

7. Distinguish between Simple Harmonic Motion and Periodic motion.
8. Explain the types of vibrations.

PART - B

Answer two questions from each set :

SET 1 : Answer any 2 questions. Each question carries 10 Marks. (2×10=20 Marks)

9. Determine the magnitude and direction of the resultant of the forces acting on the ring as shown in Fig. 2.

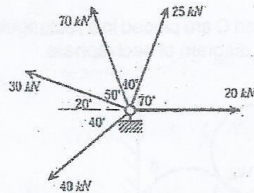


Fig. 2

10. Two smooth circular cylinders each of weight 100 N and radius 15 cm are connected at their centres by a string AB of length 40 cm and rest upon a horizontal plane as shown in below Fig. 3. The cylinder above them has a weight 200 N and radius of 15 cm. Find the force in the string AB and the pressure produced in the floor at the points of contact D and E.

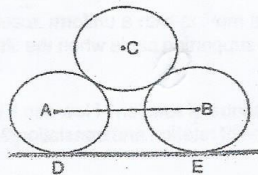


Fig. 3

11. A 5 m bar of negligible weight rests in a horizontal position on the smooth planes as shown in above Fig. 4. Determine the load P and reactions at supports.

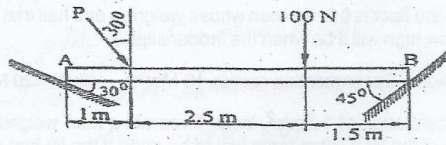
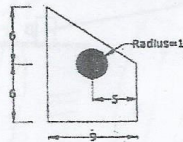


Fig. 4

SET 2 : Answer any 2 questions. Each question carries 10 Marks. (2×10 = 20 Marks)

12. a) Define radius of gyration.
b) Find the Centre of Gravity for the un-shaded composite area shown in Fig.5.



All dimensions in mm

Fig. 5

13. Determine the moments of inertia of the shaded area (Fig. 6) with respect to the x and y axes.

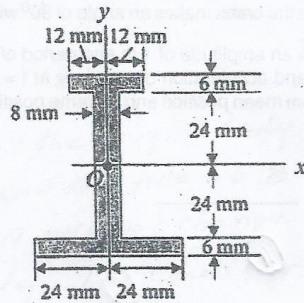


Fig. 6

10033

-4-



14. A uniform ladder of 4 m length rests against a vertical wall with which it makes an angle of 45° . The coefficient of friction between ladder and the wall is 0.4 and that between ladder and the floor is 0.5. If a man whose weight is one half that of ladder climbs up then how high will it be when the ladder slips ?

SET 3 : Answer any 2 questions. Each question carries 10 Marks. (2×10=20 Marks)

15. A lift has an upward acceleration of 1.2m/s^2 . What force will a man weighing 750 N exert on the floor of the lift ? What force would he exert if the lift had an acceleration of 1.2 m/s^2 downwards ? What upward acceleration would cause his weight to exert a force of 900 N on the floor ?

16.

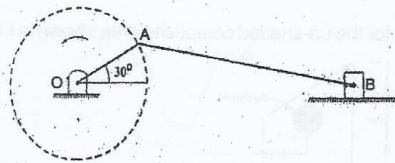


Fig. 7

In the reciprocating engine mechanism shown in Fig. 7, the crank OA rotates at a uniform speed of 300 rpm. The length of the crank and connecting rod are 12 cm and 50 cm respectively. Find the angular velocity of the connecting rod and velocity of the piston when the crank makes an angle of 30° with horizontal.

17. A body moving with SHM, has an amplitude of 1 m and period of oscillation is 2 seconds. Find the velocity and acceleration of the body at $t = 0.4$ second, when the time is measured from mean position and extreme position ?

