Pre-Calculus

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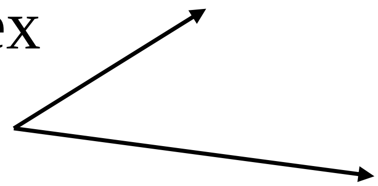
- **Pre-Calculus—Chapter 5-1**

Angle→generated by 2 rays whose endpts are the same vertex

- Vertex→

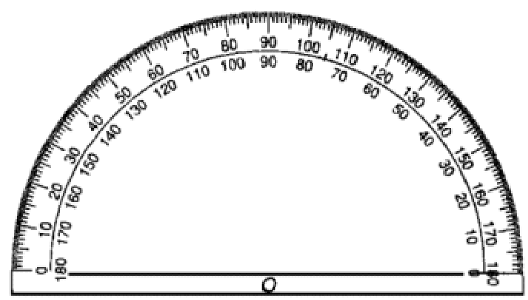


Initial side→fixed ray

Terminal side→ray that can be moved

- clockwise------neg. (-)
- counterclockwise-----pos (+)

Protractor-------

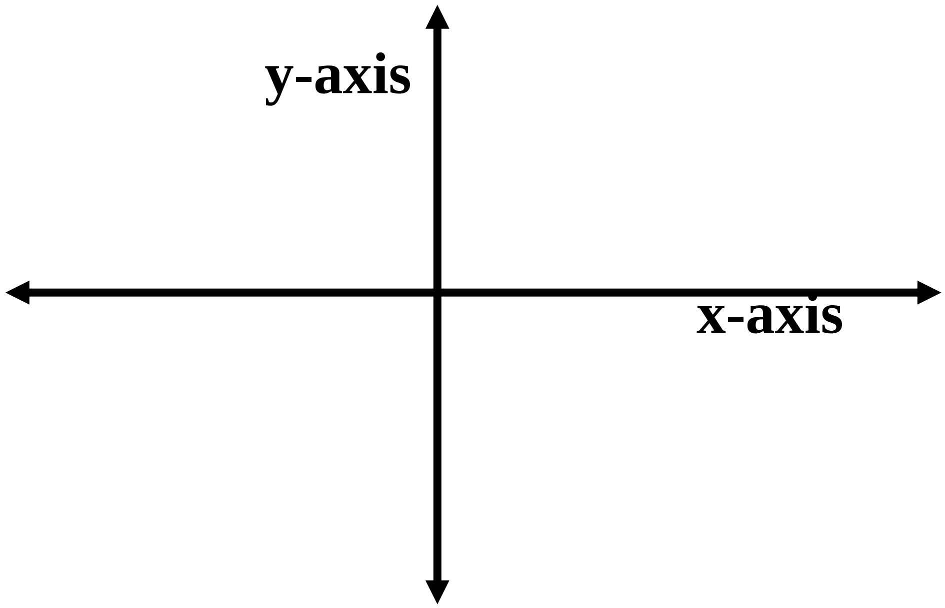


Standard Position (2 requirements)

- Vertex at the origin
- Initial side along positive x-axis

Quadrantal angle (2 requirements)

- Standard position
- Terminal side lies along one of the axis



Angles are measured in Degrees and Radians.

- Degrees are expressed in degrees, minutes and seconds
- Radians make it easier to solve problems that involve angles (James Thomson—1873—devised the radian method)
- Radians are based on the concept of the unit circle ??????
    - Notation:

Definition of a Radian→the measure of a central angle whose sides intercept an arc that is the same length as the radius of the circle

Unit Circle→see handout

Note: the radius of a unit circle is **one (1)** unit

(one unit of what????? Does it matter????? Discussion at the end of 5.1 notes)

One complete revolution measured in

- Angles----- 360 degrees
- Radians---- 2 π (this is just the measure of the circumference of a circle with radius of 1) C=2pi r

