B **B2B109** **Pages: 3**

Reg. No.____________ Name:________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

SECOND SEMESTER B.TECH DEGREE EXAMINATION, MAY 2017

Course Code: **BE 100**

Course Name: **ENGINEERING MECHANICS**

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions. 5 marks each.

1. State and explain the principle of Transmissibility of force with sketches.
2. The x,y,z components of a force are 36kN, -24kN and 24kN respectively. Find the components of this force along the line joining A(1,2,-3) and B(-1,-2,2).
3. Using first theorem of Pappus and Guldinus, find the surface area of a i) Sphere and ii) right circular cone.
4. Distinguish between (i) Static and kinetic frictions,(ii) Sliding friction and rolling friction and (iii) angle of friction and angle of repose.
5. What do you mean by instantaneous centre of rotation? How can it be located for a body moving with combined motion of rotation and translation? Refer Jul 2016
6. What is a seconds's pendulum? Derive expressions for the loss or gain of time due to changes in length of string and gravitational acceleration in the case of a simple pendulum.
7. Define the terms amplitude, period of oscillation and frequency in a simple harmonic motion.
8. State D'Alemberts principle giving equations expressing the above Principle on the motion of a lift moving upwards with an acceleration 'a' m/sec^2 carrying a weight of 'W' N.

PART B

Answer any 2 questions from each SET.

SET 1

Each question carries 10 marks.

9. ABCD is a rectangle in which AB=30mm, BC= 20mm. 'E' is the middle point of 'AB'. Forces of magnitude 16,14,18,8,10 and 20N act along 'AB', 'BC', 'CD', 'DA', 'EC' and 'DE' respectively. Find the magnitude, direction and position with respect to 'ABCD' of single force to keep the body in equilibrium. 'B' is to right of 'A' and taken in anticlockwise direction.
10. a. A cylindrical road roller of weight 1000N and radius 30 cm is pulled by a force 'F' through the centre of the wheel. While moving it comes across an obstacle of height 15 cm. Calculate the least force 'F' required to cross the obstacle . (5)

    b. A force acts at the origin of a co-ordinate system in a direction defined by the angles α_x=69.3^0 and α_z=57.9^0.Knowing that the 'Y' component of the force is -174N, determine the (i) angle α_y and (ii) the other components and the magnitude of the force. (5)

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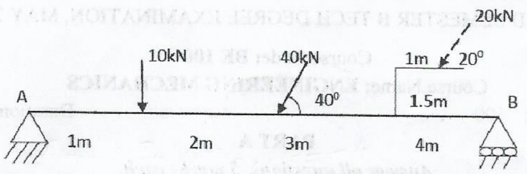
11.

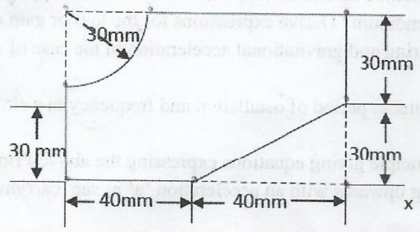


Determine the reactions at supports for the beam. (10)

SET II

Each question carries 10 marks.

12. a. Calculate the moment of inertia and radius of gyration about X axis for the sectioned area.



(7)

b. An area 'A' has the following properties, $I_x = 6.4 \times 10^6\ mm^4$, $I_y = 16 \times 10^6\ mm^4$ and $I_{xy} = 6.4 \times 10^6\ mm^4$. Calculate maximum and minimum principal moment of inertia. (3)

13. a. A ladder 5 m long and weighing 260 N is placed against a vertical wall at an inclination of 30^0 with wall. A man weighing 780 N climbs the ladder. When he is at a distance of 1.64 m along the ladder from lower end, the ladder slips, What is the coefficient of friction assuming it to be same for all contact surfaces? (5)

b. A simply supported beam of span 5 m is loaded with a concentrated load of 4kN at a distance of 1 m from right end. The beam is also loaded with a uniformly distributed load of 2kN/m length over a distance of 2m from the left end of the beam. Find the reactions at the supports of the beam using principle of virtual work. (5)

14. a. Explain with sketches how the forces involved in the lifting of a load by a wedge are analysed. (5)

b. Determine the product of inertia of the sectioned area about the X-Y axis. (5)

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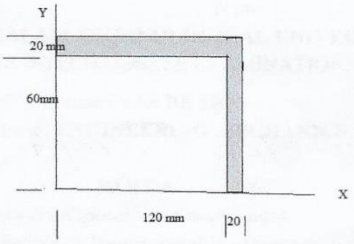


SET III

Each question carries 10 marks.

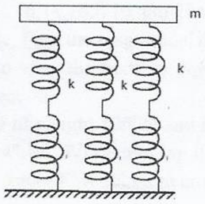
15. a. A lift is ascending with an acceleration of $0.6 m/sec^2$. A man holds a spring balance from which a parcel weighing 13.6 N is hung. What will be the reading in the spring balance? (3)

b. The crank 'OA' of a reciprocating engine mechanism is of length 15 cm and rotating at 600 rpm. The connecting rod 'AB' is 70cm long. Find the (i) angular velocity of the connecting rod, (ii) velocity of the piston 'B' and (iii) the velocity at a point 'C' on the connecting rod at a distance 50cm from the piston end when the crank makes an angle 45^0. (7)

16. a. In a system the amplitude of the motion is 1.6m and time period is 4 sec. find the time required for the particle in passing between points which are at a distance of 1.2m and 0.6m from the centre of force and are on the same side of it. (7)

b. A roller of radius 12 cm rides between two horizontal bars moving in opposite directions with velocities 2.88 m/sec and 1.92 m/sec. Calculate the distance defining the position of the path of the instantaneous centre of rotation of the roller. Assume no slip at points of contacts. (3)

17. a. A tray of mass 'm' is mounted on three identical springs as shown. The period of vibration of empty tray is 0.5 sec. After placing a mass of 1.5 kg on the tray, the period was observed to be 0.6 sec. Find the mass of the tray and stiffness of each spring. (5)



b. A body performing simple harmonic motion completes 8 oscilations in one minute. The velocity of the body is half the maximum velocity at a distance of 12 cm from the centre. Determine the amplitude and maximum acceleration. (5)

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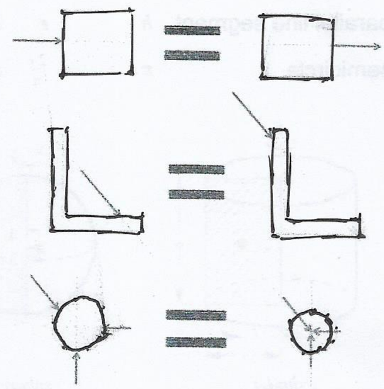
B. E. 100 E. M. May 2017

