# A Quantum Clustering based Neural Control Scheme for Visual-Motor Coordination

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## Abstract

*Visual-Motor Coordination is a problem considered analogous to the hand-eye coordination in biological systems. In this work we propose a novel approach to this problem using Quantum Clustering and an extended Kohonen's Self Organizing Feature Map (K-SOFM). This facilitates the use of the method in varying workspaces by considering the joint angles of the robot arm. Unlike previous work, where a fixed topology for the input space is considered, the proposed approach determines a topology as the workspace varies. Quantum Clustering is a method which constructs a scale-space probability function and uses the Schroedinger Equation and its lowest eigenstate to obtain a potential whose minimum gives the cluster centers. It transforms the input space into a Hilbert space, where it searches for its minimum. The motivation of this work is to identify the implicit relationship existing between the end-effector positions and the joint angles through Quantum Clustering and Neural Network methods to fine-tune the system to correctly identify the mapping.*

## 1. Introduction

This work introduces a new method based on Quantum Clustering for the visual-motor coordination of a Robotic Arm. Previous works in this area [4, 5, 6, 11, 8] gave a "black-box" like system with high training time and fixed topological space. Walter and Schulten [11] extended the Self-Organizing Feature Map Algorithm to give an improved result and added flexibility in the robot parameters. However, the fixed topology of the feature map and strong dependence on the training parameters could not be avoided. Behera and Kirubanandan [1] introduced the idea of a hybrid approach, based on Kohonen's Self Organizing Maps with an online training scheme and fixed topology. To solve the issue of fixed topology and to involve the implicit relationship between the input space and the output space, we propose a novel technique using Quantum Clustering (QC). QC was originally proposed by Horn and Gottlieb [3] and is used to determine the necessary topology of the network for the workspace.

## 2. Quantum Clustering

Quantum clustering is a non-parametric clustering technique [7] based on the scale-space algorithm [9] using the Gaussian kernel to generate the Parzen-window estimator [2]. Given a set of N data points, $x_1, \ldots, x_N$, the task is to estimate the probability density function $\Psi$(x) using the parzen-window estimator. The estimator is constructed by associating with each of the N data points a Gaussian defined as in Equation 1.

$$\Psi(x) = \sum_{i=1}^{N} e^{-(x-x_i)/2\sigma^2} \tag{1}$$

The maxima of the function $\Psi$(x) have been shown to occur at the cluster centers [9], where $1/2\sigma^2$ is the scale of the probability estimator. The function $\Psi$(x) is the Schroedinger Wave Function and performs a nonlinear transformation of the input space into a Hilbert Space. It can be seen that as $\Psi$(x) is a Gaussian kernel, it is positive definite and hence a standard inner product defined in the transformed space is also positive definite. That is, by Mercer's Theorem [10] , the nonlinear transformation $\Psi$(x) defines a Hilbert Space.

The motivation is to search for a Hamiltonian for which $\Psi$ is an eigenstate and a ground state of the operator $H$.

$$H\Psi = (-\frac{1}{2\sigma^2}\nabla^2 + V(x))\Psi = E\Psi \tag{2}$$

Equation 2 is a rescaled form of the Schrodinger Equation in Quantum Mechanics, with $\Psi$ denoting the eigenstate, $H$ the Hamiltonian, $V$ the potential energy and $E$ is the energy eigenvalue. It must be noted that $\sigma$ is the only parameter in the equation. $\sigma$ deduces the correct clustering of the space

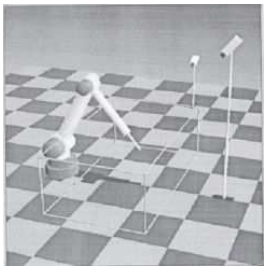

Figure 1: Schematic Diagram of the setup.

as it controls the width of the Parzen-window [9]. The parameter $\sigma$ can be controlled in a way such that the technique yields the relevant number of clusters.

