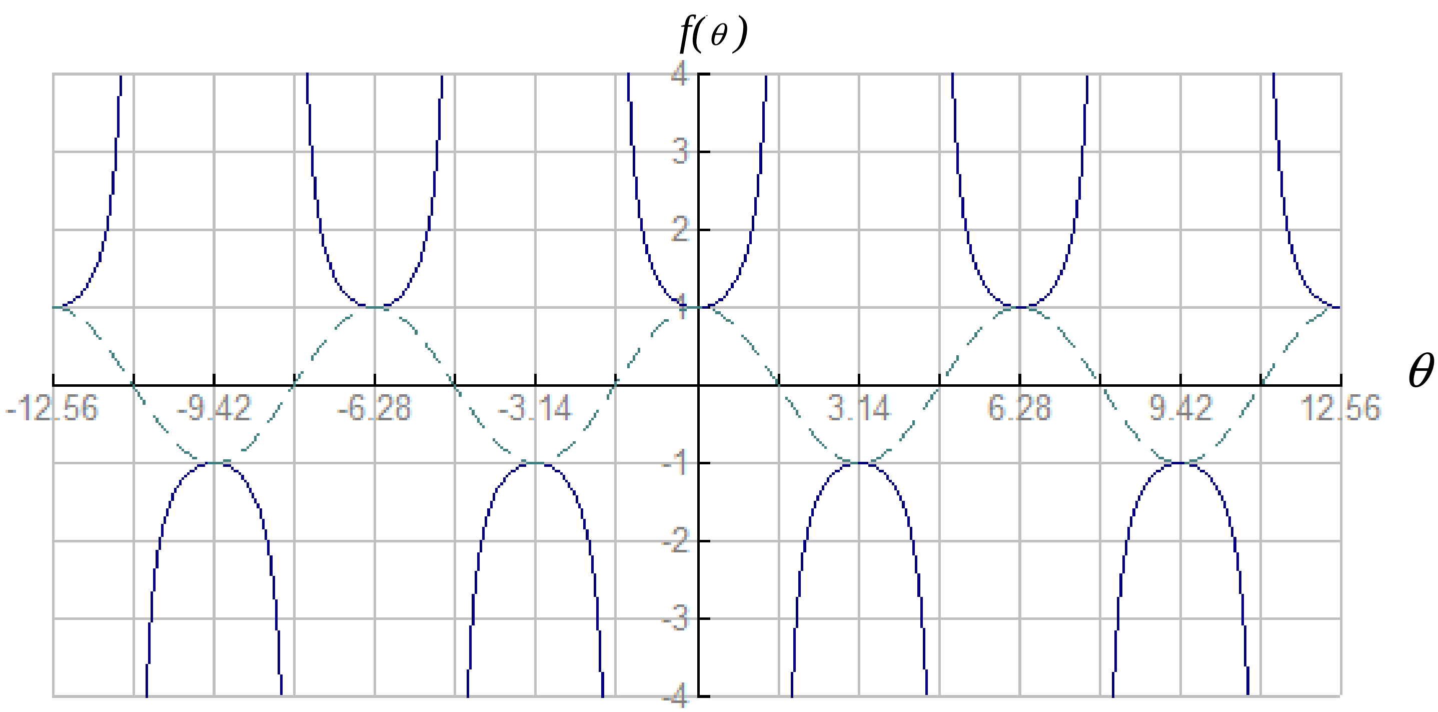
3. Tangent Function → $f(\theta) = \tan\theta$



