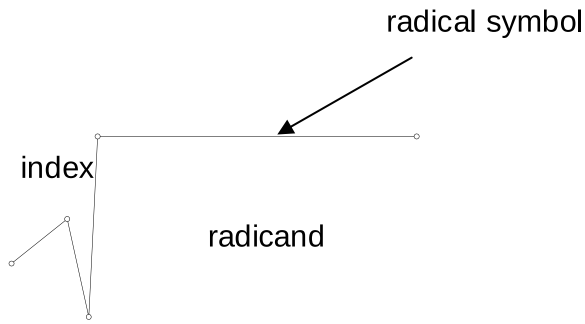
Geometry—Chapter 8

Radicals

The word radical is derived from the Latin word meaning root.



Answers will be:

- integers → 3, -5, 200,
- integers & radicals → $\sqrt[3]{65}$, or
- approximated values → 3.14157.

Know the difference between *exact* and *approximate* values.

Remember:

- Multiply / divide numbers with numbers—radicals with radicals (Imp: be sure the indices are the same)

- Add/subtract like terms—same index radicals with like radicands.

Simplifying Square Root radicals

A **square root** radical is in *simplest form* when:

- No perfect square factor other than 1 is under the radical sign.

- No fraction is under the radical sign.

- No fraction has a radical in the denominator. (rationalize the denominator)

Multiplying & Dividing Radicals

$\sqrt[n]{a} \bullet \sqrt[n]{b} = \sqrt[n]{ab}$

$\left(5\sqrt{3x}\right)\left(6\sqrt{6}\right) = 30\sqrt{18x}$ *don't forget to simplify

$\sqrt{a} \bullet \sqrt{a} = \sqrt{a^2} = a$

$\sqrt[3]{a} \bullet \sqrt[3]{a} \bullet \sqrt[3]{a} = \sqrt[3]{a \bullet a \bullet a} = \sqrt[3]{a^3} = a$

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Adding & Subtracting Radicals

$\sqrt{a} + 3\sqrt{a} = 4\sqrt{a}$

$5\sqrt{3} + 6\sqrt{27} - 8\sqrt{3} = 5\sqrt{3} + 6\sqrt{9 \bullet 3} - 8\sqrt{3} =$
$5\sqrt{3} + 6 \bullet 3\sqrt{3} - 8\sqrt{3} = 15\sqrt{3}$

Remember from Algebra—Radicals may also be expressed as ***Exponents*** which makes working with radicals much easier.

$$\sqrt[n]{a} = a^{1/n}$$

Simplify: $5\sqrt{8} - 3\sqrt{18} + \sqrt{3}$

1. Simplify $5\sqrt{8}$

Answer: $5\sqrt{8} = 5\sqrt{4 \bullet 2} = 5\sqrt{4}\sqrt{2} = 5 \bullet 2\sqrt{2} = 10\sqrt{2}$

2. Simplify $-3\sqrt{18}$

Answer: $-3\sqrt{18} = -3\sqrt{9 \bullet 2} = -3\sqrt{9}\sqrt{2} = -3 \bullet 3\sqrt{2} = -9\sqrt{2}$

3. Since the radicals in steps 1 and 2 are now the same, we can combine them.

$$10\sqrt{2} - 9\sqrt{2} = \sqrt{2}$$

4. You are left with:

$$\sqrt{2} + \sqrt{3}$$

5. Can you combine these radicals?
Answer: NO

6. Therefore, **Answer:** ------------------> $\sqrt{2} + \sqrt{3}$

Work the following:

$3\sqrt{2} - 2\sqrt{30}$

$2\sqrt{50} + 12\sqrt{8}$

-2
$4\sqrt{27} - \sqrt{75}$

$4\sqrt{3} - 2\sqrt{3} - 6\sqrt{3}$

Solving Equations with Radicals

1. Simplified all radicals in the equation.

2. If there is only one radical, isolate the radical on one side of the equation.

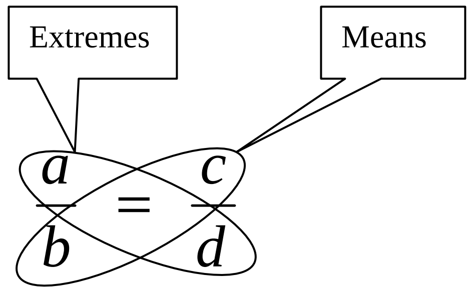
3. Square both sides of the equation. Note: Do **NOT** square each term—square all the terms on each side.

