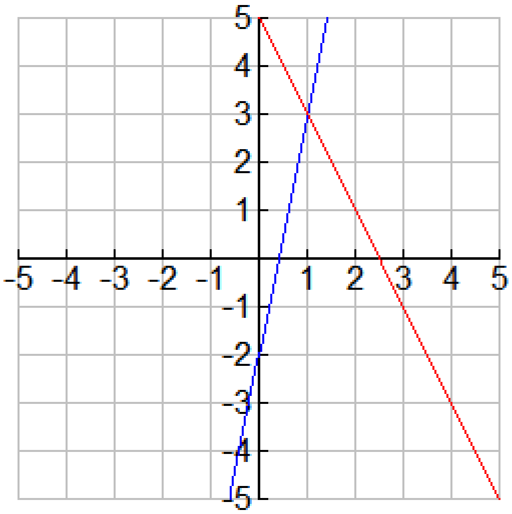
Pre-calculus—Chapter 2(1) & 3

Pre-Calculus—Chap 2.1

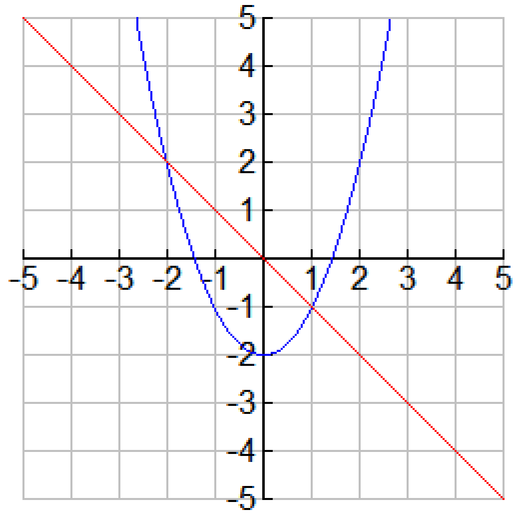
Solving Systems of Equations

Systems of equations can be solved graphically and analytically.

- **Graphically—**
  Anytime you have 2 equations you can determine the solution(s) if any by graphing the equations and the point(s) of intersection is/are the solution(s).



$$y = 5x - 2$$
$$y = -2x + 5$$



$$y = x^2 - 2$$
$$y = -x$$

Solutions--?????   Nice if the graphs intersect exactly where the grids cross but suppose the graphs do not intersect where it is easy to determine the solution or solutions.

Suppose the graphs do not cross at all—
Could this ever occur????

- Determining the solution or solutions **analytically**—two methods but sometimes it is easier to use one method than the other:
    - Elimination (using addition/subtraction) or
    - Substitution

Method of Elimination (generally only used for linear equations):
To use this method, a variable is eliminated leaving only one.
Ex:

$$3x-4y=360$$
$$5x+2y=340$$

Choose which variable would be the easiest to eliminate.

Eliminate "*y*"

$$3x-4y=360 \qquad\qquad 3x-4y=360$$
$$2(5x+2y)=2(340) \qquad \underline{10x+4y=680}$$
$$13x \quad =1040$$
$$x=80$$

Now substitute 80 for *x* in either of the original equations and solve for *y*.

Note: You could solve for ***y*** first then solve for ***x***.

Solution (only one) ordered pair → (80, -30)

Method of Substitution: Method in which you solve for ***x*** or ***y*** in one equation and then substitute that answer for that variable in the other equation.

Ex:

$$y = x^2 - 2 \qquad \text{Substitute---} \quad y = -x$$

$$y = -x \qquad -x = x^2 - 2$$

$$x^2 - x + 2 = 0$$

$$(x+1)(x-2) = 0$$

Now—what are the solutions??? Do they match the solutions that you got graphically?

The two methods discussed above are graphically and analytically. Another method is Numerically……..looking at a table of coordinates and find the *y*-value(s) that are the same then the *x*-value will be the *x*-value for the solution(s). Easy to do on the calculator.

Something to Remember: Note chart on page 57

- A Consistent system of equations—at least one solution.
- An Independent system—there is *exactly* one solution.
- A Dependent system—there are infinitely many solutions.

Pre-Calculus—Chapter 3-1

Symmetry

Many graphs have **symmetry** to them. A graph may have point symmetry, line symmetry, neither, or both.