44

1. Explain the principle of transmissibility with an example

Ans: Principle of transmissibility

The principle of transmissibility states that the point of application of a force can be transmitted along its line of action without changing the effect of the force on any rigid body to which it is applied

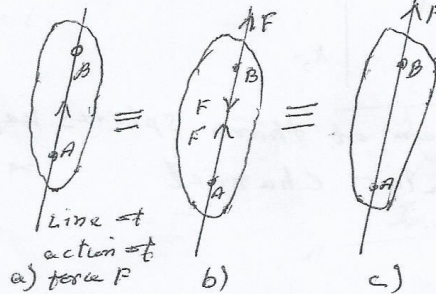


Fig 1

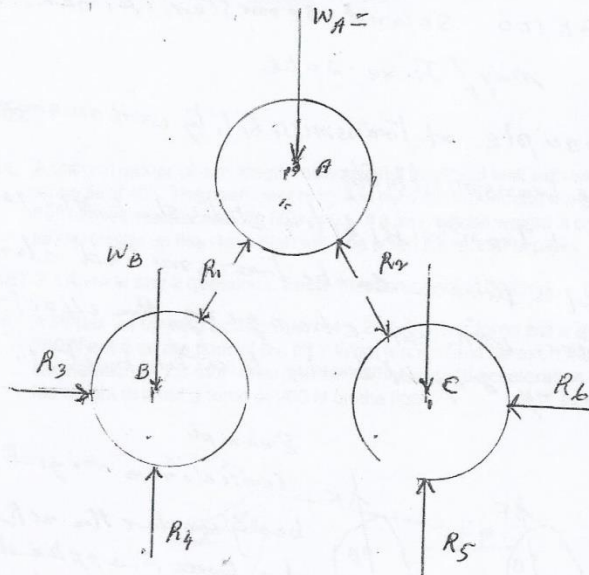
Example

Consider a rigid body under the action of a force F applied at A and acting along AB as shown in fig. 1

Two equal and opposite forces applied at B will not change the

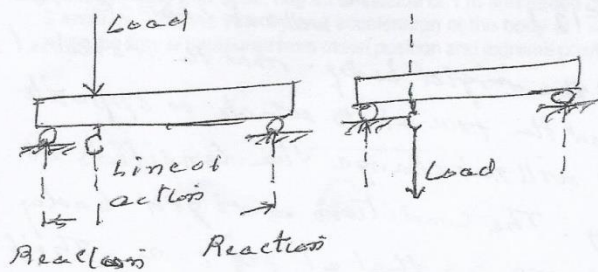
the conditions of the rigid body. Now the removal of force at A and the force at B which is opposite to the force at A will not change the conditions of the rigid body. The conditions of rigid body at fig 1-c is same as that at fig 1-a. This proves that transmission of force F from its point of application at A to another point B which is in the line of action of force F does not change the conditions of the rigid body

2)



Free body diagram of three spheres kept inside in a rectangular channel

i) Example



(6)

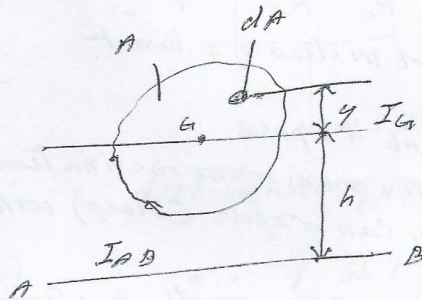
3. state and prove parallel axis theorem

Ans: parallel axis theorem?

It states that, if I_G is the moment of inertia of a plane lamina of area A , about its centroidal axis in the plane of the lamina, then the moment of inertia about any axis AB which is parallel to the centroidal axis and at a distance ' h ' from the centroidal axis is given by

$$I_{AB} = I_G + Ah^2$$

proof:



Consider an elemental area dA at a distance y from the centroidal axis. The first moment of elemental area about the axis AB as shown in fig is $dA(y+h)$. Second moment of elemental area about the axis AB is $dA(y+h)^2$. The second moment of the area about the axis AB is $\int dA(y+h)^2$

$$I_{AB} = \int dA(y+h)^2 = \int dA(y^2 + h^2 + 2hy)$$

$$= \int y^2 dA + \int h^2 dA + \int 2hy dA$$

$$= I_G + h^2 \int dA + 2h \int y dA$$

$$= I_G + h^2 A + 2h A \bar{y}$$

$$I_{AB} = I_G + Ah^2$$

\bar{y} is equal to \bar{y}' , because it is the distance of centroid G from the axis from which y is measured

(7)
 $\bar{y} = 0$
) in fig y is measured from the centroidal axis itself.

