Geometry—Chapter 7

Geometry—Chapter 7-1

Proportions & Ratios

Ratio→is a comparison of two quantities.

ex: $\frac{length(\text{model})}{length(\text{real item})} = \frac{22\,inches}{176\,inches}$

IMP: the units of the two quantities MUST be the same

Notation: *a* to *b*, $\frac{a}{b}$, *a*:*b* Note: that $b \neq 0$……Why?

The units in a ratio must ALWAYS cancel out.

Rates look like ratios but the units of Rates are different.

ex: $\frac{50\,miles}{1\,hour}$; $\frac{20\,\text{ml}}{60\,\text{sec}}$

found in nature and is quite famous

A very special ratio is the **Golden Ratio**.

ALL ratios MUST be SIMPLIFIED!!!!!

Proportion→is an <u>equation</u> stating that 2 ratios are equal.

ex: $\frac{336}{150} = \frac{224}{100}$

An <u>extended</u> proportion is an equation stating that more than 2 ratios are equal.

Cross-products—a mathematical method of solving/representing a proportion.

These products are quite happy, you just have to get rid of the fractions.

ex:

$$\frac{a}{b} = \frac{c}{d},$$

$$(bd)\frac{a}{b} = \frac{c}{d}(bd)$$

$$ad = bc$$

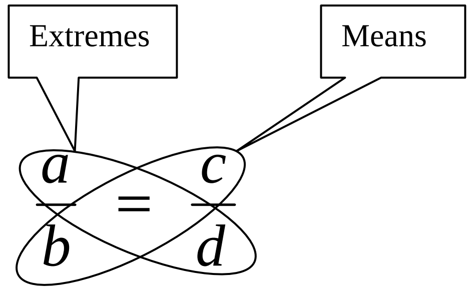
This leads to the simple method of Cross-Products….
Just multiply across and down……

$$\frac{a}{b} \times \frac{c}{d},$$

$$ad = bc$$

Only, ONLY, cross-multiply across an EQUAL sign and only when you have one term on each side. Why???

There are special names for the different positions in the proportion.



Lets be extreme and talk about the means of solving problems!

Ex: Solve for t. $\frac{3t-1}{4}=\frac{7}{8}$

Ex: In a triangle, the ratio of the measures of three sides is 8:7:5 and its perimeter is 240 centimeters. Find the measure of each side of the triangle

☺

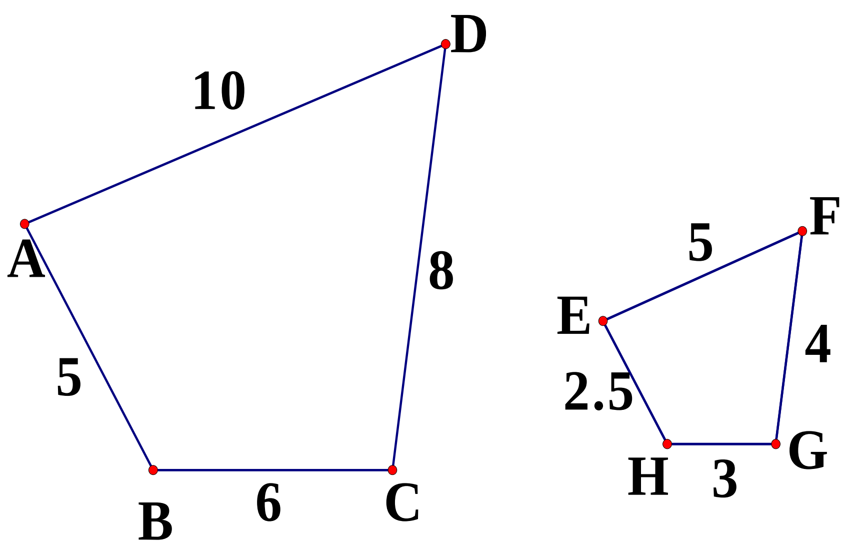
Geometry—Chapter 7-2

Similar Polygons

Remember Congruent Figures??? --same shape, same size
Well! Similar Figures→same shape, **different** size

Polygons are similar iff

- corresponding angles are $\cong$
- measures of their corresponding sides are proportional



In other words: $\measuredangle A \cong \measuredangle E$

$\measuredangle D \cong \measuredangle F$

$\measuredangle C \cong \measuredangle G$

$\measuredangle B \cong \measuredangle H$

And: $\dfrac{10}{5} = \dfrac{8}{4} = \dfrac{6}{3} = \dfrac{5}{2.5}$ ← an extended proportion

Notice that each of the ratios equal $\frac{2}{1}$ or just 2. It does not matter which polygon sides are on top, just make sure that all the sides of a polygon are either all on top or all on the bottom in the proportion.