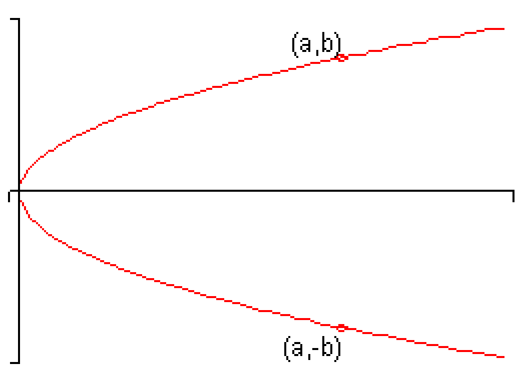
Point Symmetry→2 distinct points P and P' are symmetric with respect to a point, M, iff M is the midpoint of $\overline{PP'}$.

Example: Determine if the graph of $f(x) = x - 3$ has point symmetry about the *x*-intercept 3.

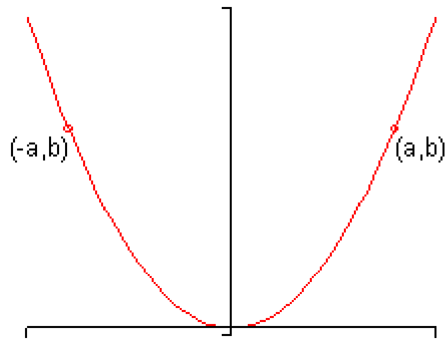
Symmetry can be useful in graphing an equation since it says that if we know one portion of the graph then we will also know the remaining portion of the graph as well. We use this fact when we are graphing parabolas.

In this section we want to look at 4 types of symmetry.

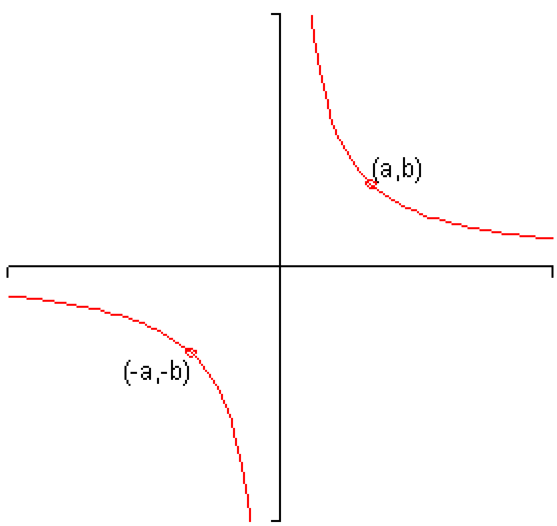
1. A graph is said to be **symmetric about the *x*-axis** if whenever (a, b) is on the graph then so is $(a, -b)$. Following is a graph that is symmetric about the *x*-axis.



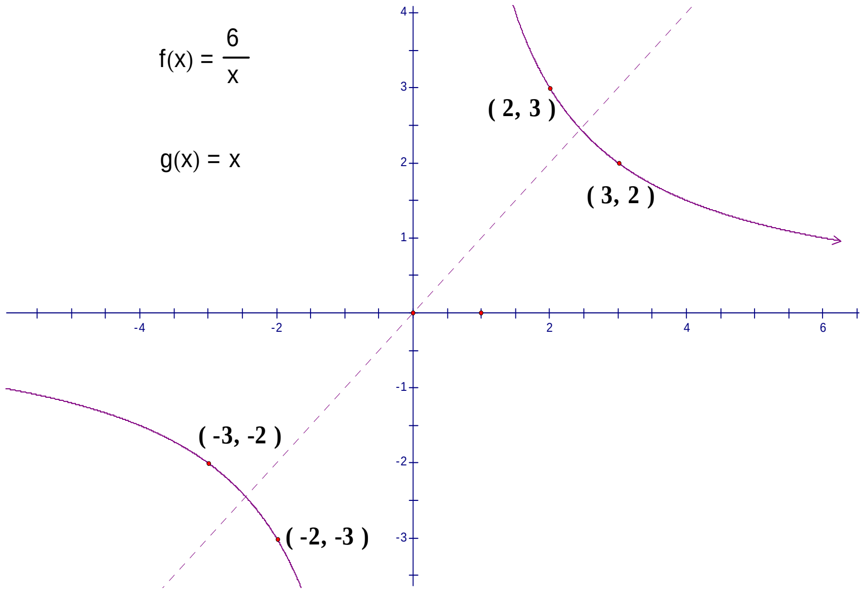
2. A graph is said to be **symmetric about the *y*-axis** if whenever (*a*, *b*) is on the graph then so is (-*a*, *b*). Following is a graph that is symmetric about the *y*-axis.