$$8\sqrt{x} + 4 = 5\sqrt{x} + 6$$

$$\sqrt{x+3} + 6\sqrt{6} = 0$$

Geometry—Chapter 8-1

Remember the special names for the different positions in the proportion?



Now----

Def: The geometric mean between 2 positive numbers is the **<u>positive</u>** number x where

$$\frac{a}{x} = \frac{x}{b}$$

which, by cross multiplying, will produce….

$$x \bullet x = a \bullet b \quad \text{or}$$

$$x^2 = ab$$

$$x = \sqrt{ab}$$

Since x is the geometric mean, occupies the means position in a proportion, its value will always be positive by definition and is called the Principal Square Root.

Note: When you are not determining the geometric mean,

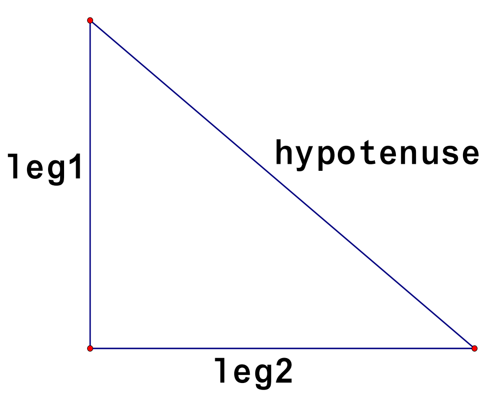
$$\text{and} \quad x^2 = \{\text{an expression}\} \quad \text{then}$$

$$x = \pm\sqrt{\{\text{the expression}\}}$$

Ex: Find the geometric mean **between** 6 and 9.

NEXT……….THE FAMOUS PYTHAGOREAN THEOREM

Pythagorean Theorem→In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.