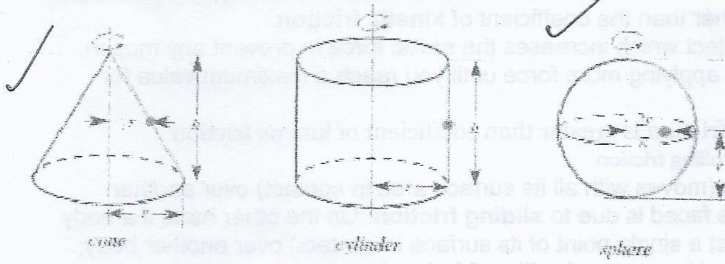
1. State and explain the principle of transmissibility of forces with sketches

The **principle of transmissibility states** that the point of application of a force can be moved anywhere along its line of action without changing the external reaction forces on a rigid body



3 Using first theorem of Pappus and Guldinus, find the surface area of a I) sphere and ii) right circular cone

Pappus's Centroid Theorem

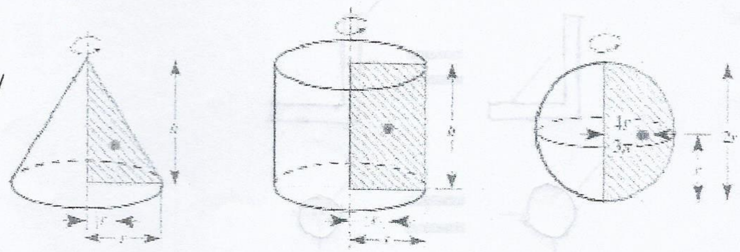


The first theorem of Pappus states that the surface area S of a surface of revolution generated by the revolution of a curve about an external axis is equal to the product of the arc length s of the generating curve and the distance d_1 traveled by the curve's geometric centroid $\bar{x}$,

$$S = s\, d_1 = 2\pi s \bar{x}$$

The following table summarizes the surface areas calculated using Pappus's centroid theorem for various surfaces of revolution.

solid	generating curve	L	$\bar{x}$	S
cone	inclined line segment	$\sqrt{r^2+h^2}$	$\frac{1}{2}r$	$\pi r\sqrt{r^2+h^2}$
cylinder	parallel line segment	h	r	$2\pi r h$
sphere	semicircle	πr	$\frac{2r}{\pi}$	$4\pi r^2$



cone cylinder sphere

4. Distinguish between

4.) i) static and kinetic friction

Static friction is friction between two or more solid objects that are not moving relative to each other. For example, **static friction** can prevent an object from sliding down a sloped surface. The coefficient of **static friction**, typically denoted as μ_s, is usually higher than the coefficient of **kinetic friction**.
rictionmoving an object which increases the **static** force to prevent any motion. However, you keep applying more force until you reach a maximum value for the **static friction**.
coefficient of static friction is greater than coefficient of kinetic friction .

ii) sliding friction and rolling friction

When a body slides (moves with all its surface area in contact) over another body, the resistance faced is due to **sliding friction**. On the other hand if a body rolls (moves with just a single point of its surface in contact) over another body, the resistance offered is a case of **rolling friction**

iii) Angle of friction and angle of repose

Angle of Friction is the angle between the normal force (N) and the resultant force (R) of normal force and friction, whereas Angle of Repose is the angle of maximum slope, where an object placed just begins to slide. In other words we can say that it is that maximum angle where a body or object can be tilted from the horizontal without falling or collapsing. to the density, surface area and shapes of the particles, and the coefficient of **friction** of the material

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Part - A.

2- Given

$F_x = 36 kN$, $F_y = -24 kN$, $F_z = 24 kN$

$A(1, 2, -3)$, $B(-1, -2, 2)$

The position vector of a line connecting two points A & B

$$\vec{r}_{AB} = (x_B - x_A)\hat{i} + (y_B - y_A)\hat{j} + (z_B - z_A)\hat{k}$$

$$= (-1-1)\hat{i} + (-2-2)\hat{j} + [2-(-3)]\hat{k}$$

$$\vec{r}_{AB} = -2\hat{i} - 4\hat{j} + 5\hat{k}$$

The length of the line AB $= \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$

$$|AB| = \sqrt{2^2 + 4^2 + 5^2} = \sqrt{4 + 16 + 25} = \sqrt{45} = 6.708 \to = 6.71$$

Unit vector along AB $= \dfrac{-2\hat{i} - 4\hat{j} + 5\hat{k}}{6.71}$

$$\vec{e}_{AB} = -0.3\hat{i} - 0.6\hat{j} + 0.75\hat{k}$$

$$F^2 = F_x^2 + F_y^2 + F_z^2 = 36^2 + (-24)^2 + 24^2 =$$

$$= 1296 + 576 + 576 = 2448$$

$$|F| = \sqrt{2448} = 49.477 \text{ kN}$$

$$\therefore \bar{F} = |F|\,\hat{e}_{AB} = 49.477(0.3\hat{i} - 0.6\hat{j} + 0.75\hat{k})$$

$$\boxed{\vec{F} = -14.84\hat{i} - 29.69\hat{j} + 37.1\hat{k}}$$

3

5. Refer Jul 2016

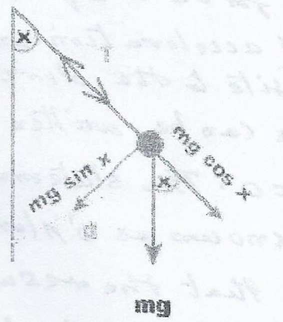
6. What is a seconds pendulum?

A **seconds pendulum** is a **pendulum** whose period is precisely two **seconds**; one **second** for a swing in one direction and one **second** for the return swing, a frequency of 1/2 Hz. A **pendulum** is a weight suspended from a pivot so that it can swing freely

Derive expression for the loss or gain of time due to changes in length of string and gravitational acceleration in the case of a simple pendulum

Simple Pendulum

When a simple pendulum swings to and fro, the acceleration of its bob is directed towards the centre point of its motion and is proportional to the distance from that point. Therefore, the motion of a simple pendulum is **SHM**.



Vivax Solutions

when the weight of the pendulum bob is resolved, the tension of the string, T, and the **mg cos x** cancel each other out, leaving **mg sin x** as the net force, as shown above. This force is responsible for bringing the bob down in a curved path.

Using F = ma for the bob,

mg sin x = ma, where a is the acceleration of the bob.