If the two requirements for similar polygons are met then

$$ABCD \sim EFGH$$

Observe the notation for polygons and the notation $\sim$ for similar polygons. Also, notice that the vertices of corresponding angles **MUST** match.

Scale Factor→is a ratio that describes the difference in size……(units must be the same)

Ex: Model of a car may have a scale factor of $\frac{1}{18}$ or $\frac{18}{1}$ which means that for every inch/foot/any measure of the model, the original or real car will be 18 times that.

Thus the scale factor of any poly will be any one of the ratios of corresponding sides reduced or simplified.

Proportions are used in math, chemistry, physics, cartography, cooking, photography, and the list could go on and on.

Remember in polygons the:
Scale factor = ratio of corresponding sides
= ratio of the respective perimeters
BUT the measures of the corresponding angles are ≅.

☺

Geometry—Chapter 7-3

Similar Triangles (Remember what you know about similar polys?)

Just as you can show that triangles are congruent, you can show that triangles are similar.

What are the ways that triangles can be shown to be $\cong$?

- Def of $\cong$ triangles
- SSS Postulate
- SAS Postulate
- ASA Postulate
- AAS Postulate

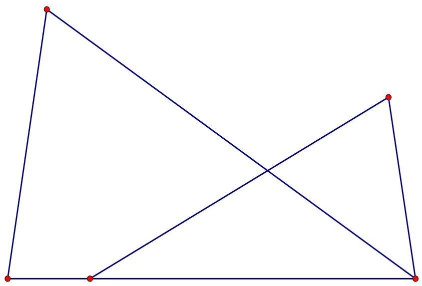
Triangles can be shown to be SIMILAR by:

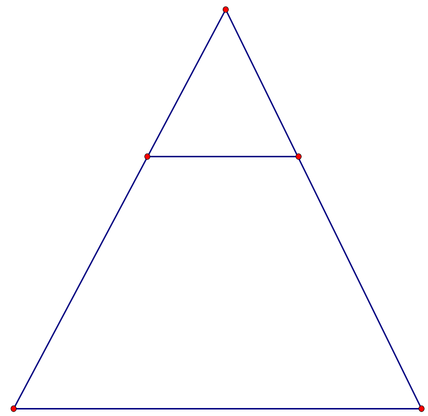
- AAA Similarity (really only need AA—Why??)
- SSS Similarity
- SAS Similarity

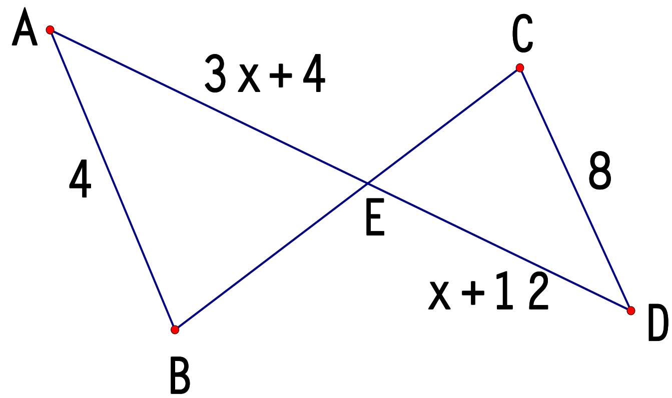
WAIT!! Do not forget that Δs are polygons, thus if Δs are similar their:

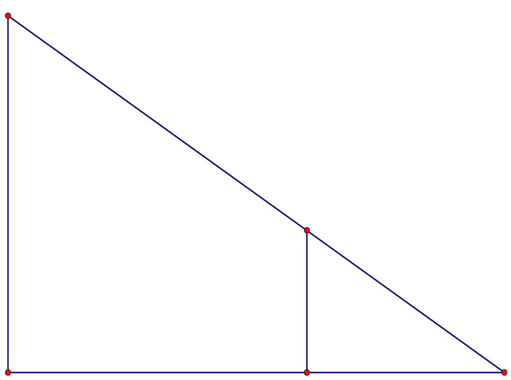
- corresponding angles are $\cong$ and
- measures of their corresponding sides are proportional

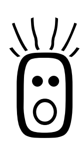
Let us view some Δs and determine it they are ∼ AND why (state a reason).