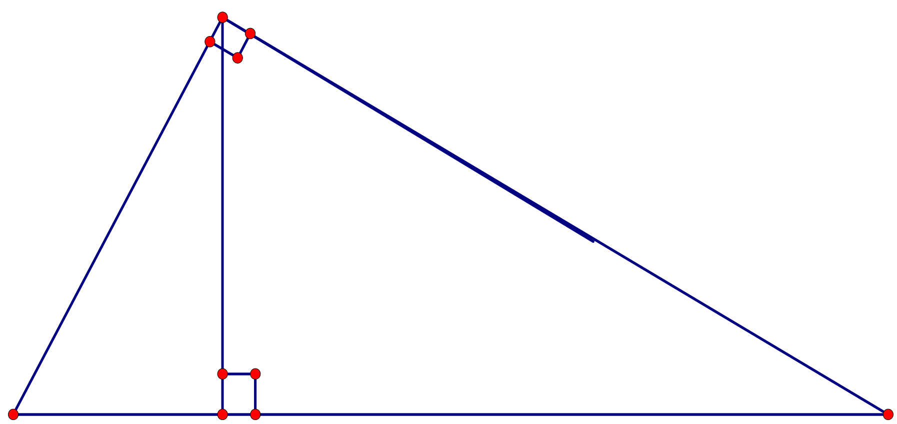


$$leg1^2 + leg2^2 = hyp^2$$

The Converse---------

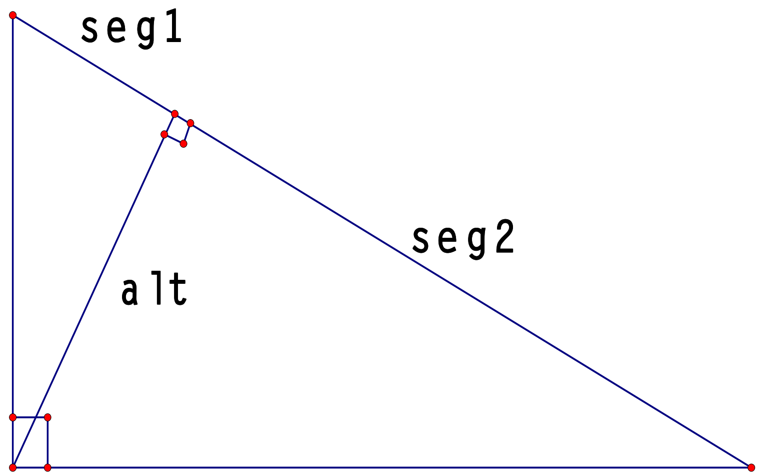
If the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse THEN
the triangle is a right triangle.

Thm: If an alt is drawn from the vertex of the *right angle* of a *right triangle* to its hypotenuse, then the two triangles formed are similar to the given triangle and to each other.



Thm: The measure of the *altitude* drawn from the vertex of the right angle of a right triangle to its hypotenuse is the *geometric mean* **between** the measures of the 2 segments of the hypotenuse.

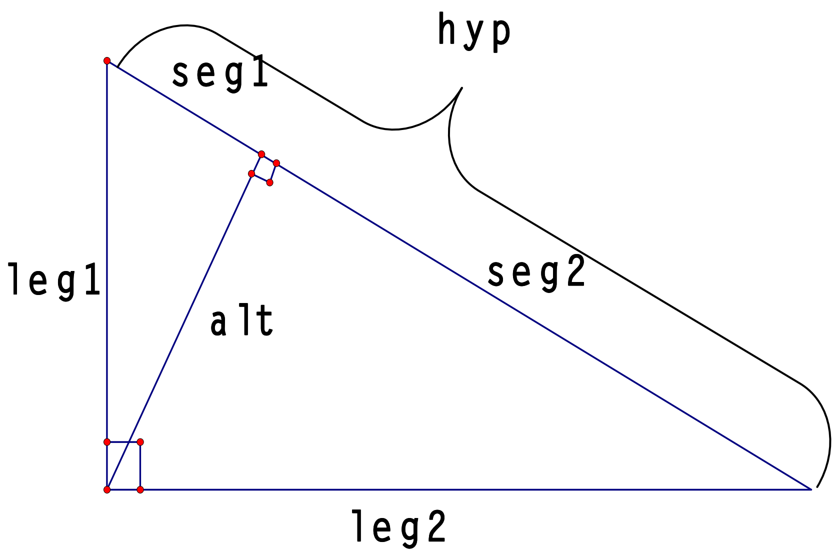
In other words: alt is the geo mean BETWEEN the segs of the hyp.



$$\frac{\text{seg1(hyp)}}{\text{alt}} = \frac{\text{alt}}{\text{seg2(hyp)}}$$

Thm: If an alt is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the measure of a *leg* of the triangle is the *geometric mean* **between** the measures of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

WHAT???? Remember each triangle has 2 legs so this theorem can be stated using 2 equations.