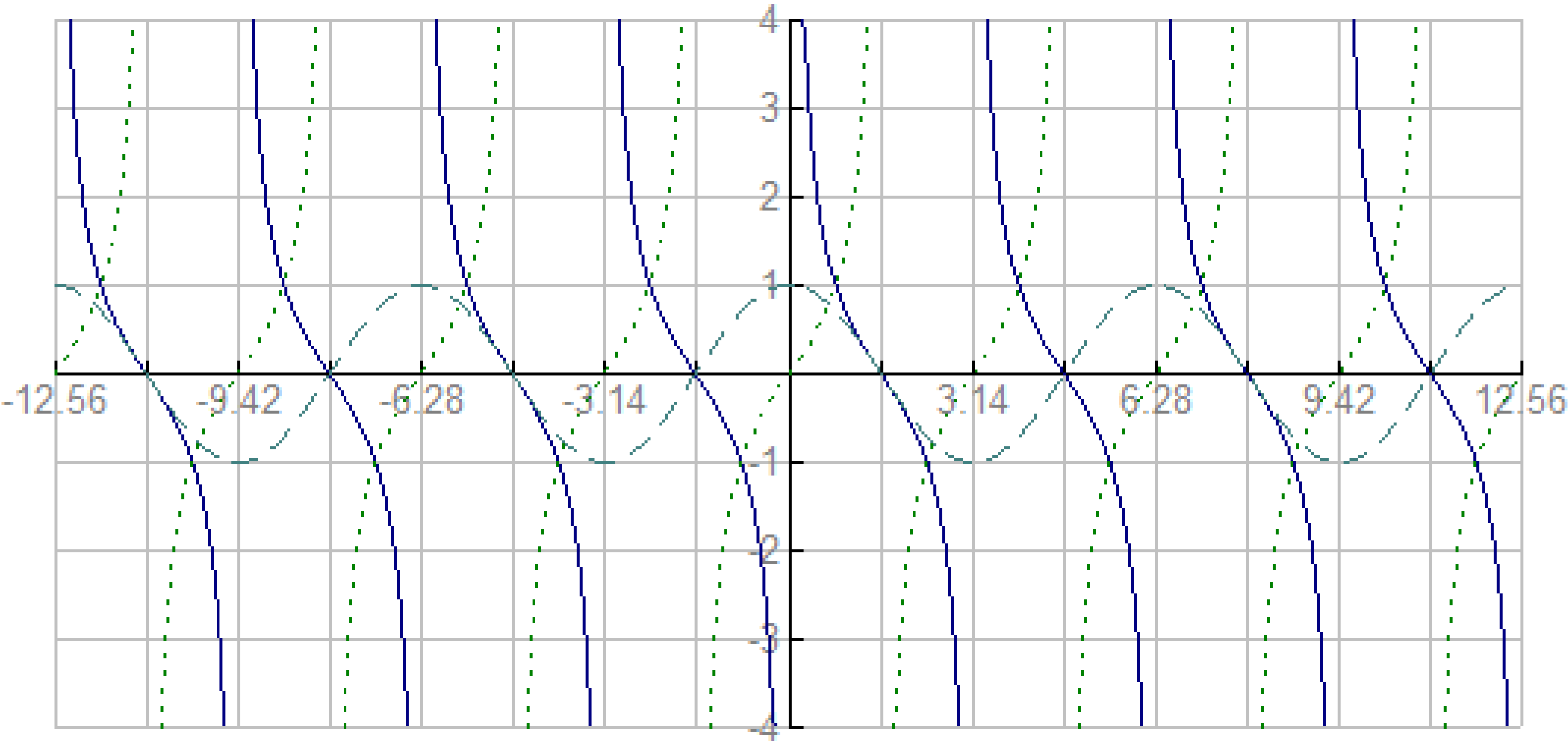
4. Cosecant Function → $f(\theta) = \csc\theta$



5. Secant Function → $f(\theta) = \sec\theta$



6. Cotangent Function → $f(\theta) = \cot\theta$



Note: The sine and cosine curves are translated 90°. This relationship is the same for the sine and cosine reciprocals (cosecant and secant) What about the tangent and cotangent graphs---?

The trig functions are ***periodic***—they repeat over and over and over and over and over and over…

The sine & cosine functions repeat every $360°$. The tangent & cotangent functions repeat every $180°$.

Thus the ***Period*** of the sine, cosecant, cosine, and secant functions is $360°$.
The ***Period*** of the tangent and cotangent functions is $180°$.

Knowing the period makes the trig functions easy to graph—all you have to know is one period or cycle and just repeat it.

Pre-calculus—Chapter 6-2

Amplitude, Period, and Phase Shift

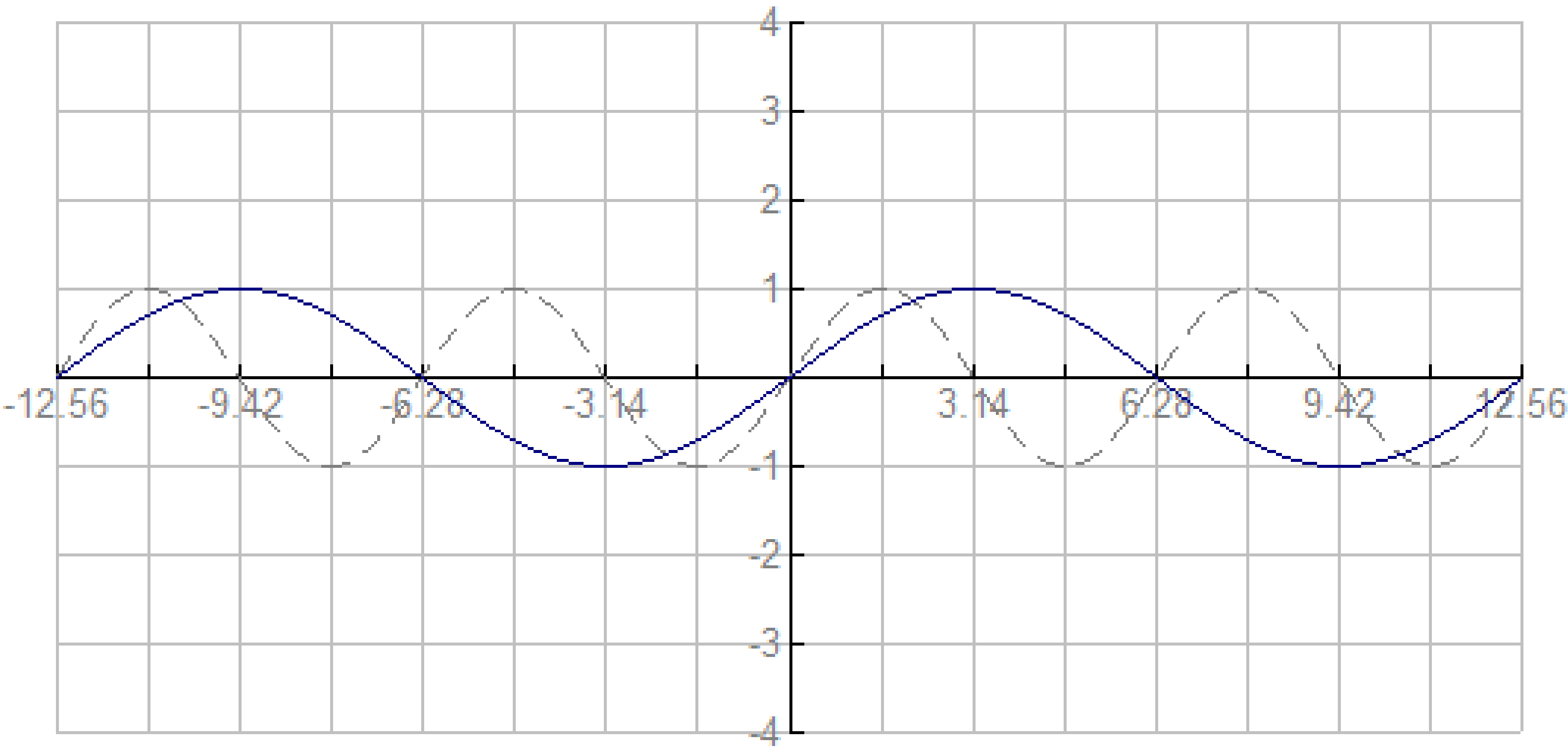
The ***amplitude*** of the functions $f(\theta) = A\sin\theta$ and $f(\theta) = A\cos\theta$ is the absolute value of A, or $|A|$.