Given $\Psi$, one can solve Equation 2 for $V$:

$$V(x) = E + \frac{(\frac{1}{2\sigma^2})\nabla^2\Psi}{\Psi} \tag{3}$$

If $V$ is positive definite, that is V$\geq 0$, $E$ maybe defined as in Equation 4.

$$E = -min\frac{(\frac{1}{2\sigma^2})\nabla^2\Psi}{\Psi} \tag{4}$$

As $\Psi$ is positive definite, it follows that it has no node and hence $E$ is the minimal eigenvalue of $V$. The lowest possible eigenvalue occurs for the harmonic potential in which case $E = \frac{d}{2}$. This leads to the inequality 5.

$$0 < E \leq \frac{d}{2} \tag{5}$$

Quantum Clustering, thus works by determining the cluster centers, with the data points assigned to a cluster based on the Euclidean Distance. In the problem of visual-motor coordination, we are interested in locating the cluster centers and hence the use of QC is preferred.

# 3 Visual-Motor Coordination Control

## 3.1 The Problem

A pair of two-dimensional 'retinal' coordinates $u_1$, $u_2$ are obtained from the two cameras respectively as shown in Figure 1. The objective is to identify the transformation $\Theta$(u) from the retinal coordinates to the joint angles $\theta$ of the three-joint arm. Here $u_{target}$ is a 4-dimensional vector obtained by grouping the retinal coordinates $u_1$and $u_2$. In this work, we consider $u_{target}$ as a 3-dimensional vector by taking into account the actual 3-D location of the end-effector in the workspace rather than through the transformed camera-plane. The first order Taylor expansion of $\Theta(\mathrm{u}_{target})$ is given by Equation 6.

$$\Theta(u_{target}) = \theta_s + A_s(u_{target} - w_s) \tag{6}$$

The idea is to discretize the workspace $U$, into nonoverlapping regions $F_r$, such that $\mathrm{w}_r \in F_r$ is the reference or weight vector, $\theta_r$ is the zero-order term and $A_r$ is the Jacobian Matrix, which determines the first order term. Equation 6 gives the local approximation of each neuron for the joint angle values of the target point $u_{target}$.

## 3.2 Discretizing the space using Quantum Clustering

We simulate the workspace using the inverse kinematics relationship between the end-effector position in the 3-dimensional world space and the joint angles. Thus, in our simulations, the end-effector position $u$ is a 3-dimensional vector. Given a set of end-effector positions $u$ and their corresponding joint-angles $\theta$ from the workspace, we consider a six dimensional vector z by grouping $u$ and $\theta$. The motivation for the proposed approach can be understood if we observe from Figure 2 that the output space formed by the joint angles is clustered into four well defined regions. Thus, the vector z also forms a clustered space. This implicit relationship between the workspace and the joint-angle space when considered when initializing and training the network, greatly improves the capability of the network while decreasing the complexity of the network topology.

Using Quantum Clustering, we obtain the cluster centers in the space represented by z. The initial values for $w_r$ and $\theta_r$ are then extracted back from the six dimensional cluster centers. It should be noted that though a given joint angle determines a unique end-effector position, it is not true otherwise. An end-effector position maybe realized by more than one joint-angle values. This implicit information must be accounted for while preparing the network. This forms one of the motivation for resorting to the proposed clustering method. It has been observed that such a selection needs a small fine-tuning of the cluster centers and hence has a very fast learning time. In addition, by an adaptive selection of clusters from the workspace, we have reduced the number of neurons significantly.

## 3.3 QC vs KSOM

In the Kohonen's Self-organizing Feature Map based method, a fixed topology for the space is used by creating a 3-dimensional lattice of neurons. These neurons then organize themselves homogeneously so that the workspace is well spanned and the Equation 6 can be used to identify the

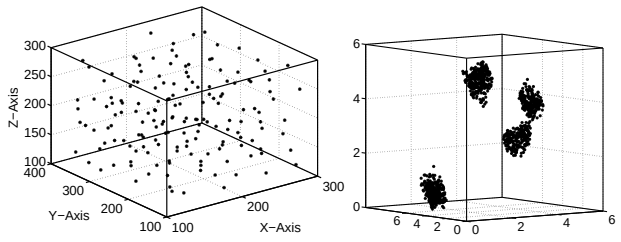


Figure 2: Plot of the reference centers of different neurons and corresponding joint angles obtained using QC-based method.

joint angles for a given end-effector position. This has two major disadvantages:

- The number of neurons have to be pre-decided and fixed, thus allowing little flexibility.
- The topology of the workspace is fixed once the lattice parameters are specified.