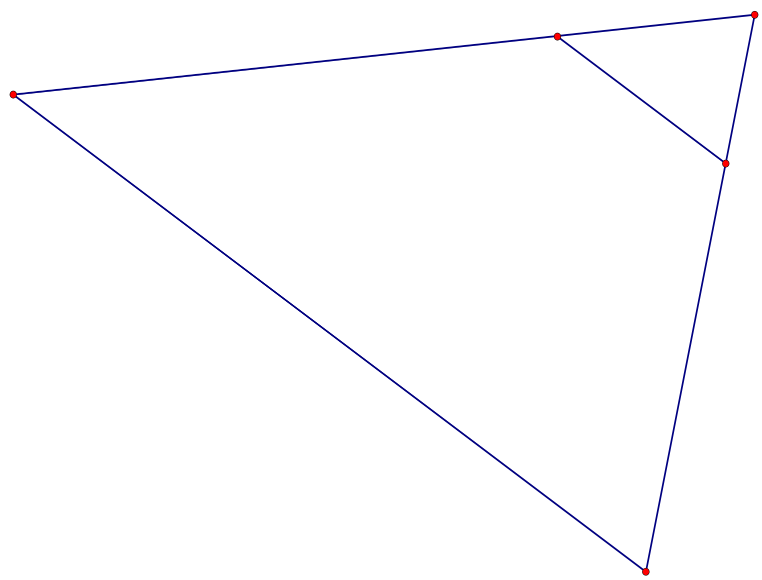
Geometry—Chapter 7-4

Parallel lines & Proportional Parts

There are 3 Theorems and 2 Corollaries that you must memorize…….

Thm: If a line is // to one side of a △ and intersects the other 2 sides, then it separates the two sides into segments of proportional lengths.

△ with // sides



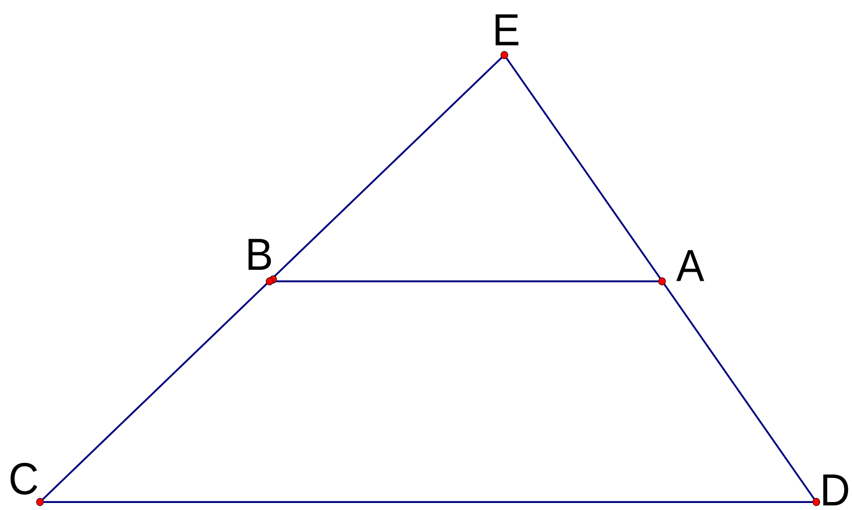
Converse---

Thm: If a line intersects 2 sides of a triangle into corresponding segments of proportional lengths, then the line is // to the 3rd side.