What happens if you graph these functions with a value for *A*??? What if A is negative?

What about the graphs of $f(\theta) = A\tan\theta$ or $f(\theta) = A\cot\theta$?

The ***Period*** of the functions $f(\theta) = A\sin k\theta$

and $f(\theta) = A\cos k\theta$ is $\dfrac{360°}{|k|}$.



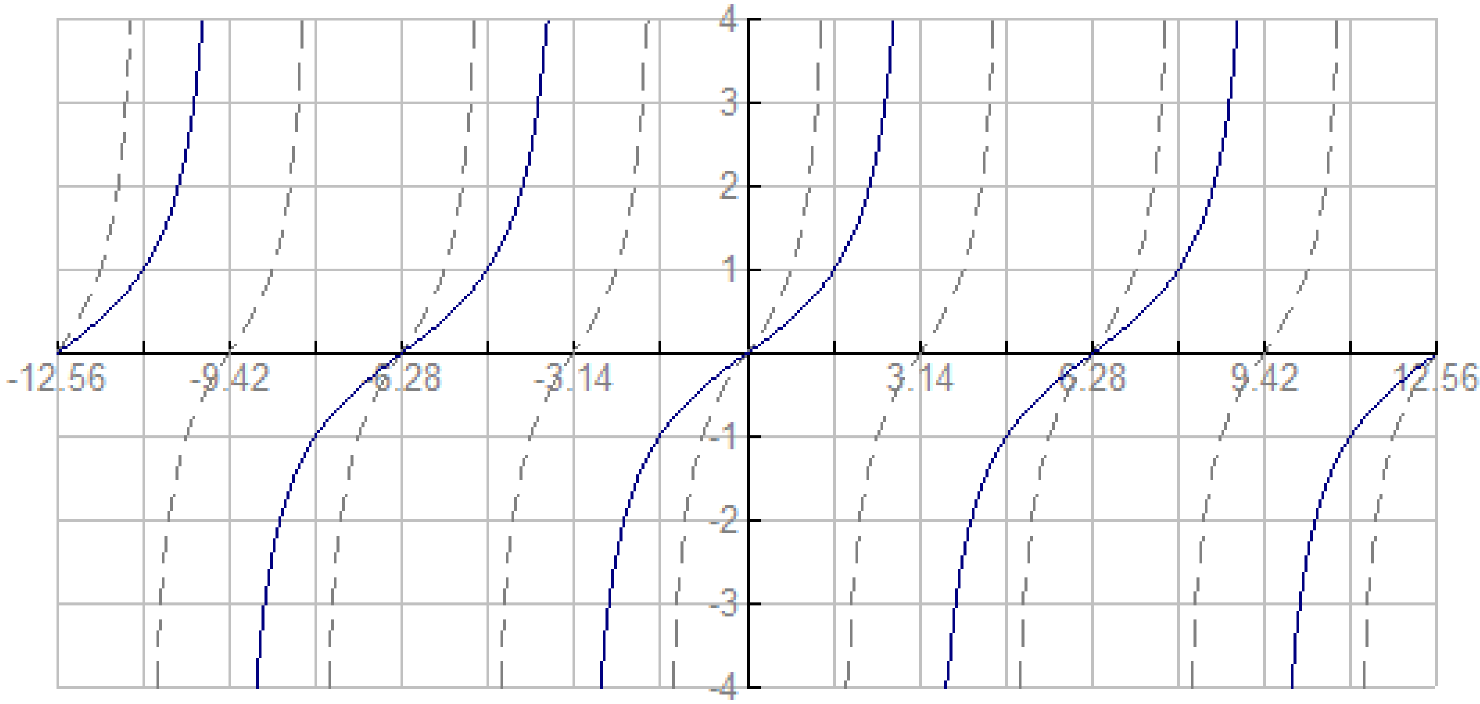
$f(\theta) = \sin\left(\dfrac{1}{2}\theta\right)$ $k = 1/2$ What is the value of A?