4. Define angle of friction and angle of repose
 prove that angle of repose is equal to angle of friction

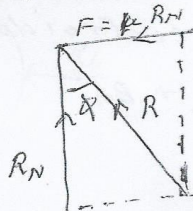
Angle of friction

It is the angle between the normal reaction at the contact surface and the resultant of normal reaction and limiting friction.

It is denoted by ϕ

$$\tan \phi = \frac{F}{R_N} = \frac{\mu R_N}{R_N} = \mu$$

$$\text{Angle of friction } \phi = \tan^{-1} \mu$$



Angle of repose

It is the maximum inclination of a plane on which a body can repose (sleep) without applying external force

When motion impends, frictional force $F = \mu \times R_N$

Resolving the forces along the inclined plane

$$\mu R_N - W \sin \alpha = 0$$

$$\therefore \mu R_N = W \sin \alpha$$

Resolving forces perpendicular to the inclined plane

$$R_N - W \cos \alpha = 0$$

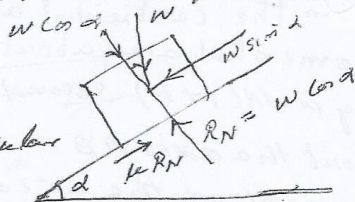
$$R_N = W \cos \alpha$$

$$\therefore \mu W \cos \alpha = W \sin \alpha$$

$$\tan \alpha = \mu = \tan \phi$$

$$\therefore \alpha = \phi$$

Angle of repose = angle of friction



(B)

5. A lift carries a weight of 3600 N and is moving with a uniform acceleration of 3.5 m/s^2 . Determine the tension in the supporting cable when the lift is moving upward. ($g = 9.8 \text{ m/s}^2$)

Given: $W = 3600 \text{ N}$
 $a = 3.5 \text{ m/s}^2$

Lift is moving upward

$$\text{Net force } \uparrow = T - W$$

$$\text{Net force} = m \cdot a$$

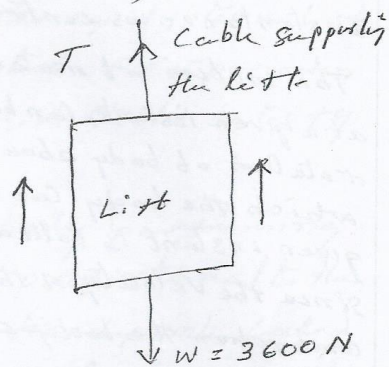
$$T - W = \frac{W}{g} a$$

$$\therefore T = W + \frac{W}{g} a$$

$$= W \left(1 + \frac{a}{g} \right)$$

$$T = 3600 \left(1 + \frac{3.5}{9.8} \right) = 3600 (1 + 0.3571)$$

$$T = 4885.56 \text{ N}$$



Lift is moving upwards

(Reaction at the lift is the same as the tension in the cable supporting the lift.)

6. What do you mean by instantaneous centre of rotation? How can it be located for a body moving with combined motion of rotation and translation?

Instantaneous centre of rotation

The motion of rotation and translation of a body at a given instant, can be considered as that of pure rotation of body about a point. This point about which the body can be assumed to be rotating at the given instant is called instantaneous centre of rotation since the velocity of this point is ~~not a fixed point~~ and when the body changes its position, the at the given instant is zero, this point is called instantaneous centre of zero velocity. This point is ~~not a fixed point~~ and when the body changes its position the position of instantaneous centre also changes. Locating positions of instantaneous centre of rotation