If the pendulum swings through a small angle and is measured in **radians**, sin x is almost equal to x.

mg. x = m a

gx = a

g d/l = a (x = d / l radians)
a = (g/l) d
a = k d
a α d
The acceleration of the bob is directly proportional to the distance from the centre point. Therefore, the motion of a simple pendulum is simple harmonic.
$k = \omega^2$ where ω is the angular speed.
$a = \omega^2 d$
$\omega^2 = g/l$
$\omega = \sqrt{\frac{g}{l}}$
If the time period is T,
$T = 2\pi/\omega$

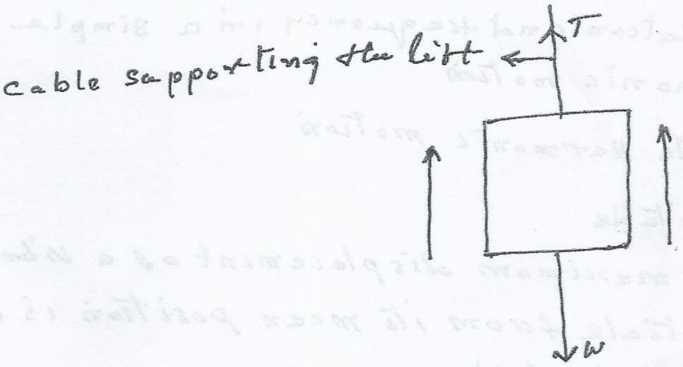
$T = 2\pi \sqrt{l/g}$

8. State D' Alemberts principle giving equations expressing the above principle on the motion of a lift moving upwards with an acceleration g m/s^2 carrying a weight of 'W' N

The magnitude of inertia force is equal to the product of the mass and acceleration and it acts in a direction opposite to the direction of acceleration. $F = ma$ can be written as $F - ma = 0$ or $F + (-ma) = 0$. The statement of the above equation is known as D'Alemberts principle which states that the resultant of a system of force acting on a body in motion is in dynamic equilibrium with the inertia of force.

Motion of a lift

Consider the motion of a lift with acceleration 'a'. Let W be the weight of a man and R be the reaction of force applied by the man on the floor of the lift



Lift is moving upwards.

When the lift is moving upwards, with an acceleration 'a' the inertia of force is downwards

For dynamic equilibrium, $\Sigma F + F_i = 0$

$$R - W - \frac{W}{g} a = 0$$

$$R = W\left(1 + \frac{a}{g}\right)$$

7. Define the terms amplitude, period of oscillation and frequency in a simple harmonic motion (6)

Simple Harmonic motion

Amplitude

The maximum displacement of a vibrating particle from its mean position is called amplitude (A)

It is a vector quantity

period of oscillation (Time period)

The time taken for one oscillation is called time period

$T = \frac{2\pi}{\omega}$, where ω angular velocity

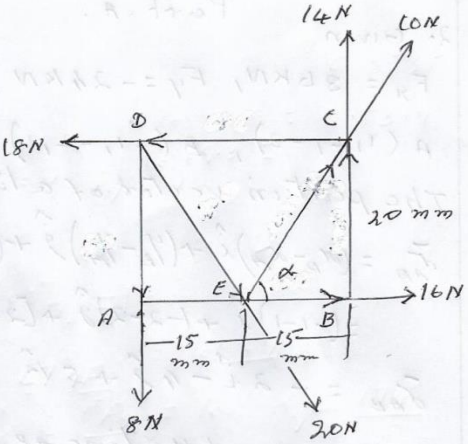
Frequency:

The number of oscillations completed by a particle in one second is called frequency

Frequency $f = \frac{1}{T}$ cycles/sec

part B

9.



$$\alpha = \tan^{-1}\frac{20}{15} = 53.15^\circ$$

$$\Sigma H = 16 - 18 + 10\cos 53.13 + 20\cos 53.13$$

$$= 16 - 18 + 10\times 0.6 + 20\times 0.6 = 16 - 18 + 6 + 12 = 16\ N$$

$$\Sigma H = 16\ N$$

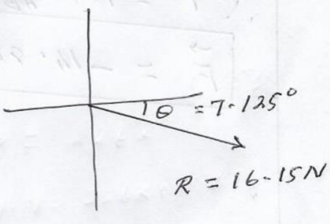
$$\Sigma V = 14 - 8 + 10\sin 53.13 - 20\sin 53.13$$

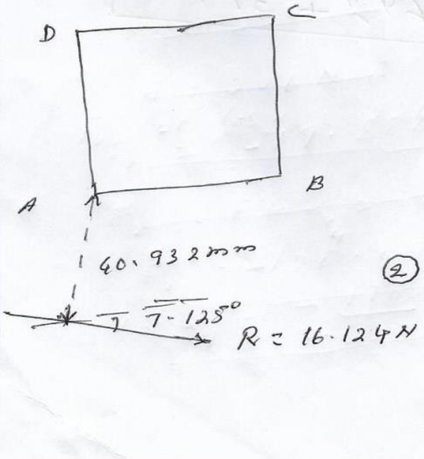
$$= 14 - 8 + 10\times 0.8 - 20\times 0.8 = 14 - 8 + 8 - 16 = -2N$$

$$\Sigma V = -2N$$

$$R = \sqrt{\Sigma H^2 + \Sigma V^2} = \sqrt{(-2)^2 + 16^2} = \sqrt{4+256} = \sqrt{260} = 16.124N$$

$$\theta = \tan^{-1}\frac{|\Sigma V|}{|\Sigma H|} = \tan^{-1}\frac{2}{16} = 7.125^\circ$$





R is in IVth quadrant : $\theta_R = 360 - 7.125 = 352.875^0$

$\theta_R = 352.875^0$

m↺ +ve
m↻ −ve

To find the position

$M_A = -14\times30 - 18\times20 + 20\cos53.13\times20 - 10\sin53.13\times15$

$= -420 - 360 + 240 - 120 = -660$ Nmm

$R \times x = M_A$, x is the perpendicular distance of resultant

$x = \frac{M_A}{R} = \frac{660}{16.124} = 40.932$ mm

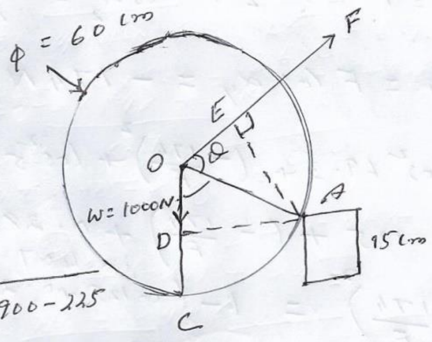
10.a

OC = 30 cm = AO

DC = 15 cm

OD = OC − DC

= 30 − 15 = 15 cm



$AO^2 = OD^2 + AD^2$

$AD = \sqrt{AO^2 - OD^2}$

$= \sqrt{30^2 - 15^2} = \sqrt{900 - 225}$

$AD = \sqrt{675} = 25.98$

$M_A = 0 \rightarrow W \times AD = F \times AE$

$1000 \times 25.98 = F \times OA \sin\theta$

$\therefore F = \frac{1000 \times 25.98}{30 \times 1}$

For F to be min, $\sin\theta$ should be max
$\sin\theta = 1$ or $\theta = 90^0$

$$\boxed{F_{min} = 866\ N}$$

10.6

$\alpha_x = 69.3^\circ$, $\alpha_z = 57.9^\circ$

$F_y = -174\,N$

$\cos 69.3 = 0.3535$

$\cos 57.9 = 0.5314$

To find i) α_y ii) $F_x = ?$, $F_z = ?$

Sol: $\cos^2\alpha_x + \cos^2\alpha_y + \cos^2\alpha_z = 1$

$F_x = F\cos\alpha_x \rightarrow \cos\alpha_x = \frac{F_x}{F} = \cos 69.3$

$F_y = F\cos\alpha_y \rightarrow \cos\alpha_y = \frac{F_y}{F} = \cos\alpha_y = -\frac{174}{F}$