Period $P = \dfrac{360°}{|k|} = \dfrac{360°}{1/2} = (360°)2 = 720$

Thus $p = 4\pi$

The ***period*** of the function

$$f(\theta) = A\tan k\theta \text{ is } \frac{180^\circ}{|k|} .$$



$$f(\theta) = \tan\left(\tfrac{1}{2}\theta\right)$$

$k = ½$

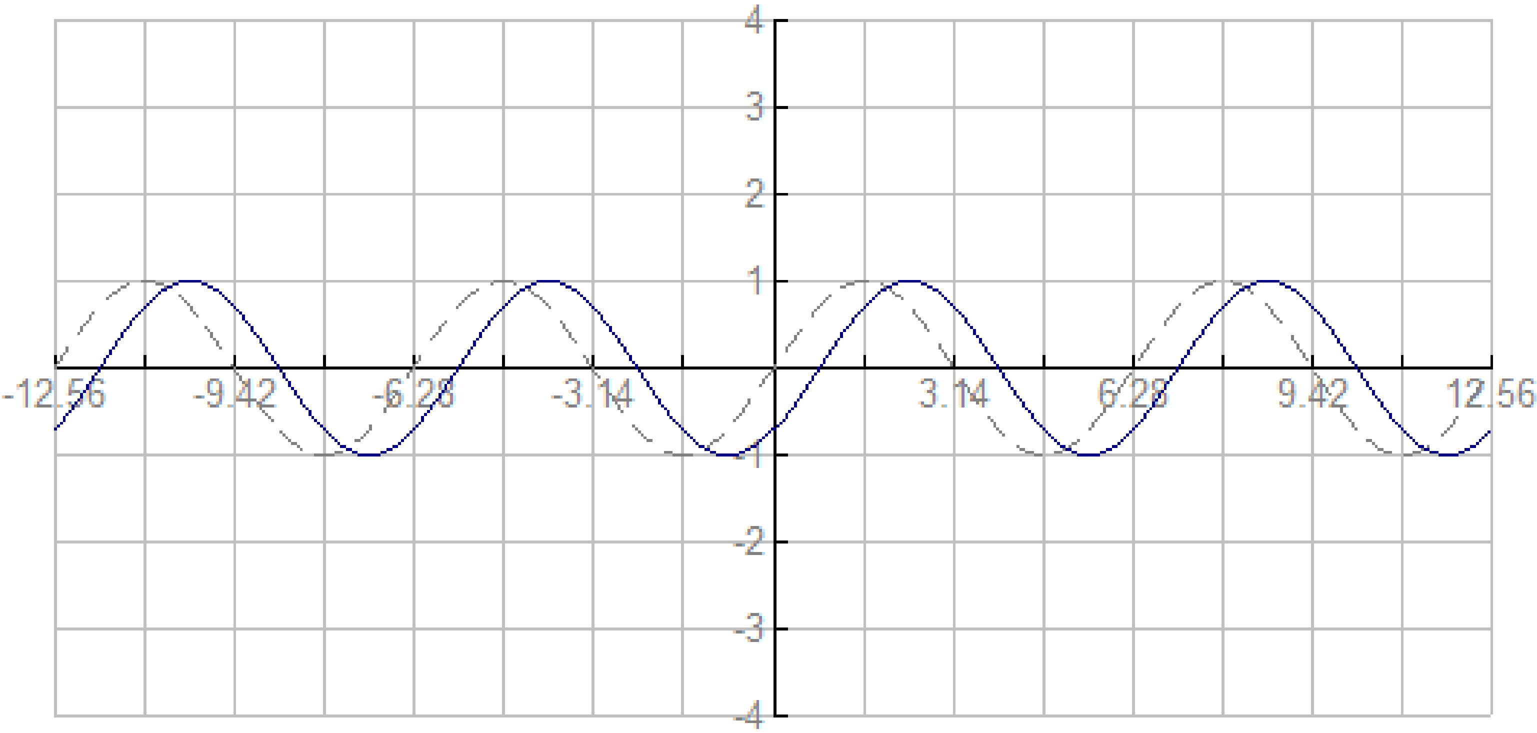
Period $P = \frac{180^\circ}{|k|} = \frac{180^\circ}{1/2} = (180^\circ)2 = 360^\circ$

Thus $p = 2\pi$

The ***Phase Shift*** of the function

$$f(\theta) = A\sin\left(k\theta + c\right) \quad \text{is} \quad -\frac{c}{k}$$

(also for the cosine function)



$$f(\theta) = \sin\left(\theta - \pi/4\right)$$

c = ???