A fixed topology of the workspace is not necessary because that constraints the neurons to occur uniformly. The receptive field dimensions of all neurons in the visual system is not same. The proposed method has the same efficiency using almost half the neurons used in the K-SOM based approach and a flexible topology. Figure 2 shows the distribution of points in a workspace and the corresponding points in the joint-angle space.

# 4 The Neural Learning Algorithm

The Learning Algorithm adopted in the present work is motivated by the extended Kohonen's Self-Organizing Feature Maps introduced by Walter and Schulten in [11]. Given an end-effector position target $u_{target}$, a winner neuron $\mu$ is selected, based on the Euclidean distance metric in the workspace. The neuron whose reference vector is closest to the target is declared winner. The arm is given a coarse movement, $\theta_0^{out}$ by the initial output of the network determined by Equation 7 resulting in the end-effector to move to a position $v_0$. This if followed by a fine movement determined by $\theta_1^{out}$, using Equation 8, giving a correcting movement to the arm and the end-effector reaching the final position of $v_1$. It has been shown that a series of similar corrective actions will converge to the correct end-effector position. However, we use only one corrective fine movement and achieve a considerable amount of accuracy.

The collective averaged output is evaluated using Equation 7.

$$\theta_0^{out} = s^{-1} \cdot \sum_k h_k^{mix}(\theta_k + A_k(u_{target} - w_k)) \qquad (7)$$

where $s = \sum_k h_k^{mix}$ and $h_k^{mix} = exp(-\frac{\|\mu - k\|}{2\sigma_{mix}^2})$, with $\mu$ representing the winner neuron such that $w_\mu$ is closest to the vector $u_{target}$.$\theta_0^{out}$ gives a coarse movement to the arm such that the end-effector reaches a position $v_0$. The correcting fine movement is evaluated using Equation 8 resulting in a final movement of the end-effector to $v_1$.

$$\theta_1^{out} = \theta_1^{out} + s^{-1} \cdot \sum_k h_k^{mix} A_k(u_{target} - v_0) \qquad (8)$$

The learning scheme used in the present work can be grouped as under:

$$\Delta v = v_1 - v_0 \qquad (9)$$
$$\Delta\theta^{out} = \theta_1^{out} - \theta_0^{out} \qquad (10)$$
$$\Delta\theta_k = \theta_0^{out} - \theta_k^{out} - A_k(v_0 - w_k) \qquad (11)$$
$$\Delta A_\mu = \|\Delta v\|^{-2} \cdot (\Delta\theta^{out} - A_\mu \cdot \Delta v)\Delta v^T \qquad (12)$$
$$w_k \leftarrow w_k + \epsilon \cdot h_{\mu k} \cdot (u_{target} - w_k) \qquad (13)$$
$$\theta_k \leftarrow \theta_k + \epsilon' \cdot h'_{\mu k} \cdot \Delta\theta_k \qquad (14)$$
$$A_k \leftarrow A_k + \epsilon' \cdot h'_{\mu k} \cdot \Delta A_k \qquad (15)$$

The functions $h_{\mu k}$ and $h'_{\mu k}$ are defined as:

$$h_{\mu k} = exp(-\frac{\|\mu - k\|}{2\sigma^2}) \qquad (16)$$
$$h'_{\mu k} = exp(-\frac{\|\mu - k\|}{2\sigma'^2}) \qquad (17)$$

with $\mu$ and k representing the three dimensional indices associated with the corresponding neuron. The parameters $\epsilon$, $\epsilon'$, $\sigma$, $\sigma'$ and $\sigma_{mix}$ vary during the training time depending on the current iteration and can be expressed using the general expression as below.

$$\eta = \eta_{initial}(\frac{\eta_{final}}{\eta_{initial}})^{(\frac{t}{t_{max}})} \qquad (18)$$

In Equation 18, $\eta \in \{\epsilon, \epsilon', \sigma, \sigma', \sigma_{mix}\}$, t is the current iteration and $t_{max}$ is the total number of iterations to be performed by the network.