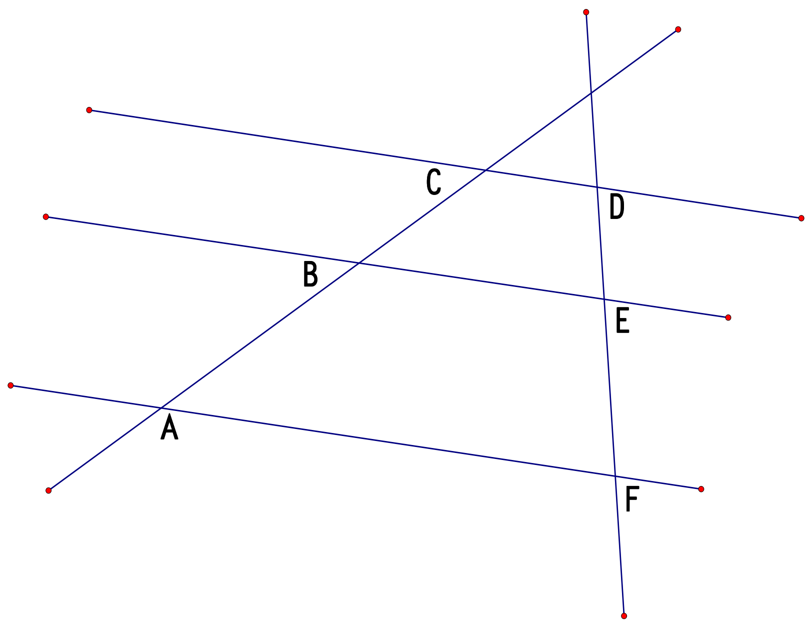
Thm: A segment whose endpts are the **midpts** of two sides of △ is

- // to the 3^{rd} side of the △, and
- its length is $\frac{1}{2}$ the length of the 3^{rd} side.

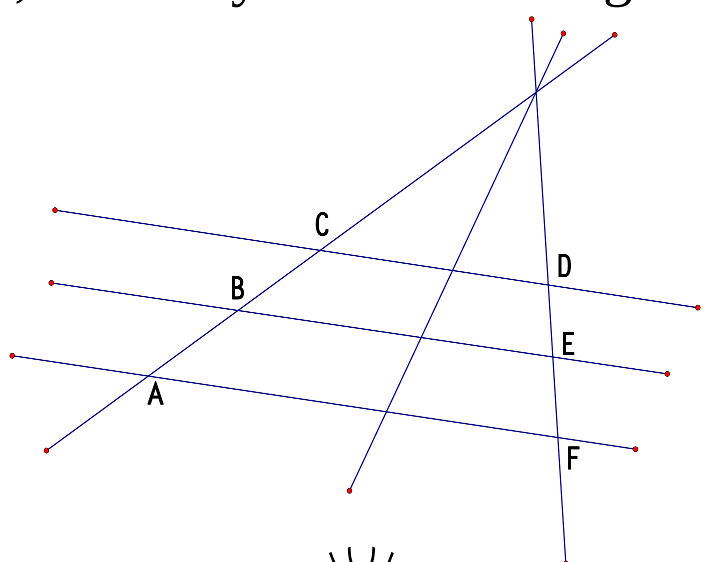
Segment in a Triangle whose endpts are the endpts.

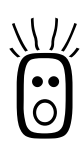


Corollary: If 3 or more // lines intersect 2 transversals, then they cut off the transversals proportionally.



Corollary: If 3 or more // lines cut off ≅ segments on one transversal, then they cut off ≅ segments on every transversal.





Geometry—Chapter 7-5

Parts of Similar Triangles

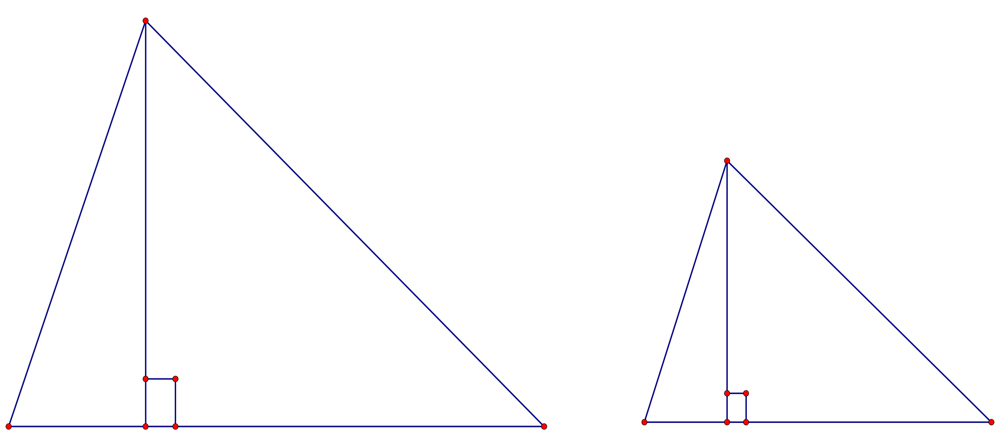
Again there are a number or Theorems (5) that you must know/memorize.