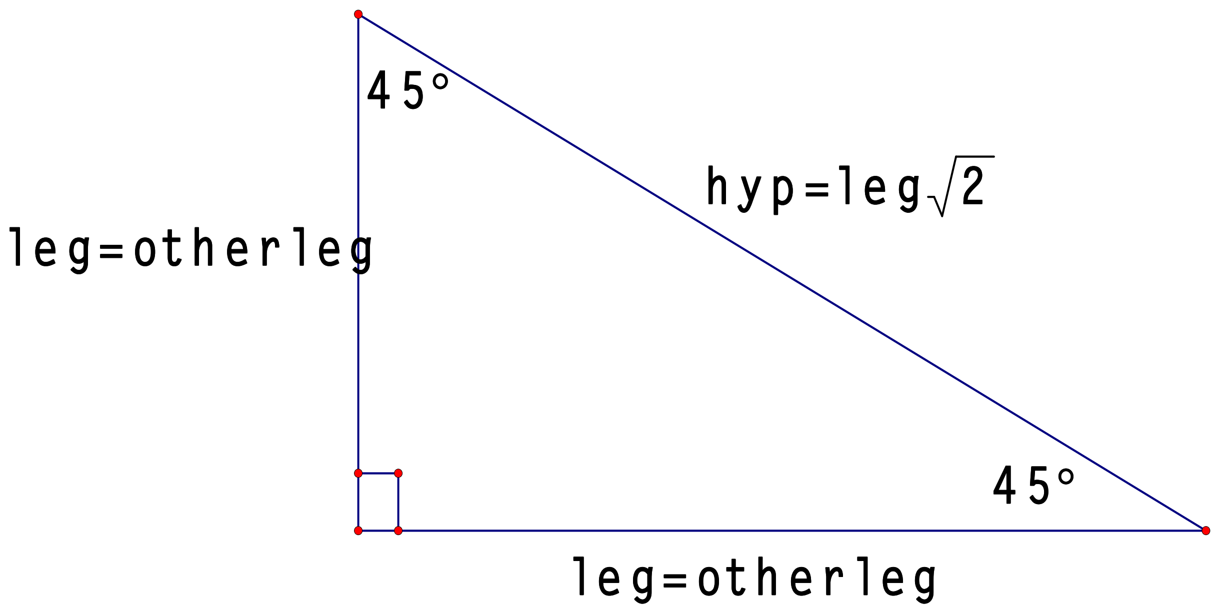
$$\frac{\text{hyp}}{\text{leg1}} = \frac{\text{leg1}}{\text{seg1}} \qquad \frac{\text{hyp}}{\text{leg2}} = \frac{\text{leg2}}{\text{seg2}}$$

☺

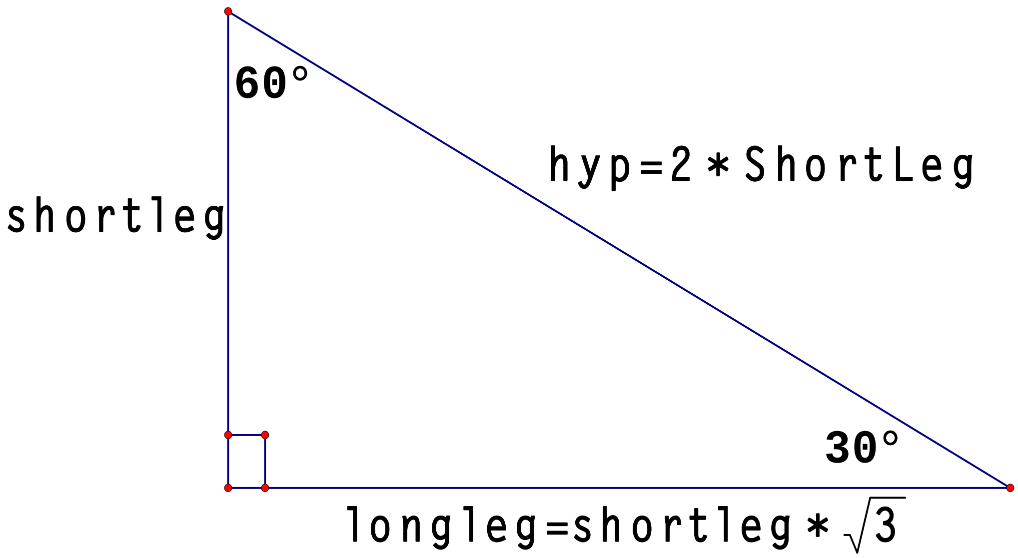
Geometry—Chapter 8-2

Special Right Triangles

Thm: In a $45°$-$45°$-$90°$ triangle, the hypotenuse is $\sqrt{2}$ times as long as either leg.