$F_z = F\cos\alpha_z \rightarrow \cos\alpha_z = \frac{F_z}{F} = \cos 57.9$

$F_y = F\cos\alpha_y = -174\,N$ —(1)

$\cos^2 69.3 + \left(-\frac{174}{F}\right)^2 + \cos^2 57.9 = 1$

$0.125 + \left(-\frac{174}{F}\right) + 0.282 = 1$

$\left(-\frac{174}{F}\right)^2 = 1 - (\underbrace{0.125 + 0.282}_{0.407}) = 0.593$

$-\frac{174}{F} = \sqrt{0.593} = 0.77$

$F = -\frac{174}{0.77} = -225.97\,N \simeq -226\,N$

$\boxed{F = -226\,N}$

$F_x = F\cos\alpha_x = 226 \times \cos 69.3 = 226 \times 0.3535$

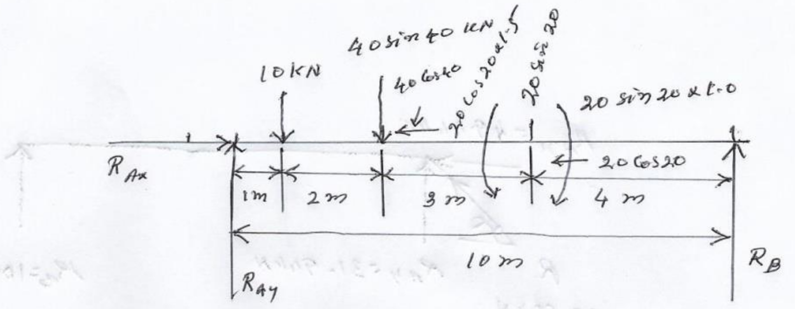
$\boxed{F_x = -79.88\,N}$

$F_z = F\cos\alpha_z = 226 \times \cos 57.9 = 226 \times 0.5314$

$\boxed{F_z = -120.09\,N}$

$\cos\alpha_y = \frac{-174}{-226} = 0.7699$; $\boxed{\alpha_y = 39.65^\circ}$

11.



$\Sigma H = 0 \rightarrow R_{Ax} - \underset{30.642}{40\cos 40} - \underset{18.794}{20\cos 20} = 0$

$R_{Ax} = 49.436$ kN —— 1 Say 49.44 kN

$\Sigma V = 0 \rightarrow R_{Ay} - 10 - \underset{25.71}{40\sin 40} - \underset{6.84}{20\sin 20} + R_B = 0$

$R_{Ay} + R_B = 42.55$ kN —— 2

$\Sigma M_A = 0 \rightarrow 10\times 1 + \underset{25.71\times 3}{40\sin 40\times 3} + \underset{6.84}{20\sin 20\times 6} + 20\sin 20\times 1$

$- 20\cos 20\times 1.5 - R_B\times 10 = 0$

$10 + 77.13 + 41.04 + 6.84 + 28.19 - 10R_B = 0$

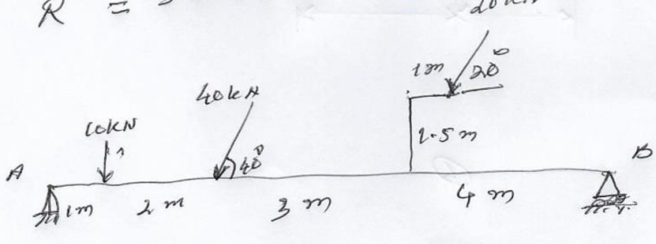
$R_B = \dfrac{106.11}{10} = 10.61$ kN

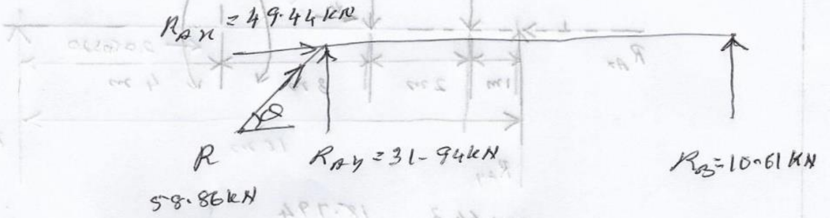
From eq. 2 : $R_{Ay} = 42.55 - 10.61 = 31.94$ kN

Resultant $= \sqrt{R_{Ax}^2 + R_{Ay}^2} = \sqrt{49.44^2 + 31.94^2}$

$R = \sqrt{2444.31 + 1020.16} = \sqrt{3464.4786}$

$R = 58.86$ kN





$$\theta = \tan^{-1}\frac{R_{Ay}}{R_{Ax}} = \tan^{-1}\frac{31.94}{49.44} = \tan^{-1}0.646 = 32^\circ 86'$$

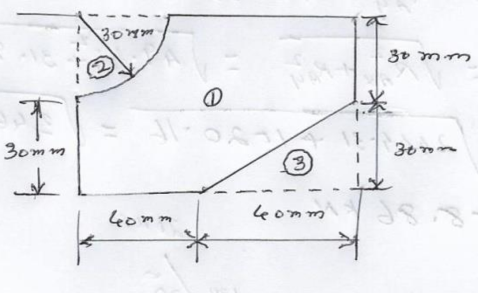
$$\theta = 32^\circ 86'$$

12. a)