180 degrees= π etc

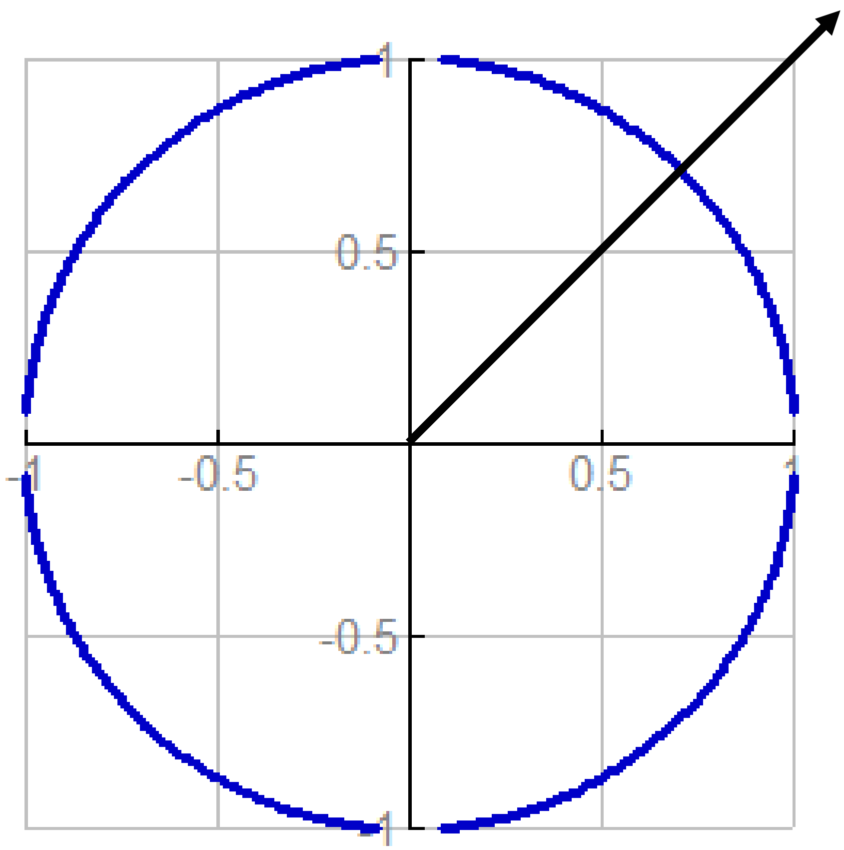
Convert:

∠ (rad)=(deg) π/180

∠ (deg)=(rad) 180/ π

Note: π should cancel out

Coterminal angles→2 angles in standard position are conterminal if they have the <u>same terminal</u> side……there are infinitely many coterminal angles….(pos / neg)

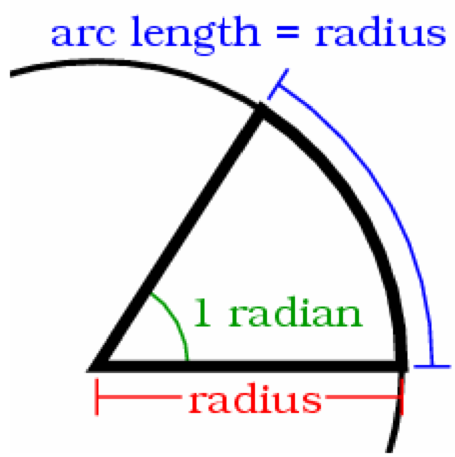


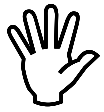
$\alpha^\circ \pm k\,(360^\circ)$ or $\alpha^R \pm k\,(2\pi)$,
where k is an integer

Reference Angle→Non-quadrantal angle in standard position

- Always an <u>acute</u> angel formed by the terminal side and the x-axis (no matter what the value/measure of the angle)

Note: How many lengths of the radius will form the circumference of a circle? Any circle?





