Kyaw Tun
HT016319E
CN 5010: Homework

$x_1' = x_2 - x_3 - t$

$x_2' = x_1 + x_2 - t^2$

$x_3' = x_1 + x_3 - 1 + t$

$t = 0 \Rightarrow x_1 = 1, x_2 = -1, x_3 = 0$

In matrix form,

$$\frac{d}{dt}\mathbf{X} = \mathbf{AX} + \mathbf{B}(t) \qquad , \mathbf{B}_0 = \mathbf{B}(0)$$

Where; $\mathbf{X} = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \qquad \mathbf{B}(t) = \begin{bmatrix} -t \\ -t^2 \\ -1+t \end{bmatrix}, \qquad \mathbf{B}_0 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

$f(\lambda) = \lambda^3 - \lambda = 0$

$$\lambda_1 = 0, \lambda_2 = 1, \lambda_3 = 1$$

$$adj(\mathbf{A} - \lambda_1 \mathbf{I}) = adj\begin{bmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 \\ -1 & -1 & 1 \\ 1 & -1 & -1 \end{bmatrix}^T$$

$$\mathbf{v}_1 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$adj(\mathbf{A} - \lambda_2 \mathbf{I}) = adj\begin{bmatrix} -1 & 1 & -1 \\ 1 & -2 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}^T$$

$$\mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\mathbf{J} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$\mathbf{AP} = \mathbf{PJ}$

$$\begin{bmatrix} 0 & 1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 & p_1 \\ 1 & 1 & p_2 \\ 1 & 1 & p_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & p_1 \\ 1 & 1 & p_2 \\ 1 & 1 & p_3 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \qquad \mathbf{P}^{-1} = \begin{bmatrix} -1 & 1 & -1 \\ 1 & -1 & 2 \\ 0 & 1 & -1 \end{bmatrix}$$

$$\mathbf{Z}_0 = \mathbf{P}^{-1}\mathbf{B}_0(t) = \begin{bmatrix} -1 & 1 & -1 \\ 1 & -1 & 2 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$$

$$\frac{d}{dt}\mathbf{X} = \mathbf{AX} + \mathbf{B}(t) \qquad , \mathbf{B}_0 = \mathbf{B}(0)$$

Let **X = PZ**

X' = AY + B(t)

PZ' = APZ + B(t)

Z' = DZ + P^{-1} **B**(t)

$$\mathbf{P}^{-1}\mathbf{B}(t) = \begin{bmatrix} -1 & 1 & -1 \\ 1 & -1 & 2 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} -t \\ -t^2 \\ -1+t \end{bmatrix} = \begin{bmatrix} -t^2+1 \\ t^2-2+t \\ 1-t-t^2 \end{bmatrix}$$

$$\frac{\partial}{\partial t}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} -t^2+1 \\ t^2-2+t \\ 1-t-t^2 \end{bmatrix}$$

$$\frac{\partial z_1}{\partial t} = -t^2 + 1$$

$$\frac{\partial z_2}{\partial t} = z_2 + z_3 - 2 + t + t^2$$

$$\frac{\partial z_3}{\partial t} = z_3 + 1 - t - t^2$$

$$z_1 - 0 = \int_0^t (-t'^2 + 1)\,dt' \qquad \Leftarrow z_1(0) = 0$$

$$z_1 = -\frac{t'^3}{3} + t' \Big]_0^t$$

$$z_1 = -\frac{t^3}{3} + t$$

$$z_3 = c_3 e^t + e^t \int_0^t (1 - t' - t'^2) e^{-t'}\, dt'$$

$$z_3 = c_3 e^t + e^t \left[e^{-t'} \left(t'^2 + 3t' + 2 \right) \right]_0^t$$

$$z_3 = c_3 e^t + \left(t^2 + 3t + 2 \right) - 2e^t$$

$$z_3 = -3e^t + \left(t^2 + 3t + 2 \right) \qquad \Leftarrow z_3(0) = -1$$

$$\frac{\partial z_2}{\partial t} = z_2 - 3e^t + \left(t^2 + 3t + 2 \right) - 2 + t + t^2$$

$$\frac{\partial z_2}{\partial t} = z_2 - 3e^t + 4t + 2t^2$$

$$z_2 = c_2 e^t + e^t \int_0^t e^{-t} \left(-3e^{t'} + 4t' + 2t'^2 \right) dt'$$

$-3t + ã^{-t}$ ■ $8 - 8$■$t - 2t^2$ ■

$$z_2 = c_2 e^t + e^t \left[-3t' + e^{-t'} (-8 - 8t' - 2t'^2) \right]_0^t$$

$$z_2 = c_2 e^t + e^t \left(-3t + e^{-t} (-8 - 8t - 2t^2) + 8 \right)$$

$$z_2 = c_2 e^t - 8 - 8t - 2t^2 + 8e^t - 3te^t$$

$$z_2 = -8 - 8t - 2t^2 + 10e^t - 3te^t \qquad z_2(0) = 2$$

$\mathbf{X} = \mathbf{PZ}$

$$\mathbf{X} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -\frac{t^3}{3} + t \\ -8 - 8t - 2t^2 + 10e^t - 3te^t \\ -3e^t + \left(t^2 + 3t + 2 \right) \end{bmatrix}$$

$$x_1 = -\left(-\frac{t^3}{3} + t \right) - 3e^t + \left(t^2 + 3t + 2 \right)$$

$$x_1 = \frac{t^3}{3} - 3e^t + \left(t^2 + 2t + 2 \right)$$

$$x_2 = -\frac{t^3}{3} + t - 8 - 8t - 2t^2 + 10e^t - 3te^t - 3e^t + \left(t^2 + 3t + 2\right)$$

$$x_2 = -\frac{t^3}{3} - 6 - 4t - t^2 - 7e^t - 3te^t$$

$$x_3 = -\frac{t^3}{3} + t - 8 - 8t - 2t^2 + 10e^t - 3te^t$$

$$x_3 = -\frac{t^3}{3} - 8 - 7t - 2t^2 + 10e^t - 3te^t \qquad \textit{Ans.}$$