point O is the instantaneous centre of rotation of the link AB

This means link AB as a whole has rotated about O

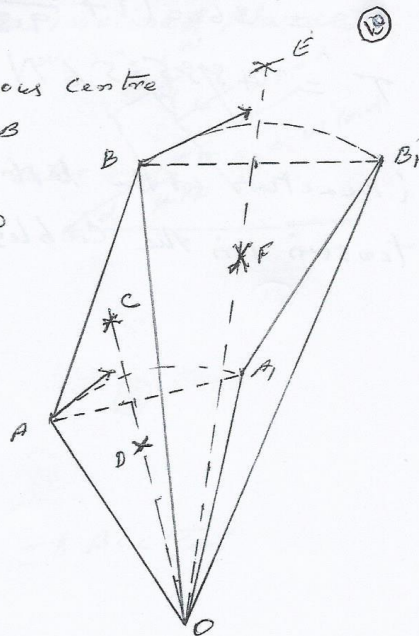
$$V_A = \omega \times OA$$

$$\omega = \frac{V_A}{OA} \quad \text{--- i}$$

$$V_B = \omega \times OB$$

$$\omega = \frac{V_B}{OB} \quad \text{--- ii}$$

$$\frac{V_A}{OA} = \frac{V_B}{OB} \quad \text{or} \quad \frac{V_A}{V_B} = \frac{OA}{OB}$$



distinguish between simple harmonic motion and periodic motion

simple harmonic motion

Simple harmonic motion (SHM) is a periodic motion

Any motion which repeats after equal interval of time is called a periodic motion. For a periodic motion to be simple harmonic, it should satisfy two general conditions

- i. The acceleration of the body performing periodic motion should be proportional to the distance of the body from a fixed point called centre of simple harmonic motion (mean position of the body)
- ii) The acceleration of the body should be directed towards the mean position