Unit Circle Handout

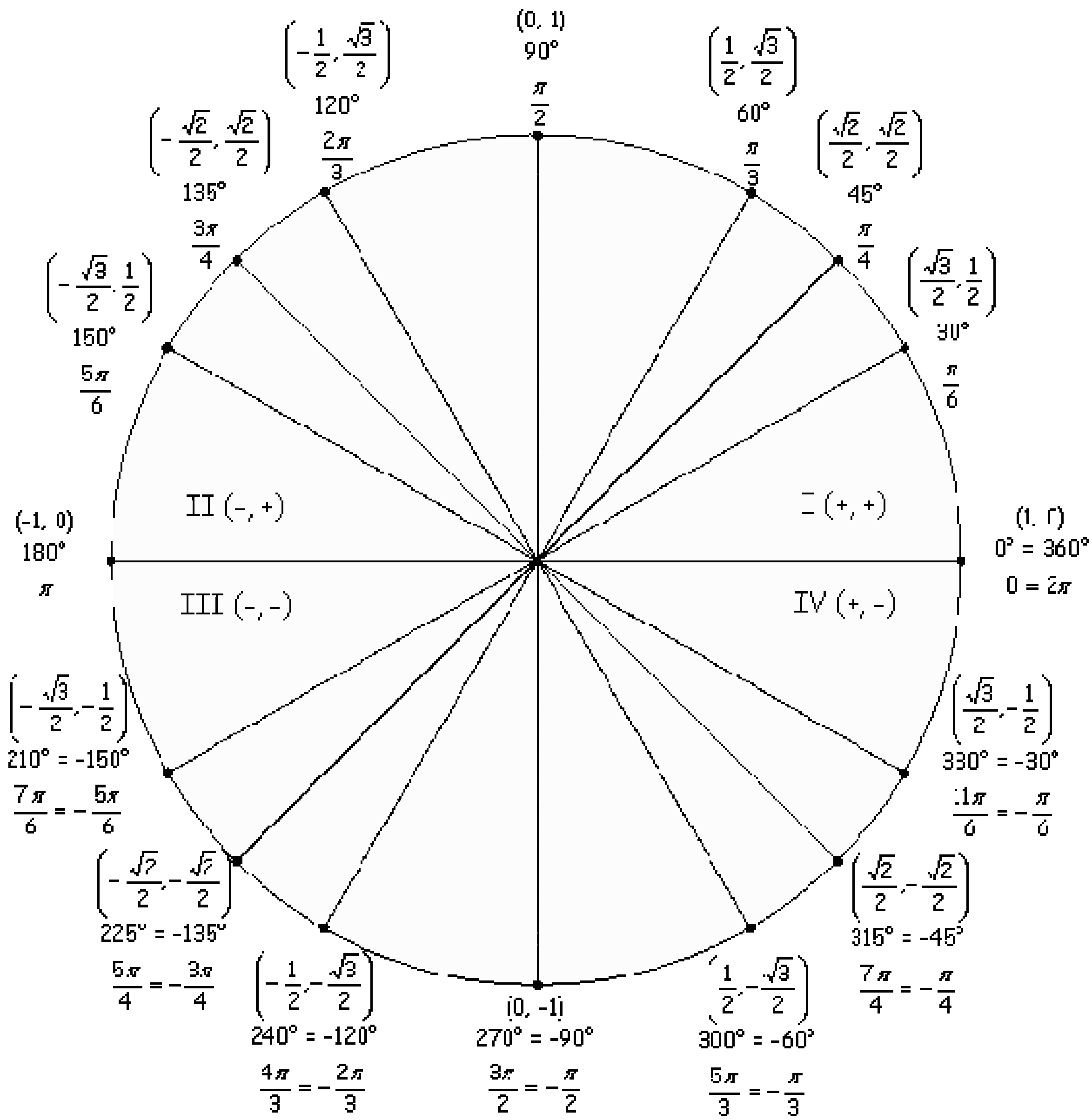
Radians and Degrees

Converting Degrees to Radians:? $\theta \cdot \frac{\pi}{180}$

Converting Radians to Degrees:? $\theta \cdot \frac{180}{\pi}$

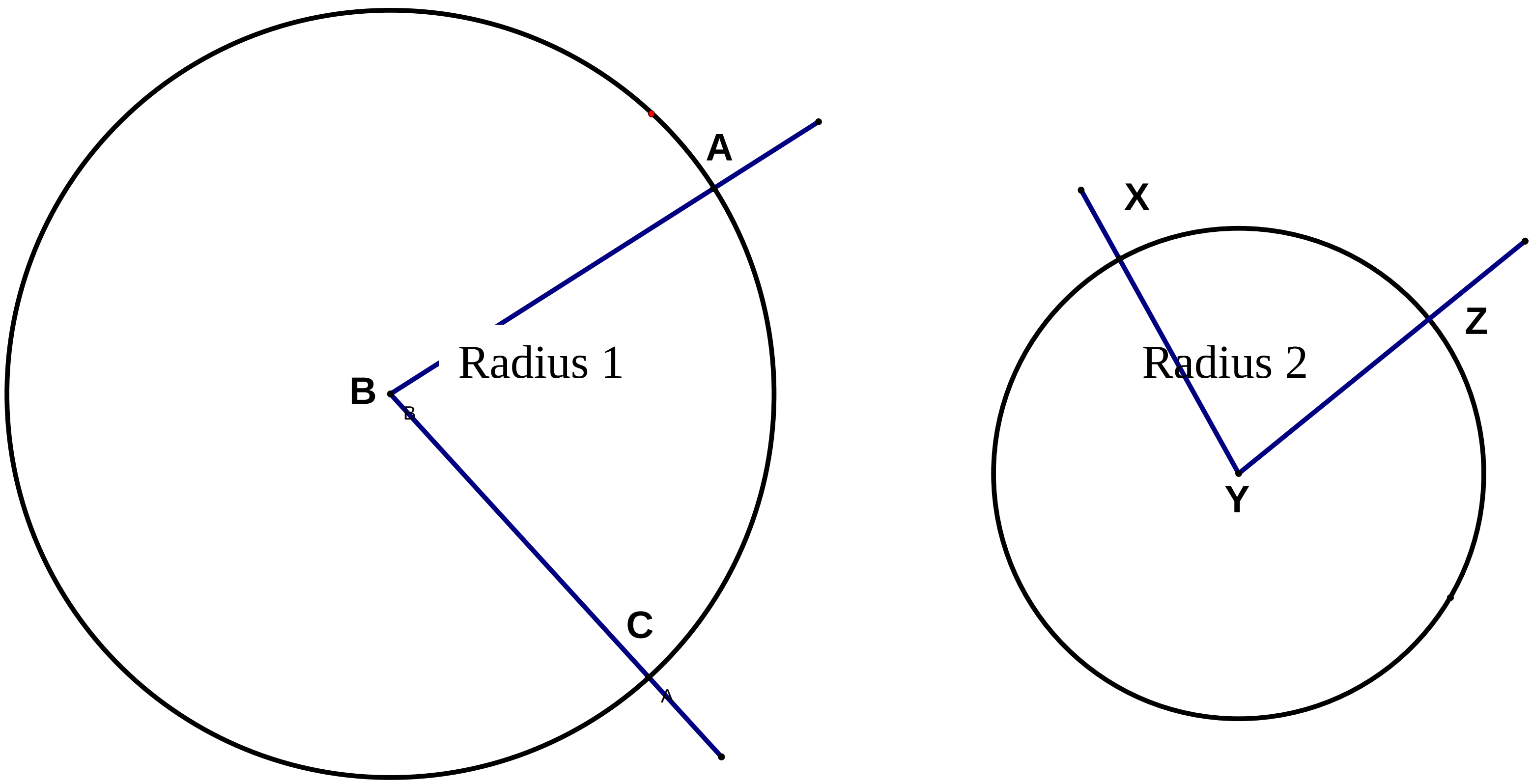
Unit Circle

(x, y) = (cos θ, sin θ)



Pre-Calculus—Chapter 5-2

Remember: Central Angles



Thm:

If 2 central angles in different circles are congruent then the length of their intercepted arcs is equal to the ratio of the measures of their radii.

$$\frac{L(\overset{\frown}{AC})}{L(\overset{\frown}{XZ})} = \frac{BC}{YZ} = \frac{R1}{R2}$$

But we know that

$$L(arc) = \frac{m(cent)\sphericalangle}{360°}\, 2\pi r$$

Suppose we substitute angles measured in Radians instead of degrees????