3. A graph is said to be **symmetric about the origin** if whenever (*a*, *b*) is on the graph then so is (-*a*, -*b*). Following is a graph that is symmetric about the origin.



4. A graph is said to be **symmetric about line** $y = x$ if whenever (a, b) is on the graph then so is (b, a). Following is a graph that is symmetric about $y = x$.



Wait!!!! What about the graph that is symmetric about the line $y = -x$?

Imp: If a function is symmetric about the line $y = x$, then the equations are inverses of each other.

Note that most graphs don't have any kind of symmetry. Also, it is possible for a graph to have more than one kind of symmetry.

Tests for Symmetry
We've some fairly simply tests for each of the different types of symmetry.

1. A graph will have symmetry about the ***x*-axis** if we get an equivalent equation when all the *y*'s are replaced with $-y$. $(x, y) \rightarrow (x, -y)$

2. A graph will have symmetry about the ***y*-axis** if we get an equivalent equation when all the *x*'s are replaced with $-x$. $(x, y) \rightarrow (-x, y)$

3. A graph will have symmetry about the **origin** if we get an equivalent equation when all the *y*'s are replaced with $-y$ and all the *x*'s are replaced with $-x$. $(x, y) \rightarrow (-x, -y)$

4. A graph will have symmetry about the **line $y = x$** if we get an equivalent equation when all the *x*'s and *y*'s are switched.
   $(x, y) \rightarrow (y, x)$

An *equivalent* equation means that it is exactly the same equation.

There is one quick fact about symmetry.

Fact
If an equation/graph has symmetry about the *x*-axis and the *y*-axis then it will also have symmetry about the origin.

Odd and Even Functions

Even→graphs are symmetric with respect to the y-axis
—the polynomial has exponents that are all EVEN.

Odd→graphs are symmetric with respect to the origin
—the polynomial has exponents that are all ODD

Pre-Calculus—Chapter 3-2

Families of Graphs

Parent Graph→an anchor graph from which other graphs in the family are derived.

Page 117 in your book—illustrate the most important parent graphs.