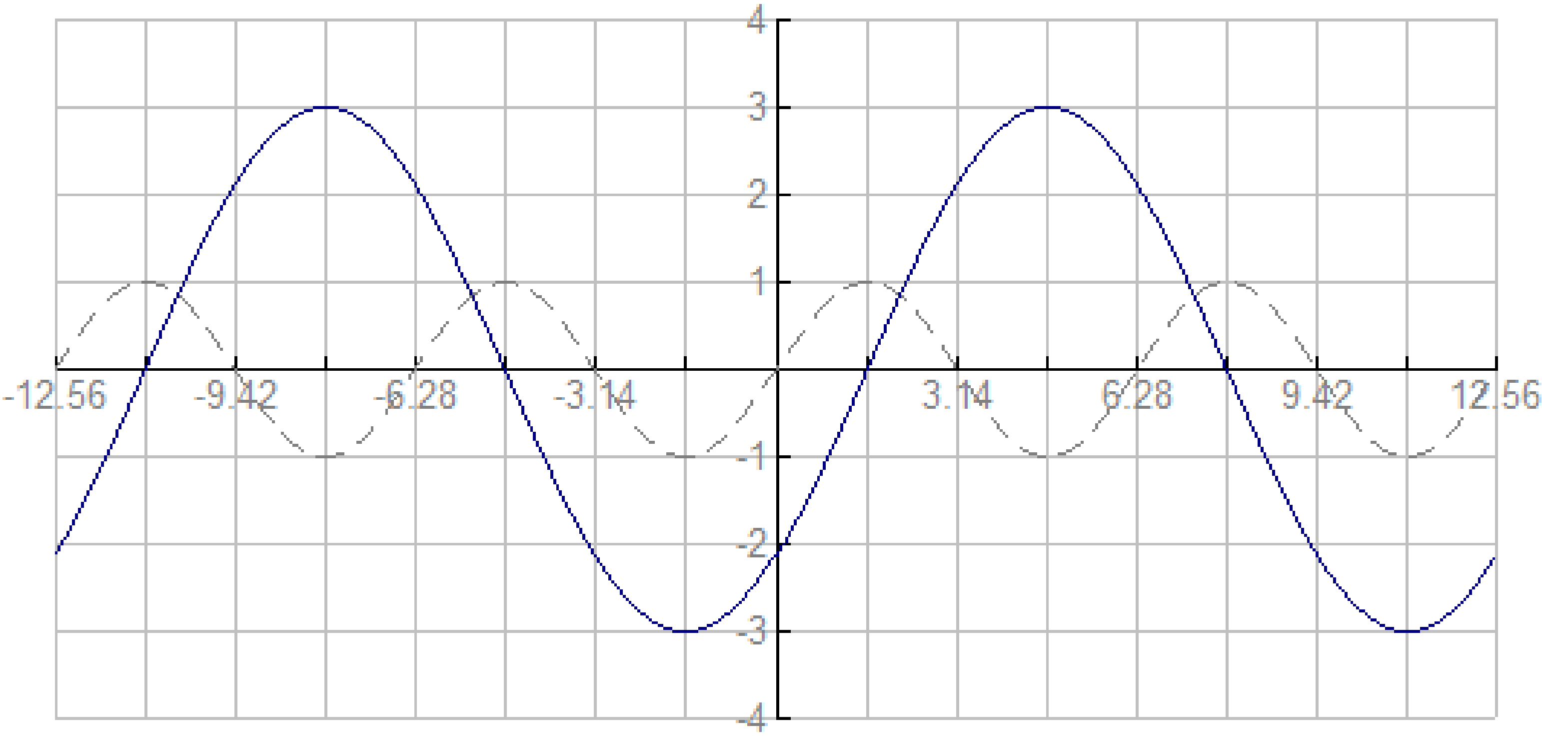
Phase Shift $\quad PS = -\frac{c}{k} = -\frac{-\pi/4}{1} = \frac{\pi}{4}$

IMP: c > 0 shift is to the left

c < 0 shift is to the right

Graph:

$$f(\theta) = 3\sin\left(\frac{1}{2}\theta - \frac{\pi}{4}\right)$$



little note→graph the parent function first then apply the different translations.

Pre-Calculus—Chapter 6-3

Graphing Trig Functions

Circular/Sinusoidal Functions
Steps to Use in Graphing

Forms:

- $f(x) = A\sin(Bx \pm C) + h$
- $f(x) = A\cos(Bx \pm C) + h$
- $f(x) = A\tan(Bx \pm C) + h$

h is the number of units graph is shifted up or down

1. Use ***A***, ***B***, and ***C*** to find the amplitude, period, and phase shift.

Amplitude $\Rightarrow$ $|A|$ maximum displacement from equilibrium

*if A is negative then invert graph.

(How would you invert a tangent graph???)

**amplitude does not affect the tangent graph (remember that you must use it in graphing)

Period $\Rightarrow$ one cycle

for the functions sine and cosine, $\text{Period} = \dfrac{2\pi}{|B|}$

for the tangent function $\text{Period} = \dfrac{\pi}{|B|}$

*Note:

If $B > 1$ than graph shrinks

If $B < 1$ than graph expands

Phase Shift $\Rightarrow$ moving cycle to the left or right

$\text{Phase Shift} = -\dfrac{C}{B}$

If $C > 0$, shift left

If $C < 0$, shift right

2. Determine the period and amplitude for the function that you are graphing.

Divide the period into 4 equal parts…….