Then: $L(arc) = \theta\ r$

But the *L(arc)* is represented by "*s*" Thus

$s = \theta\ r$ → In an Unit Circle $r = 1$

thus $\theta^R = s$ → Definition of a radian!!!

**$L(arc)$ = ? $m(cent\sphericalangle) = 42^\circ$ $r = 8cm$

Velocity of an object in a circular path

- Linear (tangential) velocity→velocity of an object as it moves along a circular path
  - Know: $v = \frac{s}{t}$; $\theta = \frac{s}{r}$

$$\frac{\theta}{t} = \frac{s}{t\,r} \; ; \quad \frac{\theta}{t} = \frac{v}{r}$$

Thus: $\frac{\theta}{t} r = v$

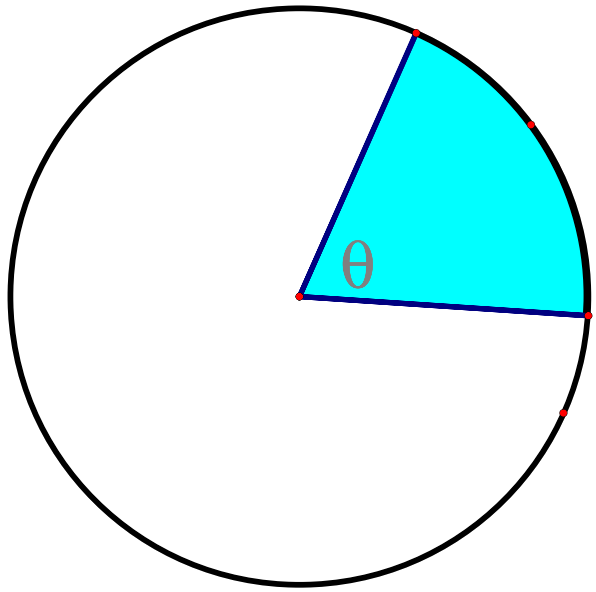
- Angular velocity→velocity that the central angle changes over time as the object travels in a circular path:

$$\text{angular velocity } = \frac{\theta}{t} \; rads / unit\, time$$

**Pulley of r = 12 cm. turns at 7 rev/sec

Linear Velocity = ? IMP: $\theta = ?$

Sector of a Circle:



Remember:

$$A(\text{sector}) = \frac{m(cent\sphericalangle)}{360^\circ}\,\pi r^2$$

$$A(\text{sector}) = \frac{\theta^R}{2\pi^R}\,\pi r^2$$

$$A(\text{sector}) = \frac{\theta^R}{2}\,r^2 = \frac{1}{2}\theta^R r^2$$

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Pre-Calculus—Chapter 5-3

Circular Functions

More Unit Circle---------
Two trig functions, sine and cosine, can be defined in terms of Unit Circle.

Review:

$$\sin\theta = \frac{opp.\ leg}{hyp}$$

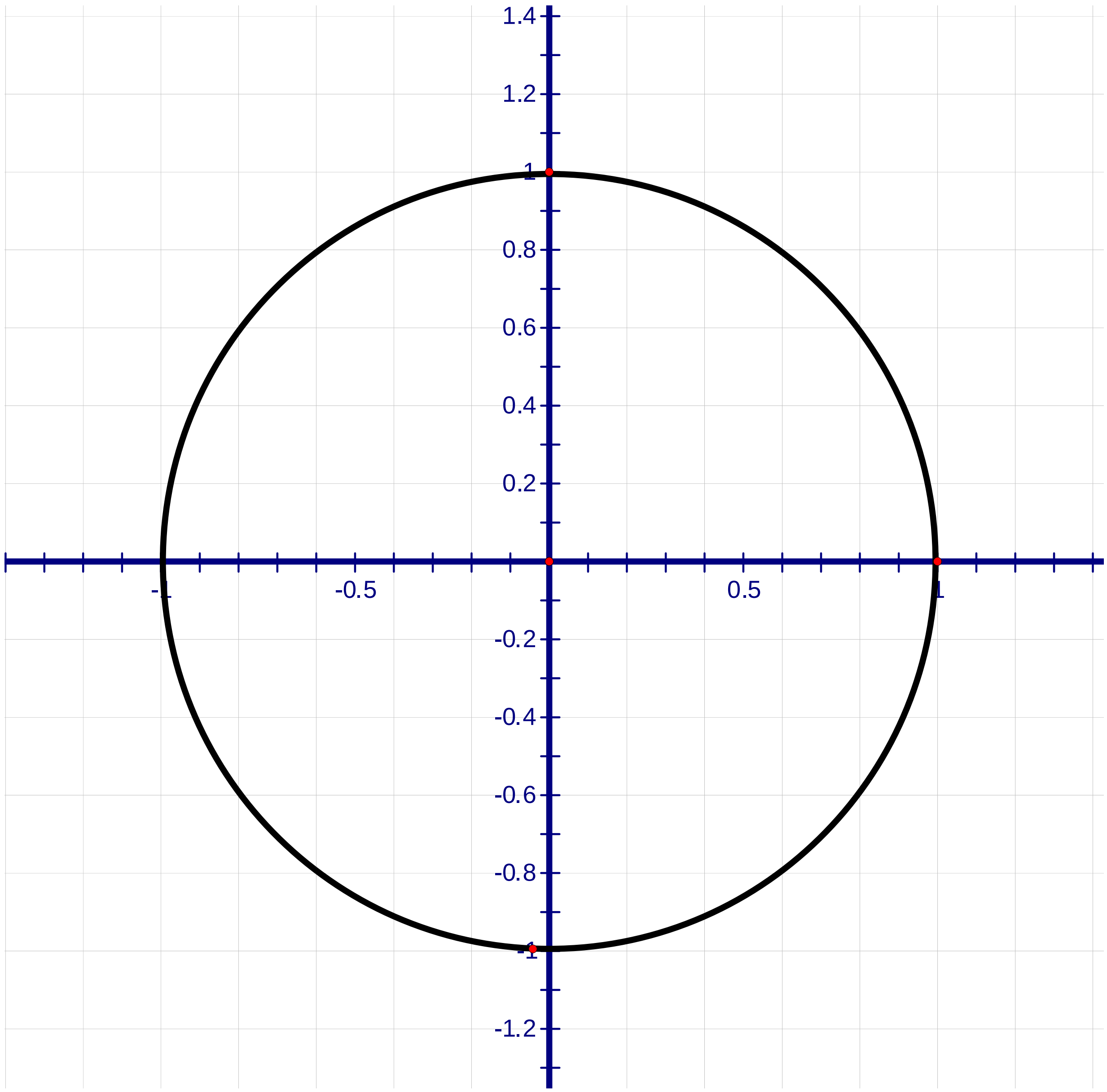
$$\cos\theta = \frac{adj.\ leg}{hyp}$$

But in the Unit Circle the hypotenuse = 1 unit.

Thus:

$$\sin\theta = \frac{opp.\ leg}{1}$$

$$\cos\theta = \frac{adj.\ leg}{1}$$



Now:

$$\sin\theta = \frac{opp.\ leg}{1} = y$$

$$\cos\theta = \frac{adj.\ leg}{1} = x$$

only in a Unit Circle

Consider, using the Unit Circle,

$\sin 90°$; $\sin \frac{\pi}{2}$, both equal 1.

Why??? What about $\cos 90°$; $\cos \frac{\pi}{2}$?

Coordinates of the point when $\theta = \frac{\pi}{2}$ are (0, 1)

Can continue around the Unit Circle duplicating the above process and the result is the Unit Circle with its coordinates.

STOP—suppose we have a circle that is <u>not</u> a Unit Circle. Then:

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\csc\theta = \frac{r}{y}$ $\sec\theta = \frac{r}{x}$ $\cot\theta = \frac{x}{y}$

Now if terminal side of angle in standard position contains points (8, -15), find all the other trig functions………

Note: first determine which quadrant the point lies (why??) ------also remember

$$r = hyp = \sqrt{x^2 + y^2}$$

*Find $\sin\theta$ when $\cos\theta = \frac{5}{13}$.

