Problem Set # 5

(No need to submit)

1. Observations on the yield of a chemical reaction taken at various temperatures were recorded as follows:

Y (%)	T (^oC)	Y (%)	T (^oC)
77.4	150	88.9	250
76.7	150	89.2	250
78.2	150	89.7	250
84.1	200	94.8	300
84.5	200	94.7	300
83.7	200	95.9	300

Fit the above data to a linear model of the form: Ý = a + b T

2. A chemical engineer is investigating the effect of two process variables (Temperature and feed rate) on the drying rate of a process. The engineer decides to run a 2^2 design with three center points.

The engineer wants to fit a multiple linear regression model for predicting the drying rate. Determine the least square estimates of the coefficients (η_i, i = 0, 1, 2, 3) in the model

$$\hat{k} = h_o + h_1 x_1 + h_2 x_2 + h_3 x_1 x_2$$

The design and the resulting yields are as follows:

Run No.	Temperature (^oC) x_1	Feed rate (kg/min) x_2	Drying rate κ
1	100	15	32
2	140	15	46
3	100	25	27
4	140	25	35
5	120	20	29

6	120	20	33
7	120	20	31

3. The water level in the North Sea is mainly determined by the so-called M_2-tide whose period is about 12 hours and thus has the form

$$H(t) = h_o + a_1 \sin\left[\frac{2pt}{12}\right] + a_2 \cos\left[\frac{2pt}{12}\right]; \quad \text{t in hours.}$$

One has made the following measurements:

t	0	2	4	6	8	10	hours
H(t)	1.0	1.6	1.4	0.6	0.2	0.8	meters

Fit H(t) to the series of measurements using the method of least squares.

4. The growth rate expression for a biochemical reaction using a substrate inhibition model is given by

$$m = \frac{m_{\max} x}{k_m + x + k_1 x^2}$$

where μ is the specific growth rate, k_1, k_m and μ_{max} are parameters, and x is the substrate concentration. Determine the parameters k_1, k_m and μ_{max} for the following data from a particular reactor.

μ, hr^{-1}	0.24	0.27	0.34	0.35	0.35	0.34	0.33	0.22
x, g/liter	0.10	0.15	0.25	0.50	0.75	1.00	1.50	3.00

5. Nitrogen and Oxygen have the atomic weights N = 14 and O = 16. Use the molecular weights of the six nitrogen oxides given below to compute the atomic weights for nitrogen and oxygen to four decimal places.

NO	30.006	N_2O	44.013	NO_2	46.006
N_2O_3	76.012	N_2O_5	108.010	N_2O_4	92.011

Q. 1

$\hat{Y} = a + bT$

$\hat{Y} = a f_1 + b f_2$

Where, $f_1 = 1, f_2 = T$

Define $f_2 = \dfrac{T - (\dfrac{300+150)}{2}}{\dfrac{300-15}{2}} = \dfrac{T-225}{75}$

$\hat{Y}$ (%)	f_2 (°C)	$\hat{Y}$ (%)	f_2 (°C)
77.4	-1	88.9	$1/3$
76.7	-1	89.2	$1/3$
78.2	-1	89.7	$1/3$
84.4	$-1/3$	94.8	1
84.5	$-1/3$	94.7	1
83.7	$-1/3$	95.9	1

We want to fit the above data to a linear model of the form,

$\hat{Y} = a + bT$

$\hat{Y} = a f_1 + b f_2$

Where, $f_1 = 1, f_2 = T$

The two equations are

$\sum < f_i, f_k > a_i = < \hat{Y}, f_k >$, for $k = 1, 2$

or, $< f_1, f_1 > a_1 + < f_2, f_1 > a_2 = < \hat{Y}, f_1 >$, for k = 1

$< f_1, f_2 > a_1 + < f_2, f_2 > a_2 = < \hat{Y}, f_2 >$, for k = 2

$$\begin{bmatrix} < f_1, f_1 > & < f_2, f_1 > \\ < f_1, f_2 > & < f_2, f_2 > \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} < \hat{Y}, f_1 > \\ < \hat{Y}, f_2 > \end{bmatrix}$$

$$< f_1, f_2 > = 12(1)^2 = 12$$

$$< f_1, f_2 > = < f_2, f_1 > = 3(1)(-1) + 3(1)(-\tfrac{1}{3}) + 3(1)(\tfrac{1}{3}) + 3(1)(1) = -3 - 1 + 1 + 3 = 0$$

