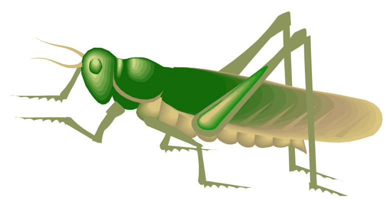
Pre-Calculus—Chapter 10

Pre-Calculus—Chapter 10-1

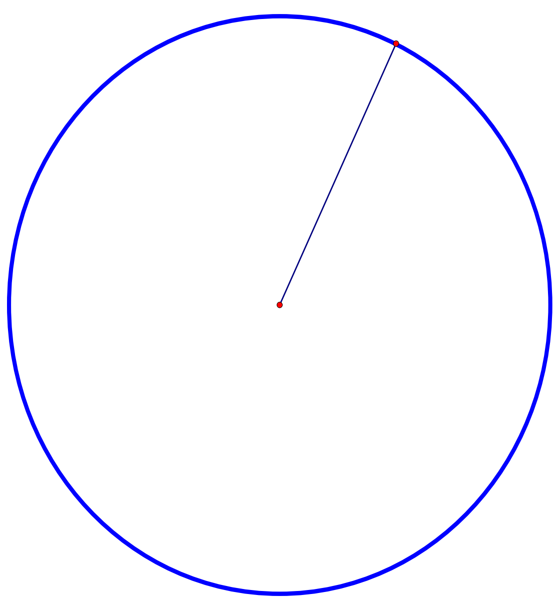
The Circle

Def: A **Circle** is the **locus** of all points in a plane at a given distance from a fixed point on the plane called the **center**.

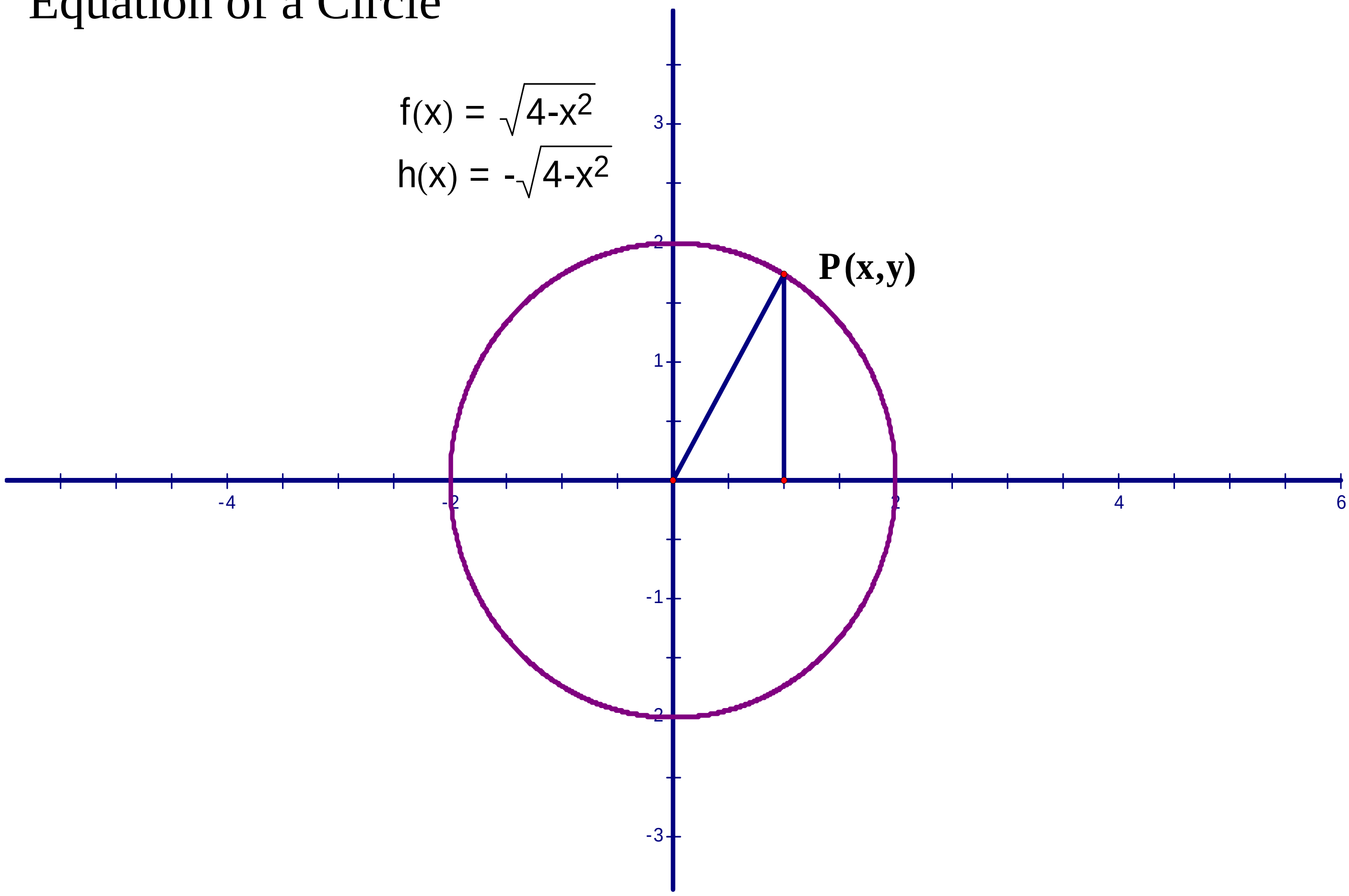
locus????? Is that like a locust????