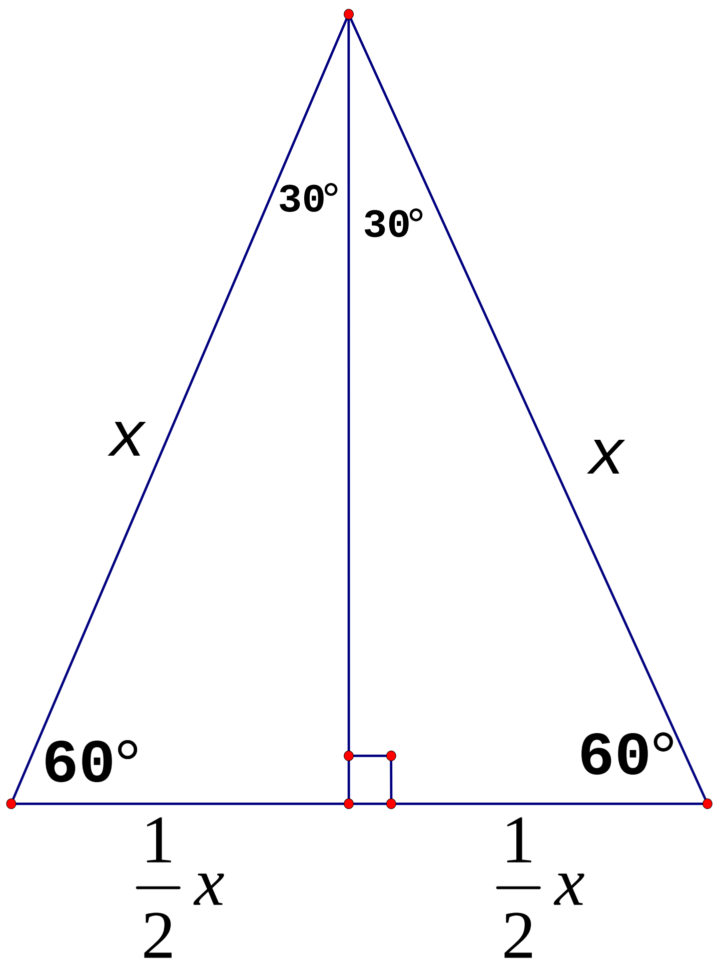


Thm: In a $30°$-$60°$-$90°$ triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.



Note: the ShortLeg = ½ hyp

Note what happens when you draw an altitude from the vertex angle of an ***equilateral*** triangle.



What would be the measure of the altitude?

☺

Geometry—Chapter 8-3

Trigonometry WOW! Cool! This section covers Trig.

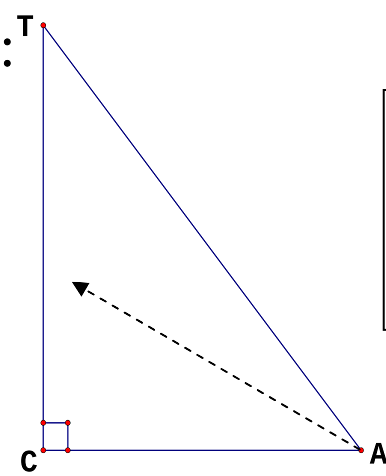
Suppose you have a right triangle and it is not one of the Special Triangles……..how would you solve the triangle?

Remember—solving a triangle means finding the values for all the angles and finding the lengths of all the sides.

The 3 most common ***ratios*** for solving a **Right Triangle** are:

sine, cosine, and tangent.

Given a right triangle:



sin, cos, tan are abbreviations for sine, cosine, and tangent

IMP: there must be an argument (∢) following sin, cos, or tan or *it will not* be a valid math operation.

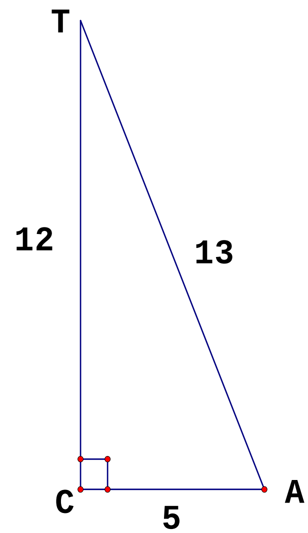
opposite leg from the ∢

adjacent leg from the ∢

NOTE the 3 ratios….

$$\sin \measuredangle = \frac{\text{opp leg}}{\text{hyp}} \qquad \cos \measuredangle = \frac{\text{adj leg}}{\text{hyp}} \qquad \tan \measuredangle = \frac{\text{opp leg}}{\text{adj leg}}$$

Given ΔCAT with measures as stated below.
Find the ratios of:



sin ∡A = ?   sin ∡T = ?

cos ∡A = ?   cos ∡T = ?

tan ∡A = ?   tan ∡T = ?