$$< f_2, f_2 > = 3(-1)^2 + 3(-\tfrac{1}{3})^2 + 3(1)^2 + 3(\tfrac{1}{3})^2 = 6.667$$

$$< \hat{Y}, f_1 > = (1037.8)(1) = 1037.8$$

$$< \hat{Y}, f_2 > = 58.16$$

$$\begin{bmatrix} 12 & 0 \\ 0 & 6.667 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1037.8 \\ 58.16 \end{bmatrix}$$

$$a_i = \frac{< \hat{Y}, f_i >}{< f_i, f_i >}$$

$$a_1 = \frac{< \hat{Y}, f_1 >}{< f_1, f_1 >}$$

$$a_2 = \frac{< \hat{Y}, f_2 >}{< f_2, f_2 >}$$

$$a_1 = \frac{1037.8}{12} = 86.48$$

$$a_2 = \frac{58.16}{6.667} = 8.72$$

$$\begin{aligned} \hat{Y} &= a_1 f_1 + a_2 f_2 \\ &= 86.48(1) + 8.72(\frac{T - 225}{75}) \\ &= 86.48 + 0.116T - 26.16 \\ &= 60.32 + .0116T \end{aligned}$$

Q 2.

$$\begin{aligned} k &= h_0 + h_1 x_1 + h_2 x_2 + h_3 x_1 x_2 \\ &= h_0 + h_1 x_1 + h_2 x_2 + h_3 x_3 \end{aligned}$$

No.	x_1	x_2	k	x_3
1	100	15	32	1500
2	140	15	46	2100
3	100	25	27	2500
4	1400	25	35	3500
5	120	20	29	2400
6	120	20	33	2400
7	120	20	31	2400

$$z_1 = \frac{x_1 - \frac{140+100}{2}}{\frac{140-100}{2}} = \frac{x_1 - 120}{20}$$

$$z_2 = \frac{x_2 - \frac{25+15}{2}}{\frac{25-15}{2}} = \frac{x_2 - 20}{5}$$

$$z_3 = \frac{x_3 - \frac{3500+1500}{2}}{\frac{3500+1500}{2}} = \frac{x_3 - 2500}{1000}$$

No.	z_1	z_2	z_3	k
1	-1	-1	-1	32
2	1	-1	-0.4	46
3	-1	1	0	27
4	1	1	1	35
5	0	0	-0.1	29
6	0	0	-0.1	33
7	0	0	-0.1	31

To find h_0, h_1, h_2, h_3 we have to solve,

$$\begin{bmatrix} <z_0, z_0> & <z_1, z_0> & <z_2, z_0> & <z_3, z_0> \\ <z_0, z_1> & <z_1, z_1> & <z_2, z_1> & <z_3, z_1> \\ <z_0, z_2> & <z_1, z_2> & <z_2, z_2> & <z_3, z_2> \\ <z_0, z_3> & <z_1, z_3> & <z_2, z_3> & <z_3, z_3> \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} <k, z_0> \\ <k, z_1> \\ <k, z_2> \\ <k, z_3> \end{bmatrix}$$

$$< z_0, z_0 >= 7(1)^2 = 7$$
$$< z_0, z_1 >=< z_1, z_0 >= (1)(-1)2 + (1)(1)(2) = 0$$
$$< z_0, z_2 >=< z_2, z_0 >= (1)(-1)2 + (1)(1)2 = 0$$
$$< z_0, z_3 >=< z_3, z_0 >= (1)(-1) + (1)(-0.4) + (1)(0) + (1)(1) + (1)(-0.1) + (1)(-0.1) + (1)(-0.1)$$
$$< z_1, z_1 >= 2(-1)^2 + 2(1)^2 = 2 + 2 = 4$$
$$< z_1, z_2 >=< z_2, z_1 >= (-1)(-1) + (1)(-1) + (1)(-1) + (1)(1) = 0$$
$$< z_1, z_3 >=< z_3, z_1 >= (-1)(-1) + (1)(-0.4) + (-1)(0) + (1)(1) = 1.6$$
$$< z_2, z_2 >= (-1)^2 + (-1)^2 + (1)^2 + (1)^2 = 4$$
$$< z_2, z_3 >=< z_3, z_2 >= (-1)(-1) + (-1)(-0.4) + (1)(0) + (1)(1) = 2.4$$
$$< z_3, z_3 >= (-1)^2 + (-0.4)^2 + (1)^2 + (-0.1)^2 + (-0.1)^2 + (-0.1)^2 = 2.19$$