① : Rectangle 80×60

② Quarter Cycle $r = 30mm$

③ Right angled triangle : base $40mm$, ht $= 30\,mm$



12. a : we have from the fig

$A_1 = 80 \times 60 = 4800\ mm^2$

$A_2 = \frac{\pi r^2}{4} = \frac{\pi \times 30^2}{4} = 706.86\ mm^2$

$A_3 = \frac{1}{2} \times 40 \times 30 = 600\ mm^2$

$A = A_1 - A_2 - A_3 = 4800 - 706.86 - 600 = 3493.14$

m.I. about xx axis

$I_{xx} = I_1 - I_2 - I_3$

$I_1 = \frac{bd^3}{12} + A_1 h_1^2$

$= \frac{80 \times 60^3}{12} + 4800 \times 30^2 = 1440000 + 4320000$

$I_1 = 576 \times 10^4\ mm^4$

$60 - \frac{4 \times 30}{3\pi} = 60 - 12.732$
$= 47.268$

$I_2 = 0.055 r^4 + A_2 h_2^2$

$= 0.055 \times 30^4 + 706.86 \times \left(60 - \frac{4r}{3\pi}\right)^2$

47.268^2

$= 44550 + 1579311.7 = 1624000$

$I_2 = 162.4 \times 10^4\ mm^4$

$I_3 = \frac{bh^3}{36} + A_3 h_3^2$

$= \frac{40 \times 30^3}{36} + 600\left(\frac{1}{3} \times 30\right)^2$

$= 30000 + 60000 = 9 \times 10^4$

$I_{xx} = I_1 - I_2 - I_3$

$I_{xx} = [576 - 162.4 - 9] \times 10^4 = 405 \times 10^4\ mm^4$

$I = Ak^2$, $k = \sqrt{\frac{I_{xx}}{A}}$ → Radius of gyration

$k = \sqrt{\frac{405 \times 10^4}{3493.14}} = \sqrt{1159.4} = \underline{34.05\ mm}$

12.6

$I_x = 6.4 \times 10^6\ mm^4$, $I_y = 16 \times 10^6\ mm^4$

$I_{xy} = 6.4 \times 10^6\ mm^4$

To find maximum and minimum principal moment of inertia

$I_{max} = I_{av} + R$ | $\frac{I_x - I_y}{2} = \frac{(6.4 - 16)10^6}{2} = -\frac{9.6 \times 10^6}{2}$

$I_{min} = I_{av} - R$

$I_{av} = \frac{I_x + I_y}{2} = \frac{(6.4 + 16)10^6}{2} = 11.2 \times 10^6\ mm^4$

$$\frac{I_{max}}{I_{min}} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$$I_{max} = 11.2 \times 10^6 + \sqrt{\left(-\frac{9.6 \times 10^6}{2}\right)^2 + (6.4 \times 10^6)^2}$$

$$= 11.2 \times 10^6 + 10^6\sqrt{(-4.8)^2 + (6.4)^2}$$

$$= [11.2 + 8]10^6 = 19.2 \times 10^6\ mm^4$$

$I_{max} = 19.2 \times 10^6\ mm^4$

$I_{min} = (11.2 - 8) \times 10^6 = 3.2 \times 10^6\ mm^4$

$I_{min} = 3.2 \times 10^6\ mm^4$

$$\sqrt{23.04 + 40.96} = \sqrt{64} = 8$$

13.a

Given : $l = 5\,m$

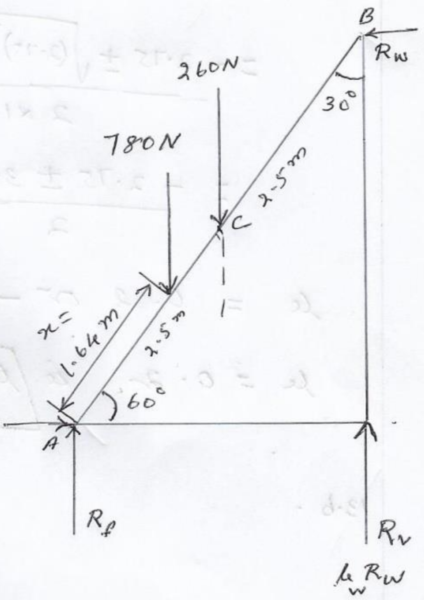
$W_l = 260\,N$

$\theta_A = 60^\circ$ & $\theta_B = 30^\circ$

$W_m = 780\,N$

$x = 1.64\,m$

$\mu = \mu_f = \mu_w = ?$



Solution

Consider the limiting equilibrium of the ladder

For $\Sigma F_H = 0$

$\mu R_f - R_W = 0$

$R_W = \mu R_f \quad —1$

For $\Sigma F_V = 0$

$R_f - 780 - 260 + \mu R_W = 0$

$R_f - 1040 + \mu \times \mu R_f = 0$

$R_f + \mu^2 R_f = 1040$

$R_f (1 + \mu^2) = 1040 \quad —2$

For $\Sigma M = 0$, taking moments about A

$780 \times (\cos 60 \times 1.64) + 260 \times (2.5 \cos 60) - \mu_w R_W \times 5 \cos 60 - R_W \times 5 \sin 60 = 0$

$780 \times \frac{1}{2} \times 1.64 + 260 \times 2.5 \times \frac{1}{2} - \mu\, R_W \times 5 \times \frac{1}{2} - R_W \times 5 \times \frac{\sqrt{3}}{2} = 0$

$639.6 + 325 - \mu (\mu R_f) \times 2.5 - \mu R_f \times 4.33 = 0$

$964.6 - 2.5 R_f \mu^2 - 4.33 R_f \mu = 0$

$\mu^2 + 2.75 \mu - 0.59 = 0$

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2.75 \pm \sqrt{(2.75)^2 - 4 \times 1 \times (-0.59)}}{2 \times 1} = \frac{-2.75 \pm \sqrt{9.92}}{2}$$

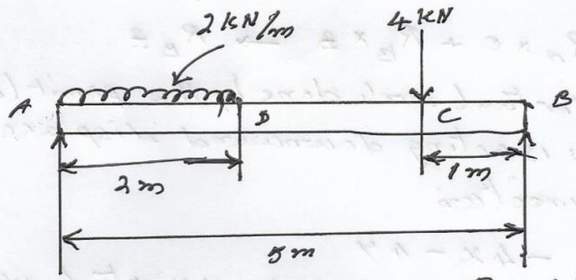
$$= \frac{-2.75 \pm 3.15}{2} = \frac{0.4}{2} \text{ or } -\frac{5.9}{2}$$

$k = 0.2$ or -2.95, neglecting $-$ value

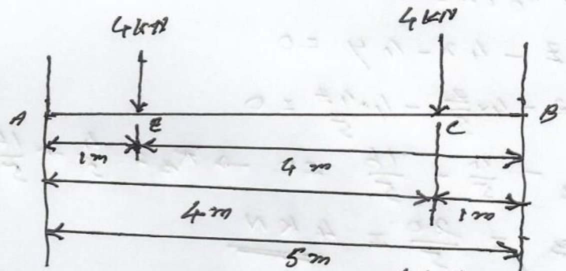
$k = 0.2$ i.e. $\boxed{k_f = k_w = 0.2}$

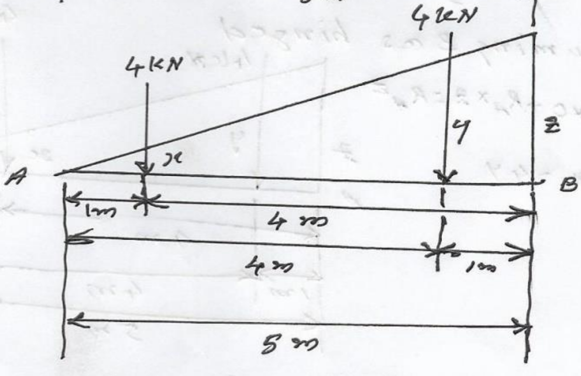
13.b .