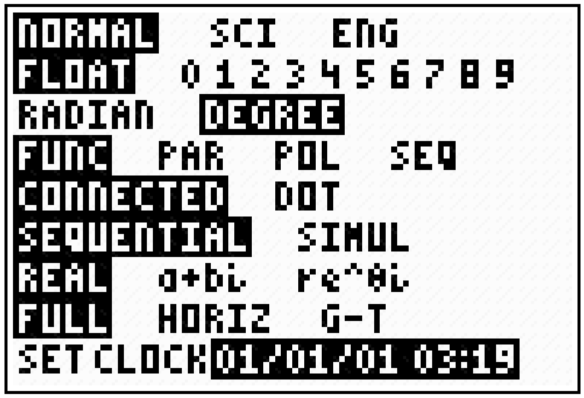
WAIT!! What about ∡C?

Never, Ever use the right ∡….

Trig ratios are related to the **acute ∡s only** of a right triangle NOT ever, never to the right angle.

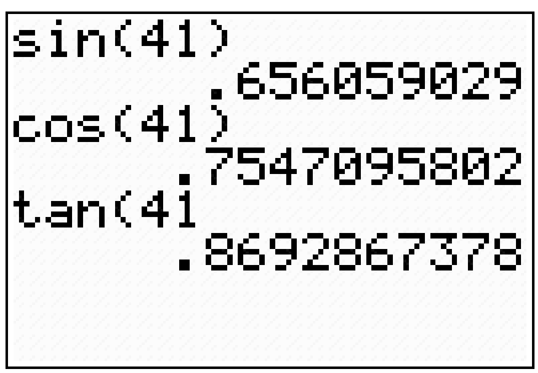
Calculator Notes:

Always—**Always**—**Always** make sure your calculator is in ***Degree Mode***.



Evaluate the following:

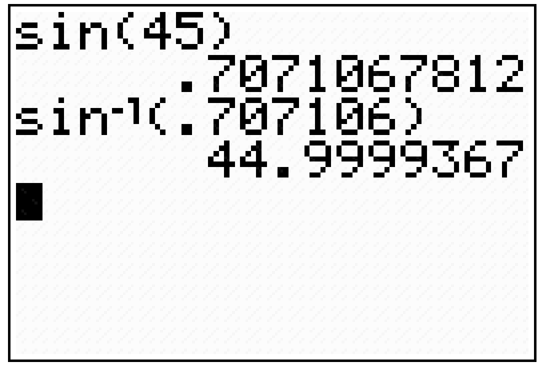
sin 41° = ? Use your calculator—Always round to the nearest ten thousandths



Answer: sin 41° ≈ 0.6561 → always round

Given the ratio, find the measure of the angle.

Again use the calculator…..



Thus: $\sin(45°) \approx 0.7071$ and

$$\sin^{-1}(0.706106) \approx 45°$$

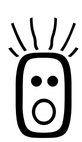
Given a triangle, how do you know which trig ratio to use?

If:

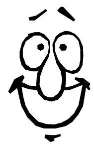
opp leg, hyp, $\measuredangle$ are listed then use sine

adj leg, hyp, $\measuredangle$ are listed then use cosine

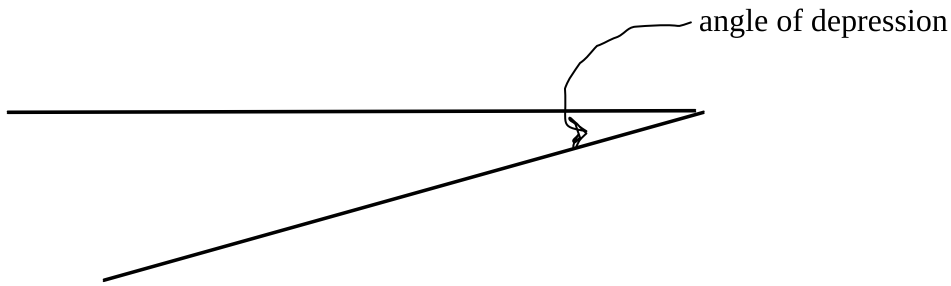
adj leg, opp leg, $\measuredangle$ are listed then use tangent