$$< k, z_0 >= 233$$
$$< k, z_1 >= (32)(-1) + 46(1) + 27(-1) + 35(1) = 22$$
$$< k, z_2 >= (-1)32 + (-1)46 + 1(27) + (1)35 = -16$$
$$< k, z_3 >= (32)(-1) + 46(-0.4) + (0)27 + (1)(25) + (-0.1)(29) + (-0.1)(29) + (-0.1)(33)$$
$$+(-0.1)(31) = -24.7$$

$$\begin{bmatrix} 7 & 0 & 0 & -0.7 \\ 0 & 4 & 0 & 1.6 \\ 0 & 0 & 4 & 2.4 \\ -0.7 & 1.6 & 2.4 & 2.19 \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} 233 \\ 22 \\ -16 \\ -24.7 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 0 & 0 & -0.7 \\ 0 & 4 & 0 & 1.6 \\ 0 & 0 & 4 & 2.4 \\ 0 & 1.6 & 2.4 & 2.12 \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} 233 \\ 22 \\ -16 \\ -1.4 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 0 & 0 & -0.7 \\ 0 & 4 & 0 & 1.6 \\ 0 & 0 & 4 & 2.4 \\ 0 & 0 & 2.4 & 1.48 \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} 233 \\ 22 \\ -16 \\ -10.2 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 0 & 0 & -0.7 \\ 0 & 4 & 0 & 1.6 \\ 0 & 0 & 4 & 2.4 \\ 0 & 0 & 2.4 & 0.04 \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} 233 \\ 22 \\ -16 \\ -0.6 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 0 & 0 & -0.7 \\ 0 & 4 & 0 & 1.6 \\ 0 & 0 & 4 & 2.4 \\ 0 & 0 & 2.4 & 1 \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} 233 \\ 22 \\ -16 \\ -15 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 0 & 0 & 0 \\ 0 & 4 & 0 & 1.6 \\ 0 & 0 & 4 & 2.4 \\ 0 & 0 & 2.4 & 1 \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} 293 \\ 22 \\ -16 \\ -15 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 2.4 \\ 0 & 0 & 2.4 & 1 \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} 293 \\ 46 \\ -16 \\ -15 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 2.4 & 1 \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} 293 \\ 46 \\ -20 \\ -15 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2.4 & 1 \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} 31.78 \\ 11.5 \\ -5 \\ -15 \end{bmatrix}$$

$$\begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} 31.78 \\ 11.5 \\ -5 \\ -15 \end{bmatrix}$$

$$\begin{aligned} k &= h_0 + h_1 x_1 + h_2 x_2 + h_3 x_1 x_2 \\ &= 31.78 + 11.5 x_1 - 5 x_2 - 15 x_3 \\ &= 31.7 + 11.5 \frac{x_1 - 120}{20} - 5 \frac{x_2 - 20}{5} - 15 \frac{x_3 - 2500}{1000} \\ &= 31.7 + 0.575 x_1 - 69 - x_2 + 20 - 0.015 x_3 + 37.5 \\ &= 20.2 + 5.75 x_1 - x_2 - 0.015 x_3 \end{aligned}$$

Q 3.

$$H(t) = h_0 + a_1 \sin\left(\frac{2pt}{12}\right) + a_2 \cos\left(\frac{2pt}{12}\right)$$

$$H(t) = a_0 f_0 + a_1 f_1 + a_2 f_2$$

where, $a_0 = h_0,\ f_0 = 1,\ f_1 = \sin\left(\frac{2pt}{12}\right),\ f_2 = \cos\left(\frac{2pt}{12}\right)$

t	f_0	f_1	f_2	$H(t)$
0	1	0	1	1
2	1	0.866	0.5	1.6
4	1	0.866	-0.5	1.4
6	1	0	-1	0.6
8	1	-0.866	-0.5	0.2
10	1	-0.866	0.5	0.8

$$\begin{bmatrix} <f_0, f_0> & <f_1, f_0> & <f_2, f_0> \\ <f_0, f_1> & <f_1, f_1> & <f_2, f_1> \\ <f_0, f_2> & <f_1, f_2> & <f_2, f_2> \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} <H, f_0> \\ <H, f_1> \\ <H, f_2> \end{bmatrix}$$