13.b. To solve the problem using the principle of virtual work



$R_A = 4\,kN$ $\quad$ $R_B = 4\,kN$





$$\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$$

$$x = \frac{z}{5} \quad \text{and} \quad y = \frac{4z}{5}$$

Assuming beam hinged at A, we will get reaction at B

From the principle of virtual work

Virtual work done by the reaction

$R_A \times 0 + R_B \times z \rightarrow R_B z$

Virtual work done by the point load and U.D.L w is acting downward displacement upward direction

$\therefore -4x - 4y$

From the principle of virtual work algebraic sum of the total work done is zero

$R_B z - 4x - 4y = 0$

$R_B z - 4 \times \frac{z}{5} - 4 \times \frac{4z}{5} = 0$

$R_B - \frac{4}{5} - \frac{16}{5} = 0 \rightarrow R_B = \frac{4}{5} + \frac{16}{5}$

$R_B = \frac{20}{5} = 4\ kN$

$\frac{y}{4} = \frac{z}{5}$

$y = \frac{4z}{5}$

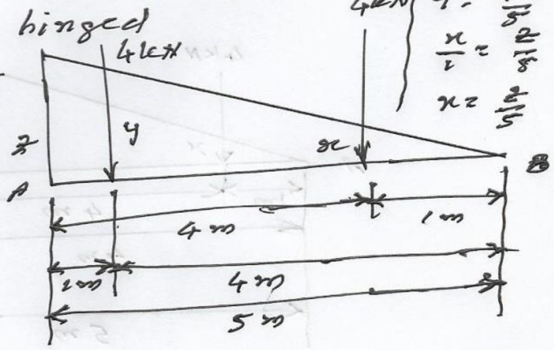
$\frac{x}{1} = \frac{z}{5}$

$x = \frac{z}{5}$

Assuming B as hinged

$R_B \times 0 + R_A \times z = R_A z$

$-4x - 4y$



$R_A z - 4x - 4y = 0$

$R_A z - 4 \times \frac{z}{5} - 4 \times \frac{4z}{5} = 0 \rightarrow R_A - \frac{4}{5} - \frac{16}{5} = 0$

$R_A = \frac{4}{5} + \frac{16}{5} = \frac{20}{5} = 4\ kN$

14-a. Explain with sketches how the forces involved in the lifting of a load by a wedge are analysed.

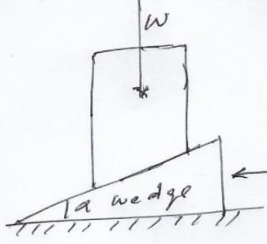
Ans: Wedge

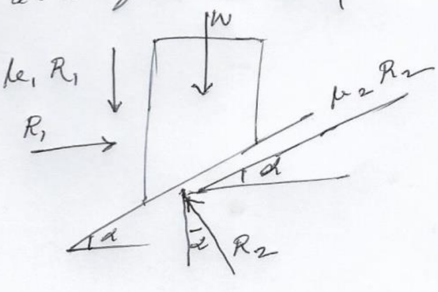
simple machines used to raise heavy loads

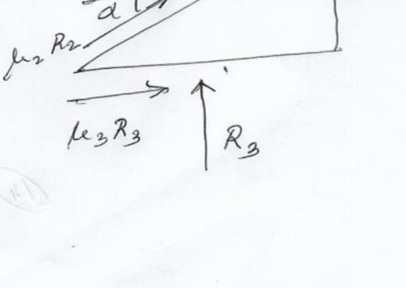
wedges are small pieces of materials with triangular or trapezoidal cross sections

The weight of the wedge is very small compared to the weight lifted. Hence generally the weight of the wedge will be neglected. The problems on wedges are generally the problems of equilibrium of bodies on inclined planes

Fig. shows the forces acting on the body







Consider the equilibrium of the body

Resolving the forces vertically

$$R_2 \cos\alpha - \mu_2 R_2 \sin\alpha - \mu_1 R_1 - W = 0$$

Resolving forces horizontally

$$R_1 - \mu_2 R_2 \cos\alpha - R_2 \sin\alpha = 0$$

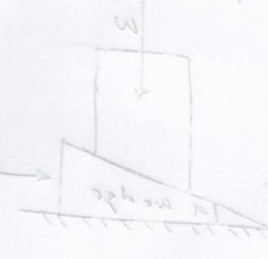
For equilibrium of the wedge

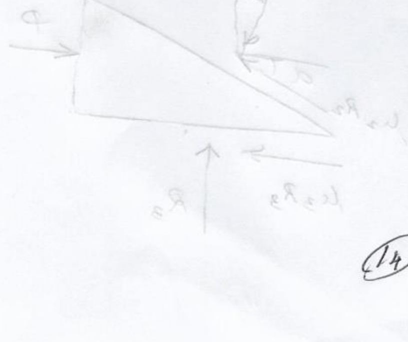
Resolving the forces horizontally

$$\mu_3 R_3 + \mu_2 R_2 \cos\alpha + R_2 \sin\alpha - P = 0$$

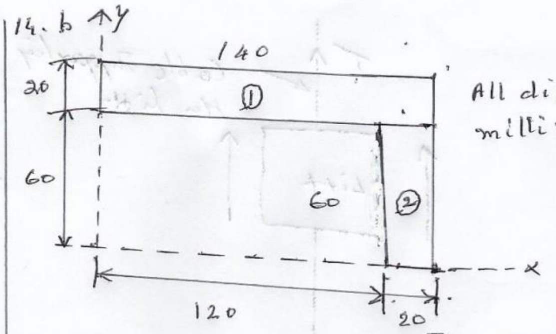
Resolving the forces vertically

$$R_3 + \mu_2 R_2 \sin\alpha - R_2 \cos\alpha = 0$$





14.b



All dimensions in millimeters

$\bar{x}_1 = 70$ mm

$\bar{y}_1 = 60+10 = 70$ mm

$A_1 = 20 \times 140$

$A_2 = 60 \times 20$

$\bar{x}_2 = 120+10 = 130$ mm, $\bar{y}_2 = 30$ mm

Parallel axis theorem: $I_{xy} = \bar{I}_{xy} + \bar{x}\,\bar{y}\,A$

Both areas (1) and (2) are symmetric at their centroidal

Axis $\rightarrow I_{xy} = 0$ for both area

therefore, for Area(1): $I_{xy1} = \bar{x}_1\,\bar{y}_1\,A_1$

$I_{xy1} = 20 \times 140 \times 70 \times 70 = 13.72 \times 10^6$ mm^4

similarly, for area(2): $I_{xy2} = \bar{x}_2\,\bar{y}_2\,A_2$

$I_{xy2} = 60 \times 20 \times 130 \times 30 = 4.68 \times 10^6$ mm^4

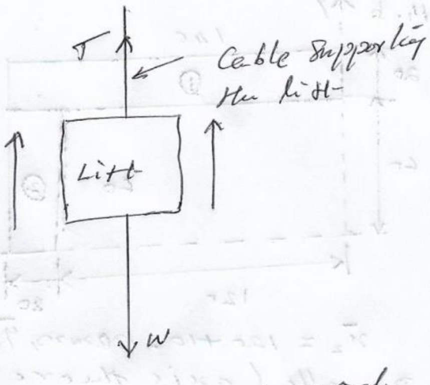
Total $I_{xy} = I_{xy1} + I_{xy2} = (13.72 + 4.68) \times 10^6$