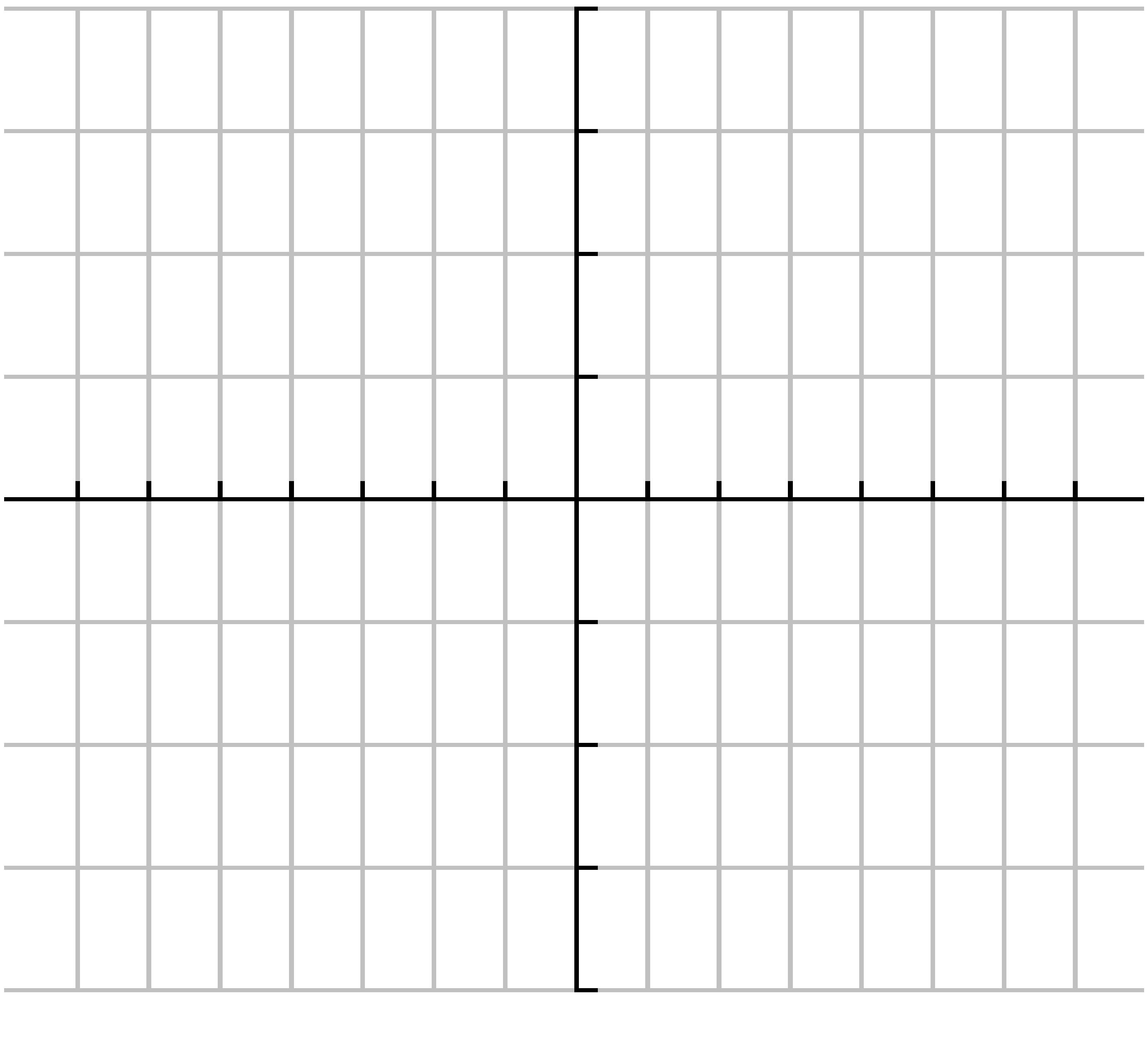
Example: Suppose $\text{Period} = 2\pi$ then mark 0, $\pi/2$, π, $3\pi/2$, and 2π

If there is no phase shift, plot the corresponding point at each value paying attention to the amplitude. Repeat if necessary for the interval.

3. If there is a phase shift, first lightly sketch the graph with out the phase shift then graph the function shifting the correct amount and direction from your original sketch.

4. Points to remember:
Sine graph crosses the x-axis at the ends and in the middle (3).
Cosine graph crosses the x-axis between the endpoints and the midpoint (2).
Tangent graph crosses the x-axis in the middle (1).
Cosecant and secant both do not cross the x-axis.

Blank graph



Pre-calculus—Chapter 6-4

Inverse Trig Functions

Notation for an inverse trigonometric function refers to an ANGLE.

Function: $y = \sin\theta$

Inverse:

$\theta = \sin^{-1} y$ or $\theta = \arcsin y$.

The inverse of a function may be found by interchanging the coordinates of the ordered pairs of the function.