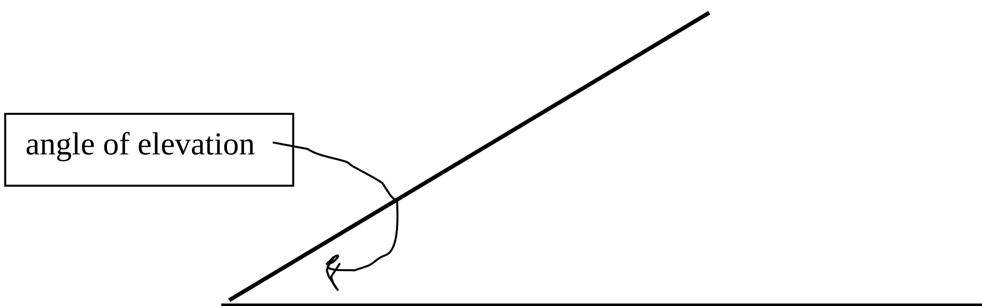


Geometry—Chapter 8-4

Angles of Elevation  and Depression  !!

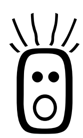
These angles are used to solve problems involving a horizontal line. (usually parallel to the horizon)





If the angle of elevation or depression is an acute angle of a right triangle then you sometimes can use the trig functions to find the sides.

Ex: A plane is 3 miles above the ground. The pilot sights the airport at an angle of depression of 15°. He sights his house at an angle of depression of 32°. What is the ground distance between the pilot’s house and the airport?

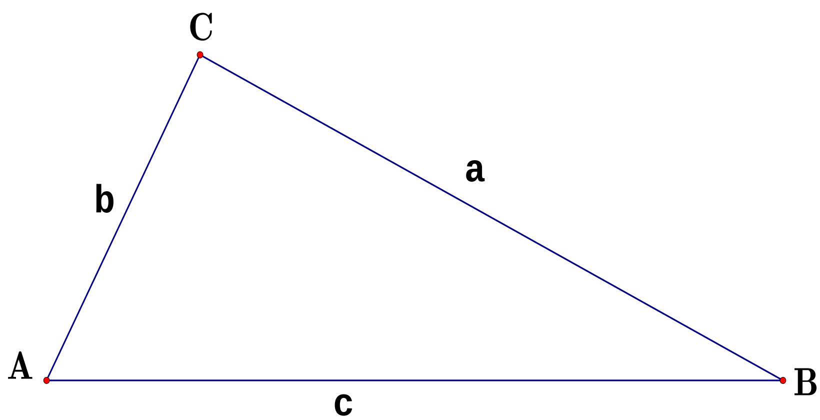


Geometry—Chapter 8-5

Law of Sines

One of the methods that you can use to solve triangles that are *not right triangles* is the Law of Sines.
Law of Cosines is another—see Chapter 9-6.

The Law of Sines states that given any triangle such as

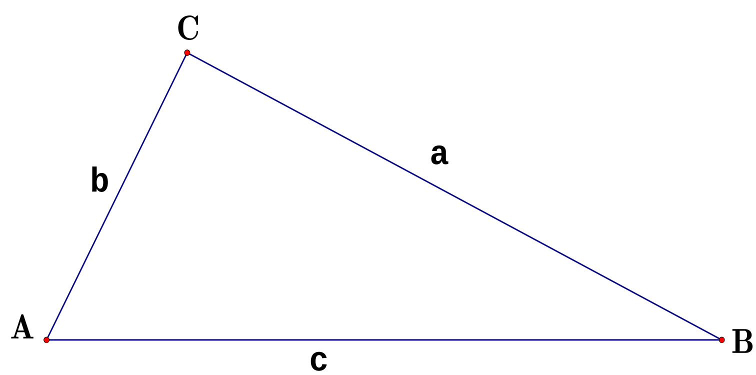


then $$\frac{\sin\sphericalangle A}{a} = \frac{\sin\sphericalangle B}{b} = \frac{\sin\sphericalangle C}{c}.$$

Remember that finding the measures of all the angles and sides is called solving the triangle.