$I_{xy} = 18.40 \times 10^6$ mm^4

15.a

$a = 0.6 \ m/s^2$

$W = 13.6 \ N$



Lift is moving upwards

Net force in the upward direction = T − W

This net force produces an acceleration 'a'

Hence using

Net force = mass × acceleration

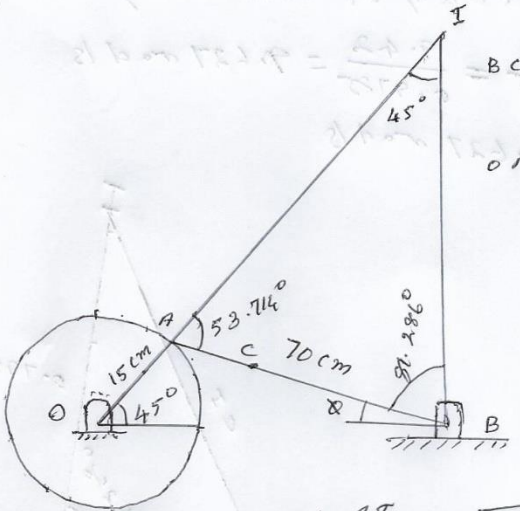
or $T - W = \frac{W}{g} \times a$

or $T = W + \frac{W}{g} a = W\left(1 + \frac{a}{g}\right)$

$T = 13.6\left(1 + \frac{0.6}{9.81}\right) = 13.6\,(1 + 0.0612)$

$T = 14.43 \ N$ → Reading in the spring balance

15. b



BC = 50 cm

OA = 15 cm
$= \frac{15}{100}$ m

$V_A = \omega_{OA} \times OA = \omega_{AB} \times AI$ —— ①

$\omega_{OA} = \frac{2\pi N}{60} = \frac{2\pi \times 600}{60} = 62.8$ rad/s

$V_A = \omega_{OA} \times OA = 62.8 \times \frac{15}{100} = 9.42$ m/s

perpendicular to OA, 45° inclined to horizontal

$V_B = V_A + V_{BA}$

Let inclination of AB with horizontal be ϕ,

then $OA \sin 45 = AB \sin \phi$

$\sin \phi = \frac{OA \sin 45}{AB} = 15 \times \frac{\sqrt{2}}{2} \times \frac{1}{70} = 0.1515$

$\phi = 8.714°$, $\angle ABI = 81.286°$

V_{BA} is perpendicular to AB or inclined $90 - 8.714$
$= 81.286°$

$\frac{AB}{\sin 45} = \frac{AI}{\sin 81.286} = \frac{BI}{\sin 53.714}$

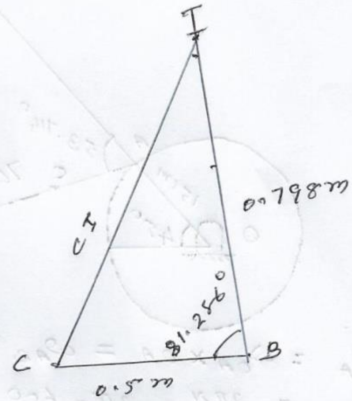
$AI = \frac{AB \sin 81.286}{\sin 45} = \frac{0.7 \times 0.988}{\left(\frac{\sqrt{2}}{2}\right)} = 0.9785$ m

$BI = \frac{AB \sin 53.714}{\sin 45} = \frac{0.7 \times 0.806}{\left(\frac{\sqrt{2}}{2}\right)} = 0.798$ m

i) Angular velocity of Connecting rod AB

$$\omega_{AB} = \frac{V_A}{AI} = \frac{9.42}{0.9785} = 9.627 \text{ rad/s}$$

$$\omega_{AB} = 9.627 \text{ rad/s}$$



ii) Velocity of piston B, $V_B = \omega_{AB} \times BI$

$$V_B = 9.627 \times 0.798 = 7.68 \text{ m/s}$$

iii) $CI = \sqrt{BC^2 + BI^2 - 2\,BC \cdot BI \cos 81.286}$

$$= \sqrt{(0.5)^2 + (0.798)^2 - 2 \times 0.5 \times 0.798 \cos 81.286}$$

$$= \sqrt{0.25 + 0.6368 - 0.1209} = \sqrt{0.7659}$$

$$CI = 0.875$$

$$\therefore V_C = CI \times \omega_{AB} = 0.875 \times 9.627 = 8.425 \text{ m/s}$$

$V_C = 8.425$ m/s, velocity at point C on the con-rod at a distance 50 cm from from the piston end.

(18)

16a.

Amplitude, $r = 1.6\ m$

Time period $t_p = 4\ s$

$$\omega = \frac{2\pi}{t_p} = \frac{2\pi}{4} = 1.57\ \frac{rad}{s}$$

For 1 oscillation

$t = t_p$ and $\theta = 2\pi$

$x = r \cos \omega t$

$x_1 = r \cos \omega t_1$

$\cos \omega t_1 = \cos(\omega \times t_1 \times \frac{180}{\pi})^{\circ}$

$x_1 = 1.2\ m$

$x_2 = 0.6\ m$

$\boxed{t = 0.2954\ s}$

Let x_1 and x_2 be the distance of the first and second points from the mean position

$$x = r \cos \omega t$$

$$x_1 = r \cos \omega t_1$$

$$1.2 = 1.6 \times \cos(1.57 \times t_1 \times \frac{180}{\pi})^{\circ}$$

$$\cos(1.57 \times t_1 \times \frac{180}{\pi}) = \frac{1.2}{1.6} = 0.75$$

$$1.57 \times t_1 \times \frac{180}{\pi} = \cos^{-1} 0.75 = 41.41$$

$$t_1 = \frac{41.41 \times \pi}{1.57 \times 180} = 0.4603\ s$$

$$x_2 = r \cos \omega t_2$$

$$0.6 = 1.6 \cos(1.57 \times t_2 \times \frac{180}{\pi})^{\circ}$$

(19)