Thus: function coordinates: (θ, y); (θ, sin θ)

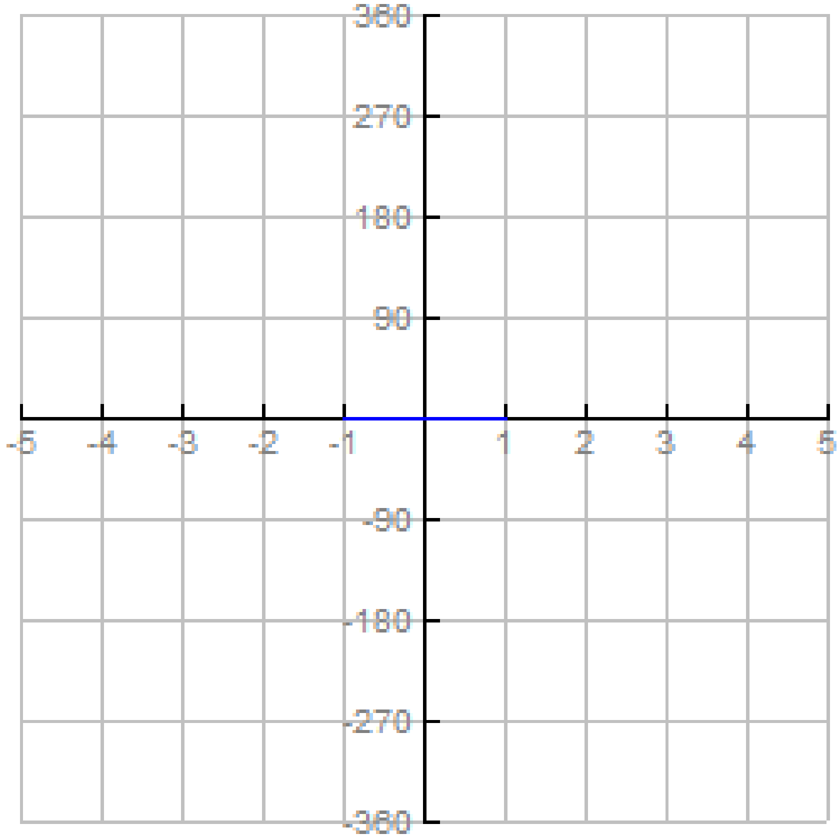
become (y, θ); (sin θ, θ)

And: $\theta = \sin^{-1} y$, cosine ?? tangent???

Note: the inverse of a trig function is not a function……..

Graph of $\theta = \sin^{-1} y$

Inverse is a Relation.



Why can't we graph this on our calculator??

Ex: find the $\cos x = \dfrac{\sqrt{3}}{2}$.

Looking for a value (in radians or degrees) in which the cosine of that value will be $\dfrac{\sqrt{3}}{2}$.

Now where did I put that Unit Circle??? 30°

Ex: Evaluate: $\tan\left(\sin^{-1}\frac{5}{13}\right)$ → θ

1st—evaluate what is in the parentheses

Method: $\theta = \sin^{-1}\frac{5}{13}$ then $\sin\theta = \frac{5}{13}$

But wait—then 5 = opp. leg and 13 = hyp

Then consider the outside…

Know that $\tan(\theta) = \frac{\sin\theta}{\cos\theta}$, figure out cosine and then you will have the answer…….

Ex: Evaluate: $\cos\left(\cos^{-1}\frac{4}{5}\right)$

$\theta = \cos^{-1}\frac{4}{5}$ then $\cos\theta = \frac{4}{5}$,

4 = adj leg, 5 = hyp.

thus $\cos\theta = ?$

Pre-Calculus—Chapter 6(5,6)

Principal Values—are values in the *Range* that will represent the negative and positive values, without repetition, of a specified trig function. By restricting the *Range* this way the inverse trig *Relation* will be a *Function*. Notation: Capitalize the first letter of the inverse trig function.

Inverse Graphs of Functions

The inverse graph of a function is the graph of the function reflected across the line $y = x$.

To find the equation of the inverse of a trig function, exchange x and y.

The inverse equation of a trig function is used to find the value of an angle if you know the value of the function.

Pre-Calculus—Chapter 6-7

Simple Harmonic Motion

Rhythmic motion of an object

- vibrations of a guitar string
- pistons of an engine
- pendulum

Remember formula for *linear* velocity??

$v = r\,\dfrac{\theta}{t}$ where $\dfrac{\theta}{t}$ is the ***angular velocity***