## 4.1 Indexing scheme for the adaptive topology of extended KSOM

In this work, we use an adaptive scheme for determining the topology of the workspace using Quantum Clustering of the joint space formed by grouping the end-effector positions and the corresponding joint angles. After the clustering step, a set of points, $w_1 \ldots w_N$, in the workspace are obtained, which are the potential cluster centers and hence denote the reference or weight vector for the neurons. The index of each neuron is then obtained using a normalized and scaled version of the corresponding weight vector. The normalization is carried out by dividing each dimension by the corresposnding linear dimension of the workspace. It should be noted that in the proposed approach, the index

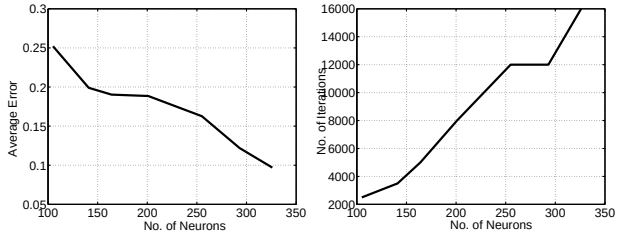


Figure 3: Plot of the Average Error and No. of iterations required as the number of neurons varies.

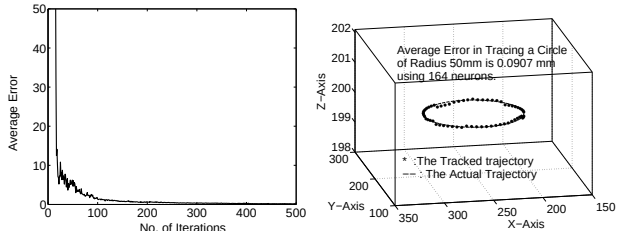


Figure 4: Plot of the Average Error with the number of iterations for 164 neurons and the tracked circle using the same.

values are real values and not necessarily integers, unlike the method in [6] and [11]. The motivation for the proposed indexing scheme is that it represents both the actual and the relative location of neurons in the workspace.

# 5 Implementation and Results

## 5.1 Parameters and Initialization

The workspace is defined by a region 200mm X 300mm X 200mm. The arm lengths are all equal to 254mm and the manipulator has a rigid wrist portion having a length equal to 50mm as used in [1]. The initialization of parameters was similar to that in [1]. The values were chosen as $\epsilon_i = 1.0$, $\epsilon_f = 0.05$, $\epsilon_i' = 0.9$, $\epsilon_i' = 0.9$, $\sigma_i = 2.5$, $\sigma_f = 0.01$, $\sigma_i' = \sigma_{mix_i} = 2.5$, $\sigma_f' = \sigma_{mix_f} = 0.01$. The subscript *i* and *f* denote the initial and final values of the corresponding parameter respectively.

## 5.2 Performance and Results

The topology of the workspace is determined by the number of clusters obtained. each cluster denotes the receptive field of a neuron. The number of clusters obtained using Quantum Clustering technique varies with the width parameter $q = \frac{1}{2\sigma^2}$. We observe that larger number of neurons give better results, but also require more training. This is evident from Figure 3. Figure 3 shows that increasing the number of neurons decrease the error. However, for faster training, it is important to have only a sufficient number of neurons. Our simulations lead us to the conclusion that the adaptive toplogy based scheme proposed in this work needs only 164 neurons for the workspace in use. This number can be compared with the work in [1], where 12x7x4 (=336) neurons are used. The error vs. iterations graph is shown in Figure 3 for an adaptive scheme of 164 neurons. Figure 4 shows a circle being tracked in the workspace with an average error of 0.09mm.

# 6 Conclusions

In this work, a new method for the Visual-Motor Coordination Control using an adaptive scheme and a flexible topology for the workspace is proposed. Fewer neurons are used with the receptive field of every neuron having a non-uniform distribution in space. The motor control is carried out through a collection of neurons firing and determining the precise movement, instead of one neuron acting independently. Quantum Clustering used in this work, determines the local maxima of the potential and identifies them as the cluster centers. It should be noted that the method still adopts a classical computation methodology.

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