$$\cos(1.57 \times t_2 \times \frac{180}{\pi}) = \frac{0.6}{1.6} = \frac{3}{8}$$

$$\cos^{-1} \frac{3}{8} = \cos^{-1} 0.375 = 67.97^{\circ}$$

$$1.57 \times t_2 \times \frac{180}{\pi} = 67.97$$

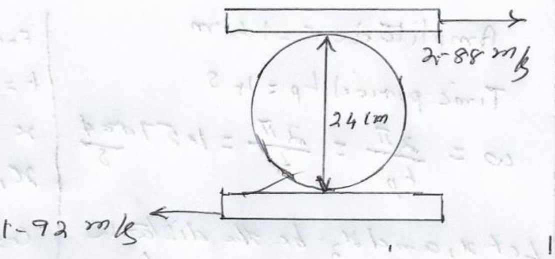
$$t_2 = \frac{67.97 \times \pi}{1.57 \times 180} = 0.7557\ s$$

Time required to pass the two points

$$t = t_2 - t_1 = 0.7557 - 0.4603 = \underline{\underline{0.2954\ s}}$$

16.b.

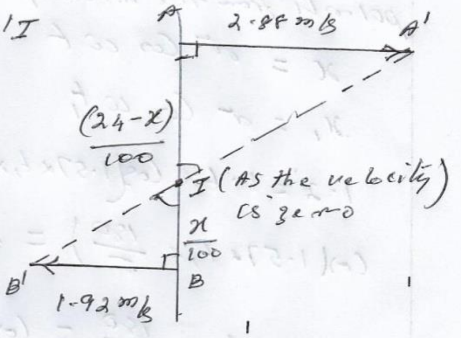
x is in cm



The triangles $AA'I$ and $BB'I$ are similar triangles

$\angle AA'I = \angle BB'I$

and $\angle AIA' = \angle BIB'$



$\therefore \frac{AA'}{BB'} = \frac{AI}{BI}$

$\frac{2.88}{1.92} = \frac{(24-x)\times 100}{100 \times x} = \frac{24-x}{x}$

$48 = 1.92 \times 250$

$2 \times 24 = 1.92 \times 250$

$2.88x = 1.92(24-x)$

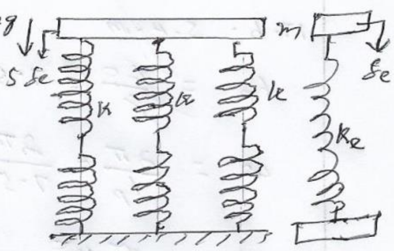
$2.88x + 1.92x = 1.92 \times 24$

$4.8x = 1.92 \times 24$

$x = \frac{1.92 \times 24}{4.8} = 9.6$ cm

Instant Centre is at 9.6 cm from bottom

17. a



t_p , $f = \frac{1}{t_p}$ — time period in sec

└ frequency in Hz

$t_p = 0.5\,s$, m

$t_{p_1} = 0.6\,s$, $(m + 1.5)$

stiffness of spring k

Spring in parallel

$\delta_e = \delta$

$k_e = k + k + k = 3k$

$t_p = 0.5\,s$ with empty tray

mass of tray $= m \rightarrow t_p = 2\pi\sqrt{\frac{m}{k_e}}$

$0.5 = 2\pi\sqrt{\frac{m}{3k}}$ —(1)

mass added $= 1.5$ kg

$t_p' = 0.6\,s$

$0.6 = 2\pi\sqrt{\frac{m+1.5}{3k}}$ —(2)

squaring and dividing (2) by (1)

$\frac{0.36}{0.25} = \frac{m+1.5}{m}$

$0.36m = (m+1.5) \times 0.25$

$0.36m = 0.25m + 0.375$

$0.11m = 0.375$

$m = \frac{0.375}{0.11} = \underline{3.41\text{ kg}}$, $\underline{k = 179.5\,\frac{N}{m}}$

$K_e = 3k$, From ① $(0.5)^2 = (2\pi)^2 \times \frac{m}{3k}$

$k = \frac{4\pi^2 \times m}{3 \times 0.25} = \frac{4\pi^2 \times 3.41}{3 \times 0.25} = 179.495\,\frac{N}{m}$

$\frac{(0.6)^2}{(0.5)^2} = \frac{\cancel{(2\pi)^2}}{\cancel{(2\pi)^2}} \times \frac{m+1.5}{\cancel{3k}} \times \frac{\cancel{3k}}{m}$

$\frac{0.36}{0.25} = \frac{m+1.5}{m}$

$0.25 = 4\pi^2 \times \frac{m}{3k}$

$\therefore k = \frac{4\pi^2 \times m}{3 \times 0.25}$

17.b. S.H.M

$t_p = \frac{60}{8} = 7.5\,s$, $x = 12\,cm = 0.12\,m$

$\omega = \frac{2\pi}{t_p} = \frac{2\pi}{7.5} = 0.838\ rad/s$

$V_{max} = r\omega$

At $x = 12\,cm$, $v = 0.5\,r\omega$

$v = \omega\sqrt{(r^2 - x^2)}$

$0.5\,r\omega = \omega\sqrt{r^2 - x^2} \rightarrow 0.5\,r = \sqrt{r^2 - 12^2}$

$0.25\,r^2 = r^2 - 144$

$0.75\,r^2 = 144$

$r = \sqrt{\frac{144}{0.75}} = 13.856\ cm = 0.138\,m$

maximum acceleration

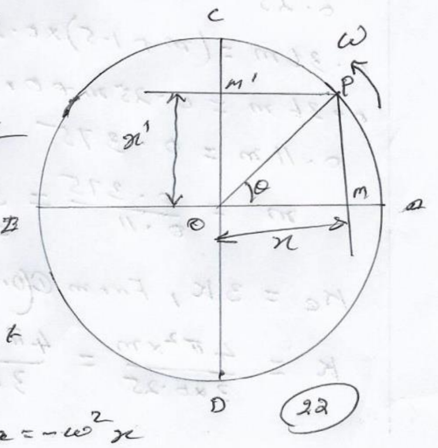
$a_{max} = -\omega^2 r$

$= -(0.838)^2 \times 0.138 = -0.097\ \frac{m}{s^2}$

Amplitude $r = 0.138\ m$

maximum acceleration

$a_{max} = -0.097\ \frac{m}{s^2}$



$\theta = \omega t$, $t = t_p$, $\theta = 2\pi$

$2\pi = \omega t_p$

$t_p = \frac{2\pi}{\omega}$, $x = r\cos\omega t$

$v = \omega\sqrt{r^2 - x^2}$

$a = -r\omega^2\cos\omega t \rightarrow a = -\omega^2 x$