ii

8. Explain types of vibrations

Classification of vibrations

1. Longitudinal vibrations
2. Transverse vibrations
3. Torsional vibrations
4. Free vibrations
5. Forced vibrations

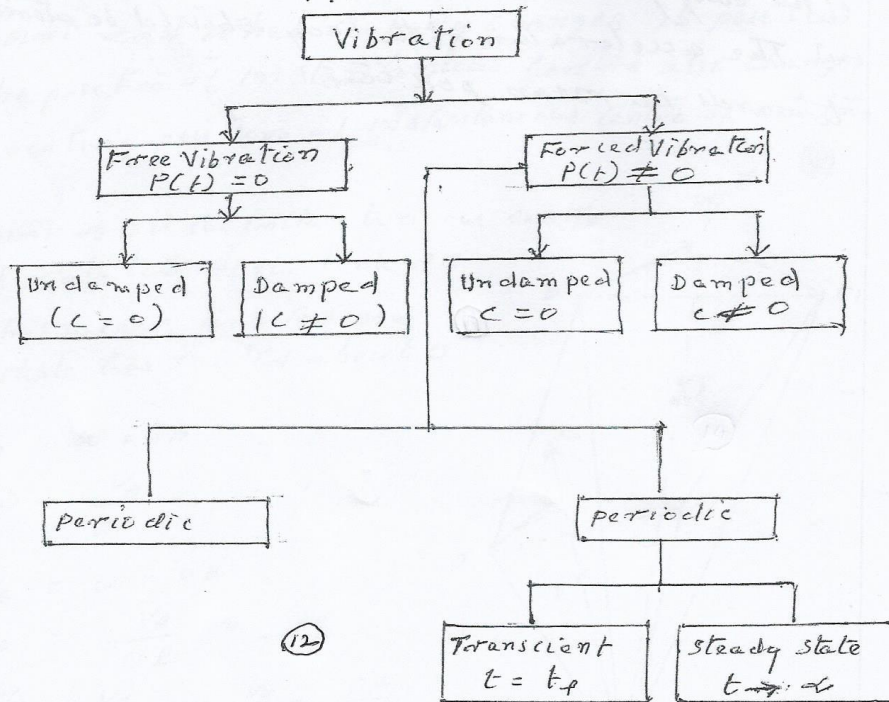
Types of vibration

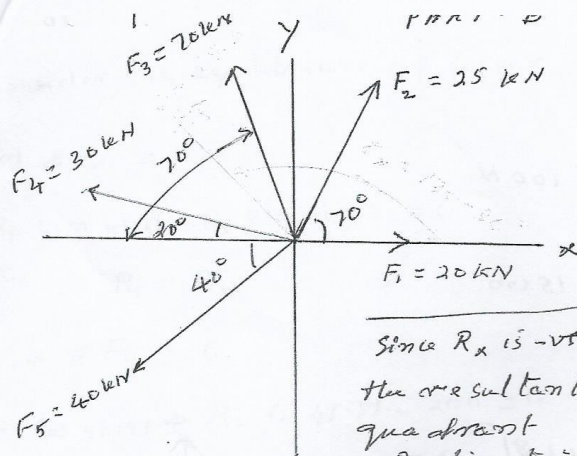
Free and forced vibrations

Linear and Non-linear vibrations

Deterministic and Random vibrations

Types of Vibration





Since R_x is -ve and R_y is +ve
the resultant is in the second
quadrant

\therefore Inclination of resultant $\theta = 180 - \theta_x = 180 - 49.66$

Component of force F_1 along x axis is 20 kN

Component of force F_2 along y axis is 0

$$\theta_x = 130.34^\circ$$

$$\therefore \text{Force } \vec{F}_1 = 20\hat{i} + 0\hat{j} = 20\hat{i} + 0\hat{j}$$

$$\vec{F}_2 = 25 \cos 70^\circ \hat{i} + 25 \sin 70^\circ \hat{j} = 8.55\hat{i} + 23.4925\hat{j}$$

$$\vec{F}_3 = -70 \cos 70^\circ \hat{i} + 70 \sin 70^\circ \hat{j} = -23.94\hat{i} + 55.779\hat{j}$$

$$\vec{F}_4 = -30 \cos 20^\circ \hat{i} + 30 \sin 20^\circ \hat{j} = -28.191\hat{i} + 10.26\hat{j}$$

$$\vec{F}_5 = -40 \cos 40^\circ \hat{i} - 40 \sin 40^\circ \hat{j} = -30.642\hat{i} - 25.712\hat{j}$$

The resultant of forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \vec{F}_4,$ and \vec{F}_5

$$\begin{aligned} \vec{R} &= \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5 \\ &= (20\hat{i} + 0\hat{j}) + (8.55\hat{i} + 23.4925\hat{j}) + (-23.94\hat{i} + 55.779\hat{j}) \\ &\quad + (-28.191\hat{i} + 10.26\hat{j}) + (-30.642\hat{i} - 25.712\hat{j}) \end{aligned}$$

$$\vec{R} = -54.223\hat{i} + 63.75\hat{j}$$

$$\text{Resultant force } R = \sqrt{(-54.223)^2 + (63.75)^2} = \sqrt{2940.13 + 4064.06}$$

$$R = \sqrt{7004.19} = 83.691 \text{ N}$$

The inclination of resultant with horizontal is given by $\cos \theta = \frac{R_x}{R} = \frac{-54.223}{83.691} = -0.6479$

$$\theta = \cos^{-1}(-0.6479) = 49.66^\circ$$

10)

Solution

Given $W_A = W_B = 100\text{ N}$

$r_A = r_B = 15\text{ cm}$

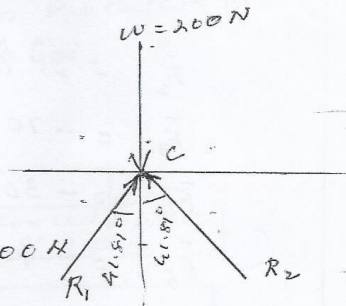
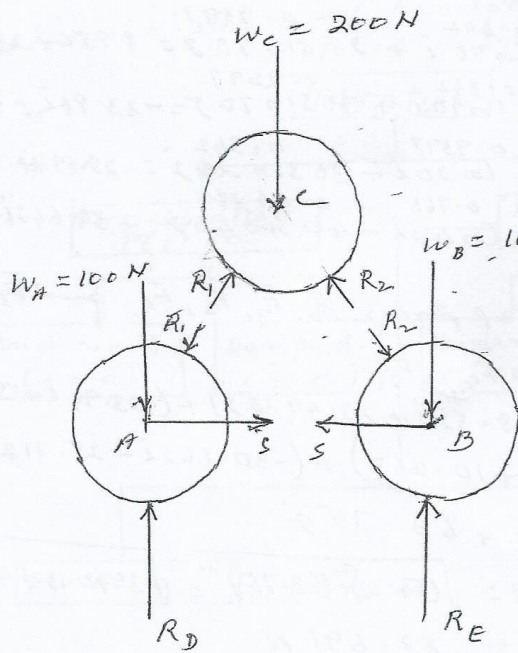
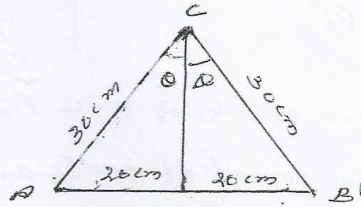
$W_C = 200\text{ N}, r_C = 15\text{ cm}$