*****Conclusion—If you know one trig value and the quadrant, you know all.. ☺

Pre-calculus—Chapter 5-4

Special Angles

The Special Angles are derived from the Special Triangles introduced in Geometry.

45-45-90

30-60-90

Determining the other trig functions----

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Pre-Calculus—Chapter 5-5

Right Triangles

Remember from Geometry?????

$$\sin \sphericalangle = \frac{opp\ leg}{hyp}$$

$$\cos \sphericalangle = \frac{adj\ leg}{hyp}$$

$$\tan \sphericalangle = \frac{opp\ leg}{adj\ leg}$$

Reciprocal functions------

Angle of elevation-------

Angle of depression------

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Pre-Calculus Chapter 5-6 & 5-7

Law of Sines and Cosines

Know how to solve right triangles……but suppose the triangle is NOT right???? How would you determine the unknown angles and sides?

Law of Sines

$$\frac{a}{\sin \measuredangle A} = \frac{b}{\sin \measuredangle B} = \frac{c}{\sin \measuredangle C}$$

ex: Solve:

$m\measuredangle A = 29°10';\quad m\measuredangle B = 62°20';\quad c = 11.5$

1st step—draw

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A \qquad \text{or} \qquad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac\cos B \qquad \text{or} \qquad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

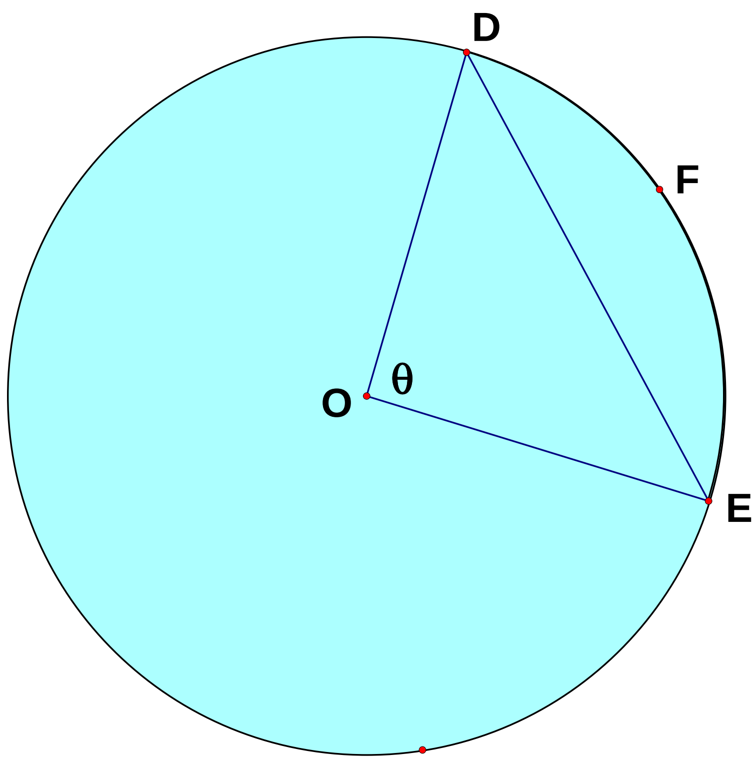
$$c^2 = a^2 + b^2 - 2ab\cos C \qquad \text{or} \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

STOP!!!!! Check out the Handout.

Pre-Calculus Chapter 5-8

Area of Triangles→ See Handout------

One more area formula-----



$$A(\text{segment}) = A(\text{sector}) - A(\text{triangle})$$

Thus:

$$A(segment) = \frac{1}{2} r^2 \theta - \frac{1}{2}\, d\, e\ \sin\theta$$

But note: d = radius = r and
e = radius = r

Thus:

$$A(segment) = \frac{1}{2} r^2 \theta - \frac{1}{2}\, r\, r \ \sin\theta$$

Finally:

$$A(segment) = \frac{1}{2} r^2 \left(\theta - \ \sin\theta\right)$$

END of CHAPTER 5