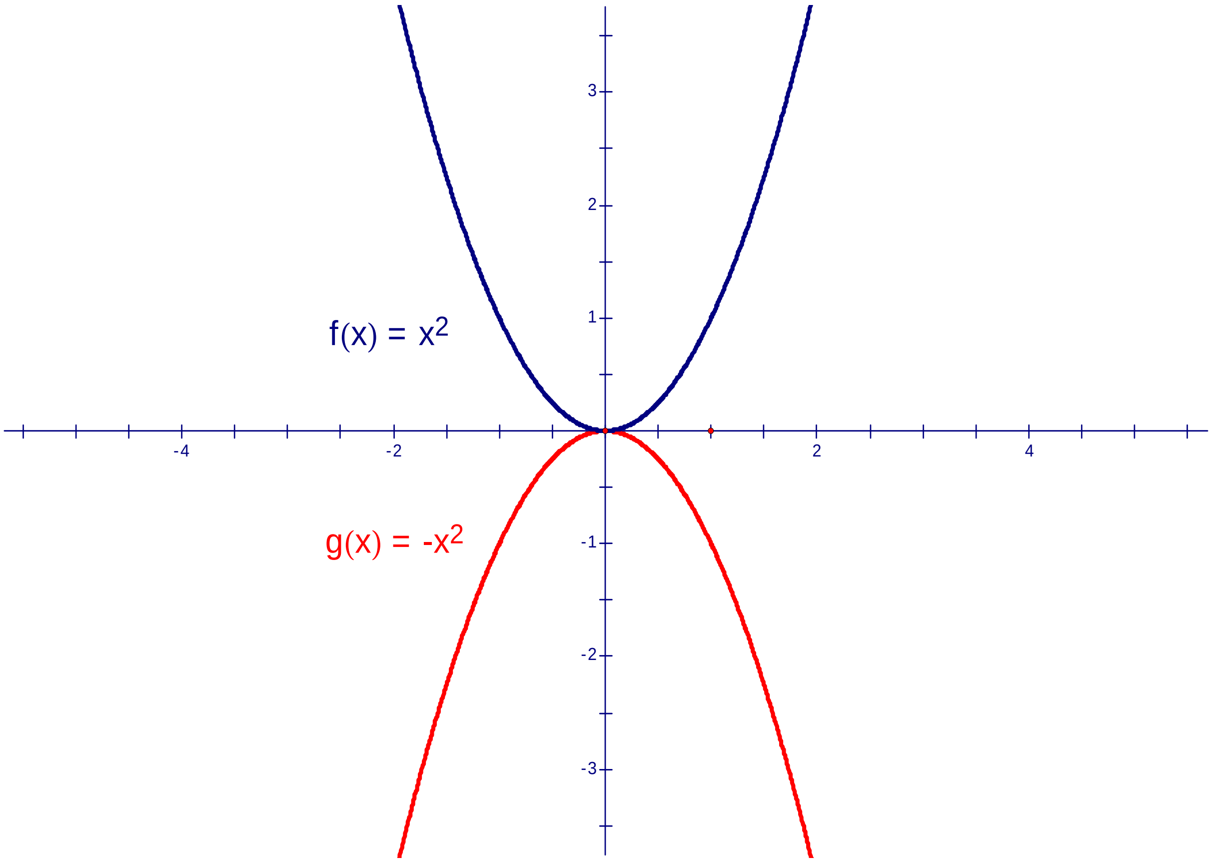
Know those Parent Graphs

Changes to the parent's graph affect the appearance of the child's graph……..BUT the child's graph will still resemble the parent's.

Changes

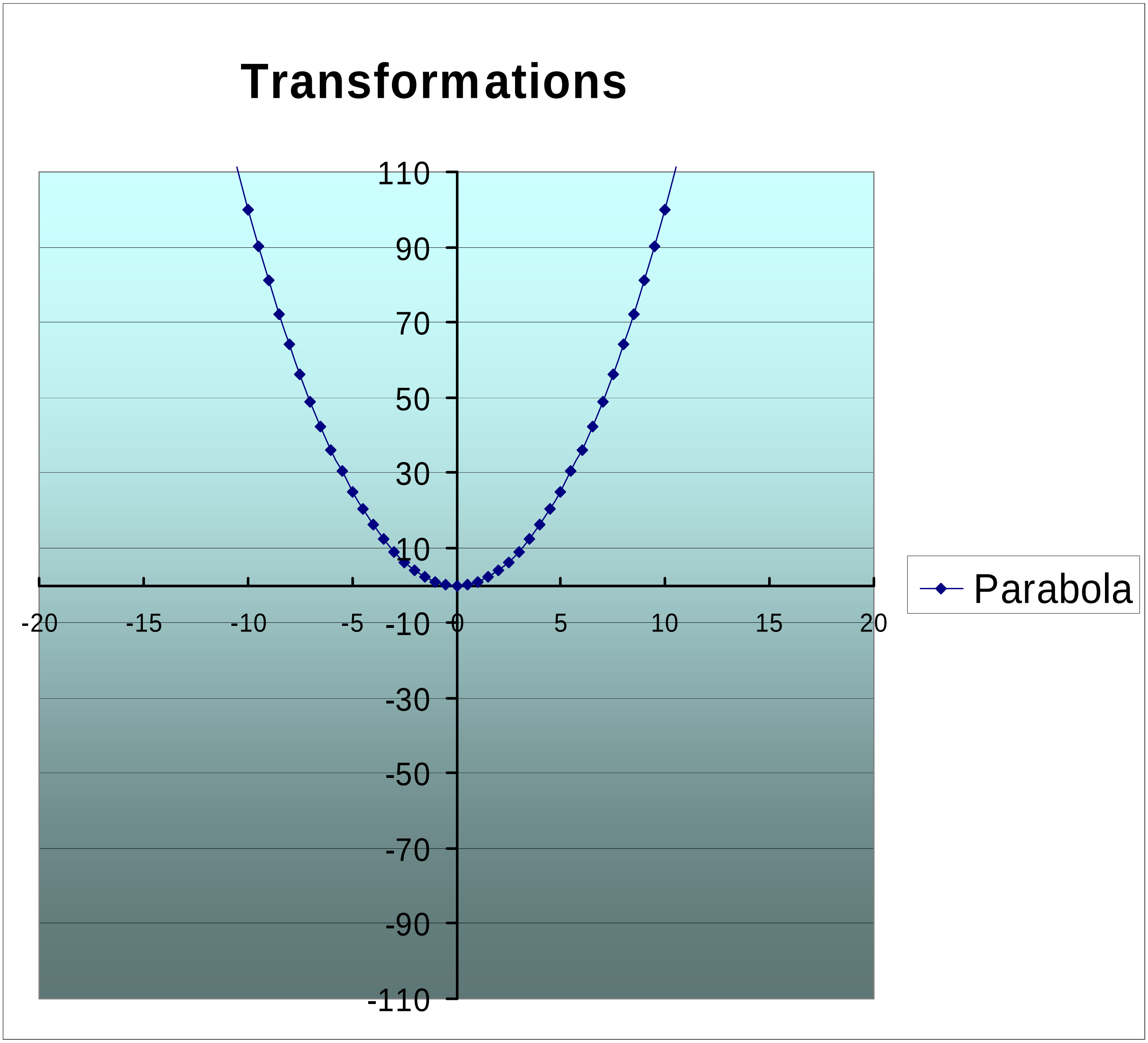
- Reflections
- Translations
- Geometric transformation (sometimes called dilations)