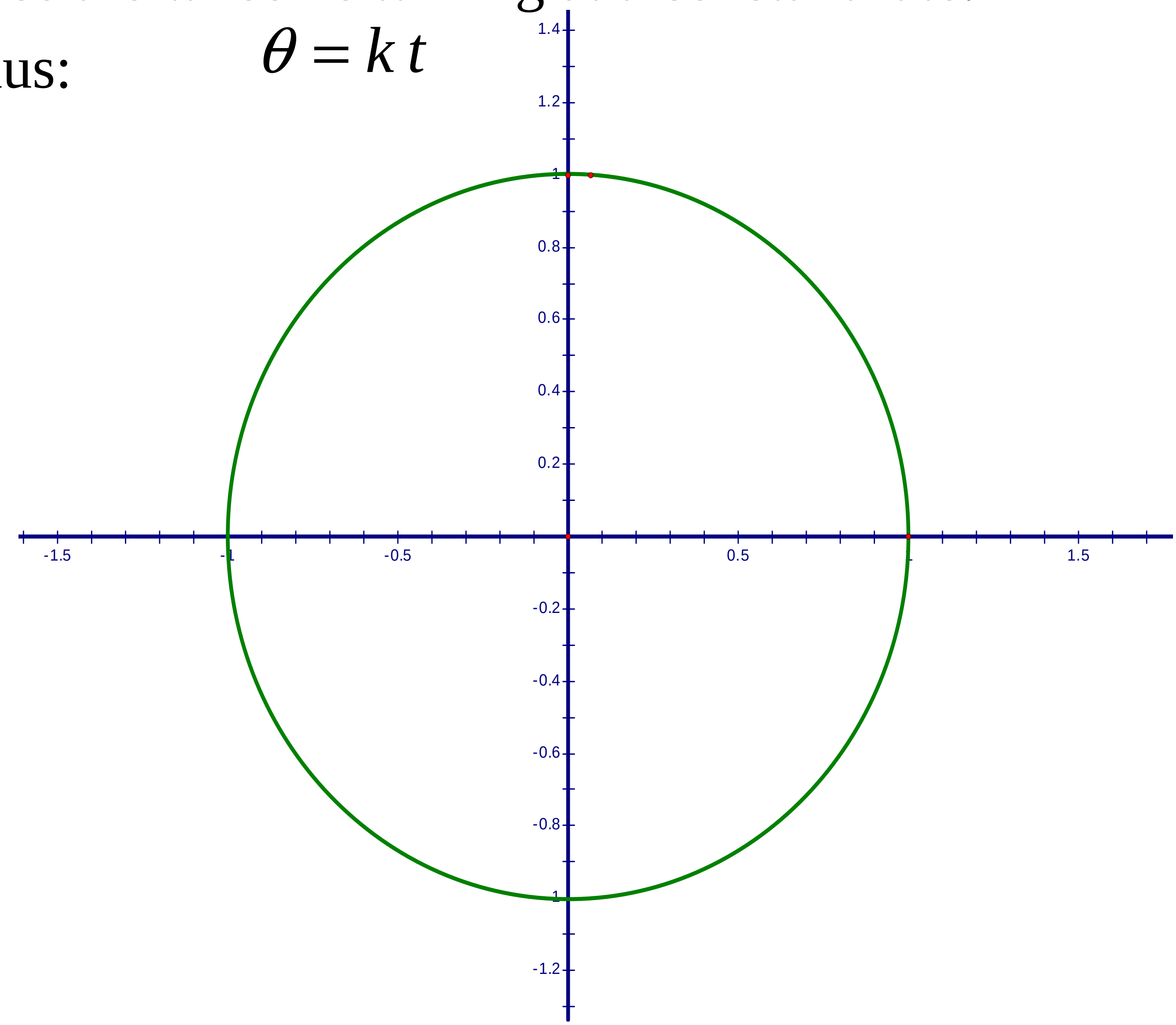
Angular Velocity→number of degrees/radians that a point moves in a unit of time.

Suppose a wheel is turning counterclockwise at a constant angular velocity then:

$$k = \theta / t$$

where k represents a constant→angular velocity since the wheel is turning at a constant rate.

Thus: $\theta = k\,t$



Sketchpad—Angular Velocity Sketch

Select a point *P* moving counterclockwise at a constant angular velocity with initial coordinates (*A*, 0) where *A* is the length of the *radius*.

Now:

$\cos\theta = x/A$ and $\sin\theta = y/A$

Substitute $\theta = k\,t$ in each equation----

$\cos kt = x/A$ $\sin kt = y/A$

But wait!!!

$x = A\cos kt$ is the horizontal motion

$y = A\sin kt$ is the vertical motion

with the initial position → (*A*, 0)

When P is in the **initial position**,
$kt = 0$ at time $t = 0$.

If the angle measures c radians at time $t = 0$, then the angle $\theta = (kt + c)$ will represent the angle as a function of time.

Stop—think about these two equations in terms of amplitude, period, and phase shift.

thus

$$x = A\cos(kt + c) \qquad\qquad y = A\sin(kt + c)$$

Note: the sine and cosine curves having the same amplitude and period can be made to coincide by a phase shift.

Ex: A buoy in a lake moves 6 ft. from a high point to a low point, then returns to its high point every 10 seconds.

Write two possible equations to describe the motion of the buoy.

1st—find the amplitude, period, and phase shift of the function.

amplitude = A = $6/2 = 3\,ft.$ Why half?

period = 10 sec thus $10\sec = 2\pi/k$

$$k = 2\pi/10 = \pi/5$$

Since the angle at time = 0 is 0 (in radians) there is no phase shift.

Now is the function sine or cosine????

Imp: Always use x if the graph is cosine and y if the graph is sine. Remember we solve for these two variables.

Finally:

$$x = 3\cos\left(\frac{\pi}{5}t\right)$$

and knowing that ¼ of a cycle will move the cosine graph over to look like the sine graph if both the amp. and period are the same.
Thus 10/4 will give you the **time** before the cycle begins or 2.5 seconds. Since it is before PS must be negative…….and c, positive.

$$y = 3\sin\left(\frac{\pi}{5}t + \frac{\pi}{2}\right)$$

Frequency is the number of cycles per unit time.

$f = \dfrac{1}{P}$, where P is the period. (How many cycles in 1 seconds)