$<f_0, f_0> = 6(1)^2 = 6$

$<f_0, f_1> = 0$

$<f_0, f_2> = 0$

$<f_1, f_0> = 0$

$<f_1, f_1> = 3$

$<f_1, f_2> = 0$

$<f_2, f_0> = 0$

$<f_2, f_1> = 0$

$<f_2, f_2> = 3$

$<H, f_0> = 5.6$

$<H, f_1> = 1.732$

$<H, f_2> = 0.8$

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 5.6 \\ 1.732 \\ 0.8 \end{bmatrix}$$

$$6a_1 = 5.6$$
$$a_1 = 0.933$$

$$3a_2 = 1.732$$
$$a_2 = 0.577$$

$$3a_3 = 0.8$$
$$a_3 = 0.267$$

$$H(t) = a_1 f_1 + a_2 f_2 + a_3 f_3$$
$$= a_1(1) + a_2 \sin\left(\frac{2pt}{12}\right) + a_3 \cos\left(\frac{2pt}{12}\right)$$
$$= 0.933(1) + 0.577 \sin\left(\frac{2pt}{12}\right) + 0.267 \cos\left(\frac{2pt}{12}\right)$$

Q 4.

$$m = \frac{m_{\max} x}{k_m + x + k_1 x^2}$$

$$\frac{1}{m} = \frac{k_m + x + k_1 x^2}{m_{\max} x}$$

$$\frac{1}{m} = \frac{k_m}{m_{mzx} x} + \frac{1}{m_{\max} x} + \frac{k_1 x^2}{m_{\max} x}$$

$$\frac{1}{m} = \frac{1}{m_{\max}} + \frac{1}{m_{\max}} \frac{1}{x} + \frac{k_1}{m_{\max}} x$$

$$\frac{1}{m} = a_1 f_1 + a_2 f_2 + a_3 f_3$$

where, $a_1 = \dfrac{1}{m_{\max}}$ $\quad , f_1 = 1$

$a_2 = \dfrac{k_m}{m_{\max}}$ $\quad , f_2 = \dfrac{1}{x}$

$a_3 = \dfrac{k_1}{m_{\max}}$ $\quad , f_3 = x$

f_1	f_2	f_3	$\frac{1}{m}$
1	10	0.1	4.167
1	6.67	0.15	3.704
1	4	0.25	2.941
1	2	0.5	2.857
1	1.333	0.75	2.857
1	1	1.00	2.941
1	0.667	1.5	3.030
1	0.333	3.0	4.545

$$\begin{bmatrix} <f_1,f_1> & <f_2,f_1> & <f_3,f_1> \\ <f_1,f_2> & <f_2,f_2> & <f_3,f_2> \\ <f_1,f_3> & <f_3,f_2> & <f_3,f_3> \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} <\frac{1}{m},f_1> \\ <\frac{1}{m},f_2> \\ <\frac{1}{m},f_3> \end{bmatrix}$$

$<f_1,f_1>=8(1)^2=8$

$<f_1,f_2>=<f_2,f_1>=26.003$

$<f_1,f_3>=<f_3,f_1>=7.25$

$<f_2,f_2>=167.82$

$<f_2,f_3>=<f_3,f_2>=7.9$

$<f_3,f_3>=13.1575$

$<\frac{1}{m},f_1>=27.042$

$<\frac{1}{m},f_2>=94.126$

$<\frac{1}{m},f_3>=26.399$

$$\begin{bmatrix} 8 & 26 & 7.25 \\ 26 & 167.82 & 7.9 \\ 7.25 & 7.9 & 13.158 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 27.042 \\ 94.126 \\ 26.399 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 8 & 26 & 7.25 & 27.042 \\ 0 & 24.346 & -4.88 & 1.195 \\ 0 & -17.31 & 7.224 & 1.99 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 8 & 26 & 7.25 & 27.042 \\ 0 & 24.346 & -4.88 & 1.195 \\ 0 & 0 & 3.63 & 2.61 \end{array}\right]$$

$$a_2 = 0.73 = x_3$$

$$a_1 = \frac{1}{U_{22}}(g_2 - U_{23}x_3) = 0.21 = x_2$$

$$a_3 = \frac{1}{U_{11}}(g_1 - U_{12}a_2 - U_{13}x_3) = x_1$$

$$a_0 = \frac{1}{m_{\max}}, m_{\max} = 0.49$$

$$a_1 = \frac{k_m}{m_{\max}}, k_m = 0.103$$

$$a_3 = \frac{k_1}{m_{\max}}, k_1 = 0.36$$

$$m = \frac{0.49}{2.103 + x + 0.36x^2}$$

Since, $a_1 = \dfrac{1}{m_{\max}} \Rightarrow m_{\max} = 0.494$

$a_2 = \dfrac{k_m}{m_{\max}} \Rightarrow k_m = (0.212)(0.494) = 0.10$

$a_3 = \dfrac{k_1}{m_{\max}} \Rightarrow k_1 = 0.363$

Q 5.