Reflections→A reflection flips a figure over a line called the axis of symmetry. A reflections is a ***Linear Transformation*** which is a relocation of the graph on the coordinate plane, not changing its shape or size.



Another type of linear transformation is a ***Translation*** which slides the graph vertically and/or horizontally, not changing the shape.

A parent graph can be modified by a ***Geometric Transformation*** which shrinks or stretches the graph vertically or horizontally.



On page 121, you will find a chart of some of the transformations that could occur on parent functions. You need to know what changes in an equation will result in what changes for the graph.

Pre-Calculus—Chapter 3-3

Graphing Inverse Functions and Relations

Relations are inverse relations whenever (*a*, *b*) is an element of one relation and (*b*, *a*) is an element of the other.

Notation: $f^{-1}(x)$

Important to remember that an equation and its inverse are symmetric about the line $y = x$ and that $(x, y) \rightarrow (y, x)$.

Remember the Vertical Line Test????

Now consider the

Horizontal Line Test

If a relation or function passes the Horizontal line test then its inverse will be a function.

Pre-Calculus—Chapter 3-4

Rational Functions and Asymptotes

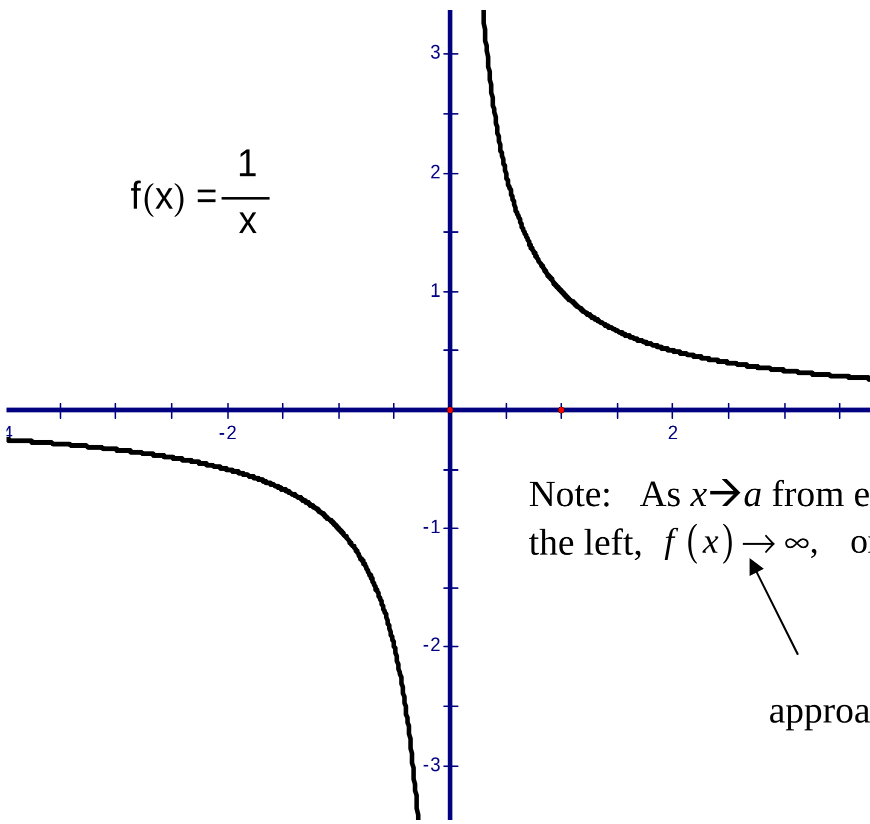
A ***Rational Function*** is the quotient of two polynomials.