A locus is just a set of points that satisfy a given condition. In a circle the condition is that all points are a given distance (radius) from the fixed point (center).



Equation of a Circle



Pythagorean theorem:

$r^2 = x^2 + y^2$ → equation of the parent graph of ALL ⊙s.

Now suppose that the center is NOT at the origin…..
but at (*h*, *k*).

You would use the **Standard Form** of the equation of a circle:

$$r^2 = (x-h)^2 + (y-k)^2$$

This is just moving the center of the circle and of course the radius follows and defines the circle. Remember translations?

Gosh! How would we solve for ***y***??? Substitute a value for the radius, take the square root of both sides and then solve for *y*. This is the equation that is used for graphing.

$$r^2 = (x-h)^2 + (y-k)^2$$

$$r^2 - (x-h)^2 = (y-k)^2$$

$$\pm\sqrt{r^2 - (x-h)^2} = \pm\sqrt{(y-k)^2}$$

$$\pm\sqrt{r^2 - (x-h)^2} + k = y$$

The standard form can be expanded to produce the **General Form** of the equation.

$$(x-h)^2+(y-k)^2=r^2$$

$$\left(x^2-2hx+h^2\right)+\left(y^2-2ky+k^2\right)=r^2$$

$$x^2-2hx+h^2+y^2-2ky+k^2=r^2$$

$$x^2+y^2+(-2h)x+(-2k)y+\left(h^2+k^2-r^2\right)=0$$

$$x^2+y^2+Dx+Ey+F=0$$

Where D, E, and F are constants. Note: the coefficients of x^2 and y^2 must be **1**. Manipulate the equation using division so that the coefficients are one.

An equation given in the general form can be rewritten so that it is in standard form by using completing the square.

$$x^2+y^2-10x+4y+17=0$$

$$\left(x^2-10x+?\right)+\left(y^2+4y+?\right)=-17$$

$$\left(x^2-10x+25\right)+\left(y^2+4y+16\right)=-17+25+16$$

$$(x-5)^2+(y+2)^2=12$$

Center of the ⊙ is located at ?????

Radius of the ⊙ is ??????

Note that there are 3 unknowns in the general equation. Given 3 or more ordered pairs (will only use 3), the equation can be solved for the standard equation.

Ex: Find the equation of the circle that passes thru (-2,3), (6,-5), and (0,7). Determine the center and radius of the circle.

Set up 3 equation using the general equation.

$$\left.\begin{aligned} (-2)^2 + 3^2 - 2D + 3E + F &= 0 \\ 6^2 + (-5)^2 + 6D - 5E + F &= 0 \\ 0^2 + 7^2 + 0D + 7E + F &= 0 \end{aligned}\right\} \text{Simplify the system}$$

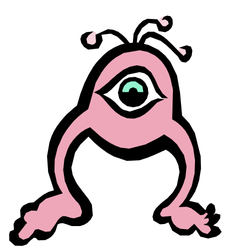
Solution→ D = -10, E = -4, F = -21

Equation of the circle is……

$$x^2 + y^2 - 10x - 4y - 21 = 0$$

$$(x-5)^2 + (y-2)^2 = 50$$

Center (?,?) Radius = ?



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