$$M = a_1 f_1 + a_2 f_2$$

where, M = molecular weight

a_1 = atomic weight of nitrogen

a_2 = atomic weight of oxygen

f_1 = number of atoms of nitrogen

f_2 = number of atoms of oxygen

	$f_0(N_2)$	$f_1(O_2)$	M
NO	1	1	30.006
N_2O_3	2	3	76.012
N_2O	2	1	44.013
N_2O_5	2	5	108.010
NO_2	1	2	46.006
N_2O_4	2	4	92.011

$$\begin{bmatrix} <f_1,f_1> & <f_2,f_1> \\ <f_1,f_2> & <f_2,f_2> \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} <M,f_1> \\ <M,f_2> \end{bmatrix}$$

$$<f_1,f_1> = (1)^2 + (2)^2 + (2)^2 + (2)^2 + (1)^2 = 18$$
$$<f_1,f_2> = <f_2,f_1> = (1) + (2)(3) + (2)(1) + (2)(5) + (1) + (2)(4) = 29$$
$$<f_2,f_2> = (1)^2 + (3)^2 + (1)^2 + (5)^2 + (2)^2 + (4)^2 = 56$$
$$<M,f_1> = (1)(30.006) + (2)(76.012) + (2)(44.013) + 2(108.010) + 1(46.006) + 2(92.011)$$
$$= 716.104$$
$$<M,f_2> = (1)(30.006) + (3)(76.012) + (1)(44.013) + 5(108.010) + 2(46.006) + 4(92.011)$$
$$= 1302.151$$

$$\begin{bmatrix} 18 & 29 \\ 29 & 56 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 716.104 \\ 1302.251 \end{bmatrix}$$
$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{167}\begin{bmatrix} 18 & -29 \\ -29 & 56 \end{bmatrix}\begin{bmatrix} 716.104 \\ 1302.161 \end{bmatrix}$$
$$= \frac{1}{167}\begin{bmatrix} 2339.155 \\ 2671.88 \end{bmatrix} = \begin{bmatrix} 14 \\ 15.9 \end{bmatrix}$$

Q 6.

$$C(t) = C_{SS} + ae^{-0.47t} + be^{-0.06t}$$
$$C(t) - C_{SS} = ae^{-0.47t} + be^{-0.06t}$$
$$\hat{y} = af_1 + bf_2$$

where, $\hat{y} = C(t) - C_{SS}$

$f_1 = e^{-0.47t}$, $\quad f_2 = e^{-0.06t}$

t	f_1	f_2	$\hat{y}$
3	0.244	0.835	2
9	0.146	0.583	2.2
12	3.553×10^{-3}	0.487	1.8
18	2.118×10^{-4}	0.34	1.3
24	1.262×10^{-5}	0.237	1.0
30	7.524×10^{-7}	0.165	0.6

To find a and b, we have

$$\begin{bmatrix} <f_1,f_1> & <f_2,f_1> \\ <f_1,f_2> & <f_2,f_2> \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} <\hat{y},f_1> \\ <\hat{y},f_2> \end{bmatrix}$$

$$\begin{bmatrix} 0.06 & 0.214 \\ 0.214 & 1.473 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0.5268 \\ 4.6072 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{0.0426}\begin{bmatrix} 1.473 & -0.214 \\ -0.214 & 0.06 \end{bmatrix}\begin{bmatrix} 0.5268 \\ 4.6072 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -4.93 \\ 3.844 \end{bmatrix}$$

$$C(t) - C_{SS} = -4.93e^{-0.47t} + 3.844e^{-0.06t}$$

$$C'(t) = -4.93e^{-0.47t}(-0.47) + 3.844e^{-0.06t}(-0.06)$$

$$C'(t) = 2.3171e^{-0.47t} - 0.230e^{-0.06t}$$

$$0 = 2.3171e^{-0.47t} - 0.230e^{-0.06t}$$

$$2.3171e^{-0.47t} = 0.230e^{-0.06t}$$

$$e^{-0.47t} = 10.048e^{-0.06t}$$

$$-0.06t = \ln 10.048 + (-0.47t)$$

$$0.41t = 2.3074$$

$$t_{\max} = 5.63\text{s}$$