$$f(x) = \frac{g(x)}{h(x)}, \quad h(x) \neq 0$$

If a rational function is not defined when $x = a$, then the line with the equation $x = a$ is a ***vertical asymptote***.

Rational function $f(x) = \dfrac{1}{x}$ has a vertical asymptote at ???

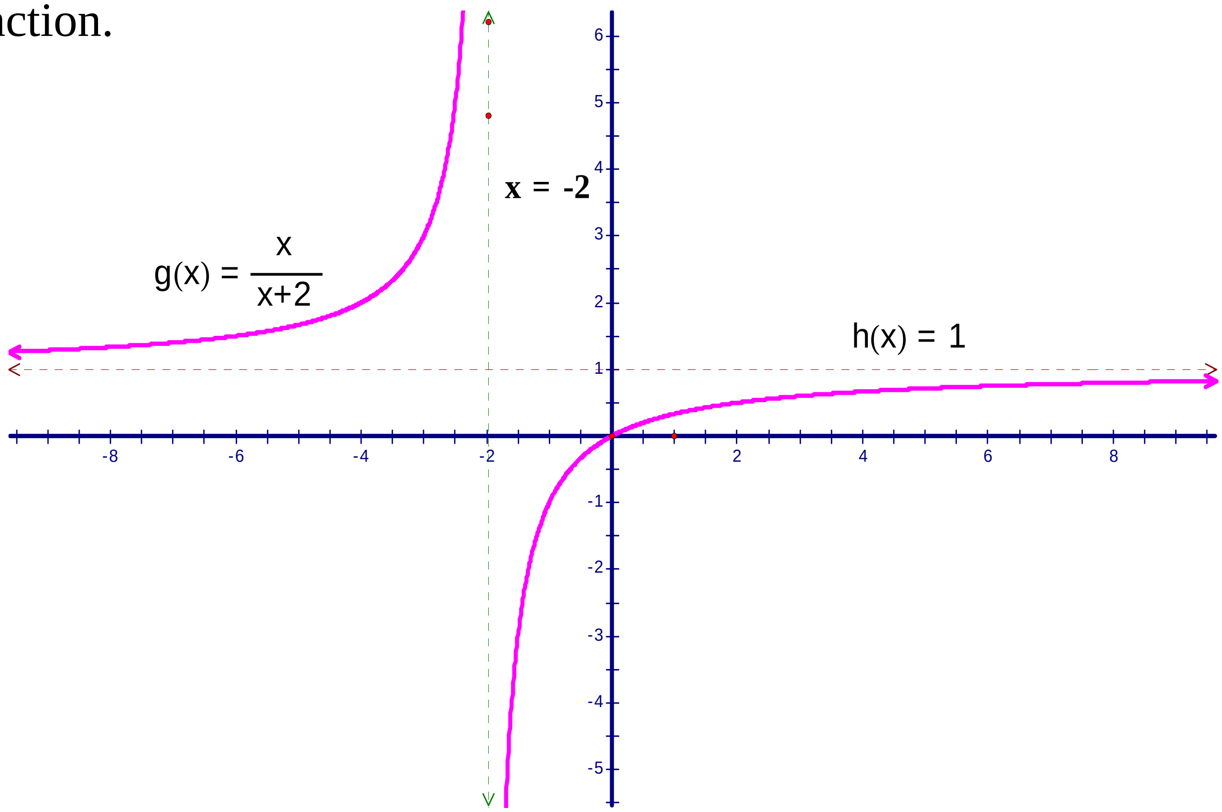
Graph:



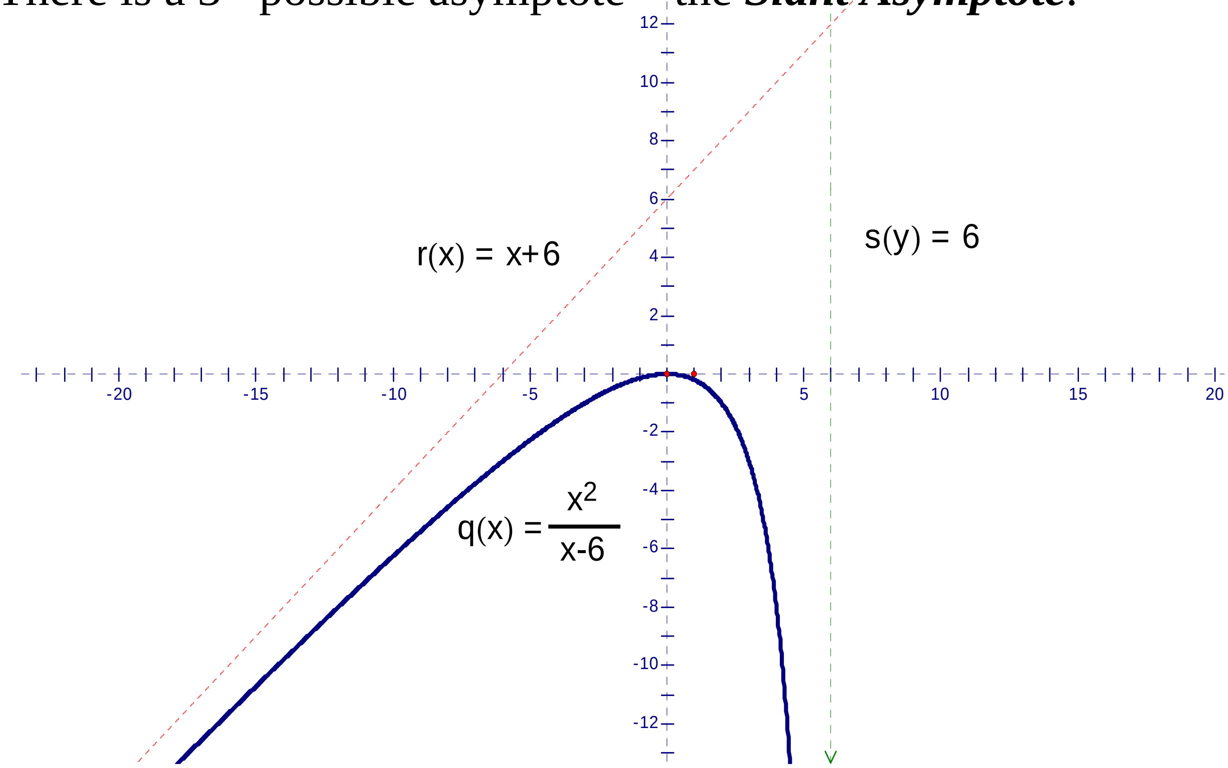
Note: As $x \rightarrow a$ from either the right or the left, $f(x) \rightarrow \infty$, or $f(x) \rightarrow -\infty$

approaches

If the graph of a rational function approaches $y = a$, as $x \to \pm\infty$, then the line $y = a$ is called the ***Horizontal Asymptote*** of the function.



There is a 3rd possible asymptote—the ***Slant Asymptote***.



Pre-Calculus—Chapter 1-6

Parallel and Perpendicular Lines

- 2 non-vertical lines are parallel iff their slopes are equal.
- any 2 vertical lines are parallel
- 2 non-vertical lines are perpendicular iff their slopes are negative reciprocals   (or the product of their slopes = -1)
- any vertical line and any horizontal line are perpendicular