Thm: If 2 △s are similar, then the perimeters are proportional to the measures of the corresponding sides.
(also = to the scale factor)

Scale Factor = ratio of the corres. sides = ratio of the perimeters

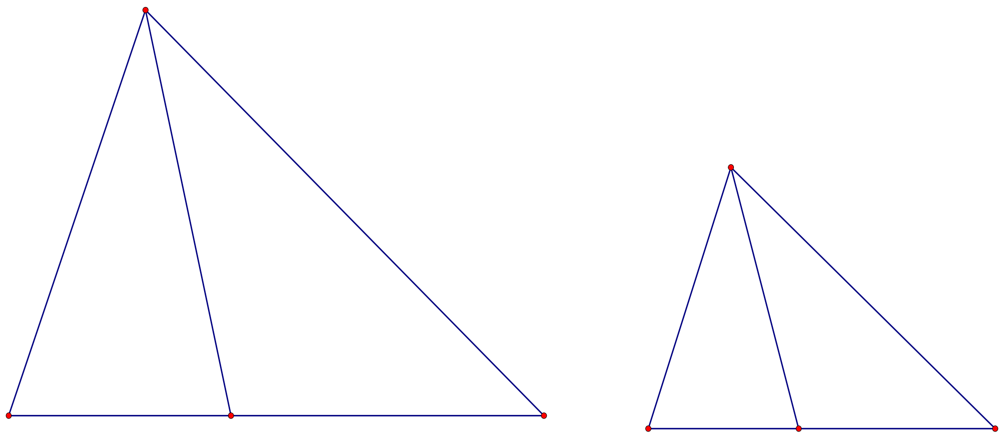
Thm: If 2 △s are similar, then the measures of the corres. alts. are = to the Scale Factor.
(or proportional to the measures of the corres sides)

Scale Factor = ratio of the corres. sides = ratio of the corres. alts.



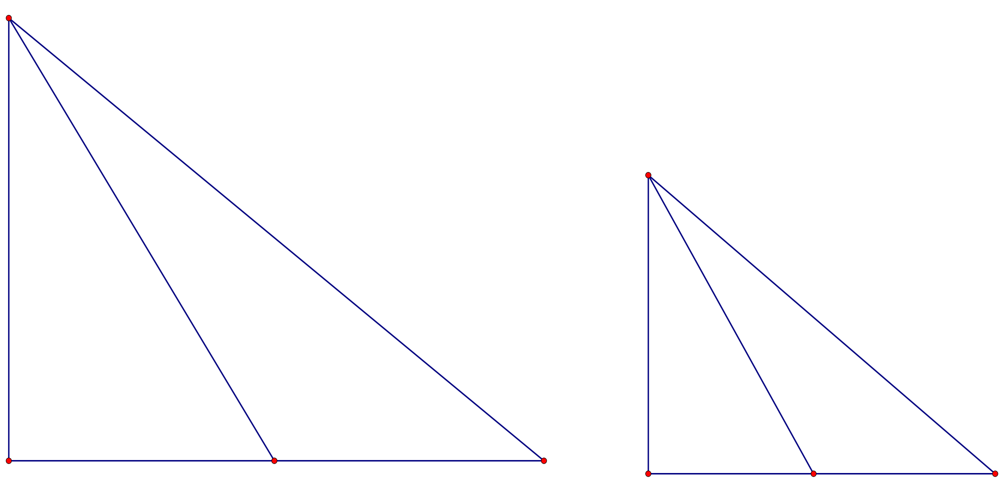
Thm: If 2 △s are similar, then the measures of the corres. ∢ bisectors of the triangles are = to the Scale Factor.
(or proportional to the measures of the corres sides)

Scale Factor = ratio of the corres. sides = ratio of the ∢ bisectors



Thm: If 2 △s are similar, then the measures of the corres. medians of the triangles are = to the Scale Factor.
(or proportional to the measures of the corres sides)

Scale Factor = ratio of the corres. sides
= ratio of the corres. medians



Thm: An ∢ bisector in a △ separates the opp side into segments that have the same ratio as the other two sides.