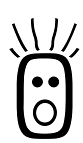
The Law of Sines can be used to solve a triangle when one of the cases below exist.

- the measures of 2 angles and any side are given
- the measures of 2 sides and an angle *opposite* one of the two sides are given



Ex: $m\measuredangle A = 33°$, $m\measuredangle B = 47°$ $b = 14$
Solve the triangle.

Ex: $m\measuredangle B = 102°$, $b = 24$, $a = 18$
Solve the triangle.



Geometry—Chapter 8-6

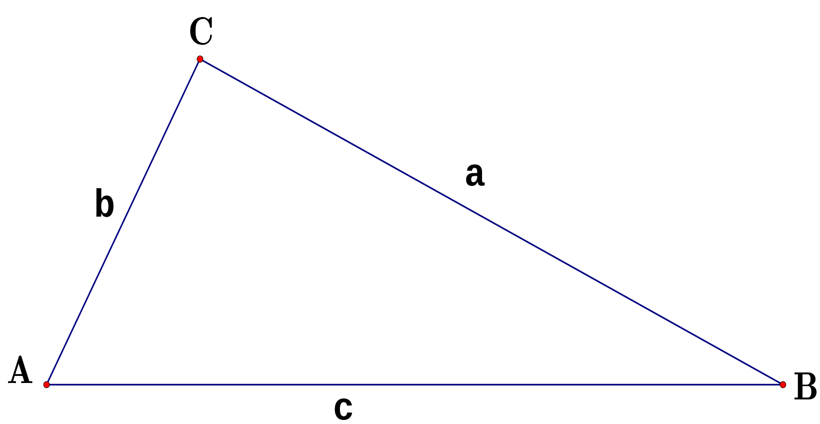
Law of Cosines

Another method to solve a triangle that is not *right*.

Wait! If it is not right is it wrong?

And if it is wrong HOW can the solution ever be right???? ☺

The Law of Cosines states that given any triangle such as



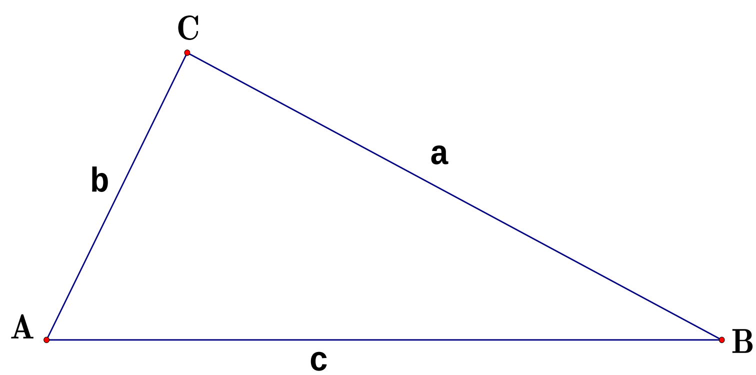
then $a^2 = b^2 + c^2 - 2bc\cos\measuredangle A$

$b^2 = a^2 + c^2 - 2ac\cos\measuredangle B$

$c^2 = a^2 + b^2 - 2ab\cos\measuredangle C$

Just as there are cases in which you can use the Law of Sines, there are cases in which you can use the Law of Cosines.

- the measures of 2 sides and the included angle are given
- the measures of the 3 sides are given



Ex: $m\measuredangle B = 52°$, $c = 7$, $a = 4$
Solve the triangle.

Ex: $a = 5$, $b = 6$, $c = 7$
Solve the triangle.