$L = AB = 40\text{ cm}$

$\sin \theta = \frac{20}{30}$

$\theta = \sin^{-1} \frac{20}{30} = 41.81^\circ$

$\angle CAB = \angle CBA = 90 - 41.81$
 $= 48.19^\circ$



(14)

Consider the equilibrium of upper cylinder

$$\sum F_x = 0$$

$$R_1 \sin 41.81 - R_2 \sin 41.81 = 0$$

$$R_1 = R_2$$

$$\sum F_y = 0$$

$$R_1 \cos 41.81 + R_2 \cos 41.81 - 200 = 0$$

$$2R_1 \cos 41.81 - 200 = 0$$

$$R_1 = \frac{200}{2 \cos 41.81} = 134.16 \text{ N}$$

$$R_1 = R_2 = 134.16 \text{ N}$$

Consider equilibrium of lower cylinder

$$\sum F_x = 0$$

$$S - R_1 \sin \theta = 0$$

$$S - 134.16 \sin 41.81 = 0$$

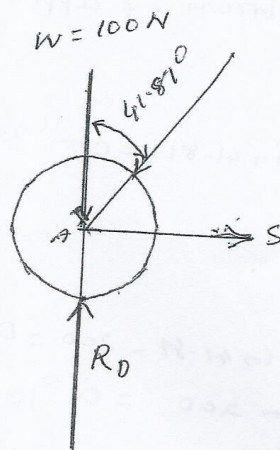
$$S = 134.16 \times \sin 41.81 = 89.44 \text{ N}$$

$$\sum F_y = 0$$

$$R_D - 100 - 134.16 \cos 41.81 = 0$$

$$R_D = 100 + 134.16 \cos 41.81 = 200 \text{ N}$$

(15)



Because of symmetrical arrangement of cylinders, the reaction at E will be same as that at D. Therefore $R_E = R_D = 200 \text{ N}$

$$F_{\text{net}} \Sigma F_H = 0$$

$$S - R_1 \sin 41.81^\circ = 0$$

$$S = R_1 \sin 41.81^\circ = 134.16 \sin 41.81^\circ = 89.439$$

$$S = 89.44 \text{ N}$$

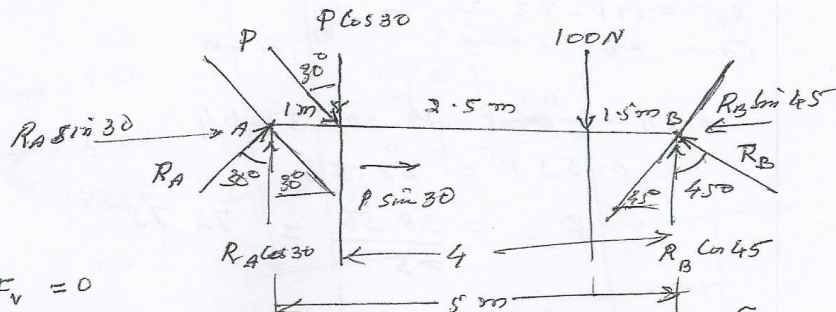
Force in the string AB, $S = 89.44 \text{ N}$

pressure produced in the floor at the points of contact D and E

$$\underline{R_D = R_E = 200 \text{ N}}$$

(16)

Solution



$$\sum F_x = 0$$

$$R_A \cos 30 - P \cos 30 - 100 + R_B \cos 45 = 0$$

$$R_A - P - \frac{100}{\cos 30} + \frac{R_B \cos 45}{\cos 30} = 0$$

$$R_A - P + 0.8165 R_B = 115.47 \quad \text{--- I}$$

$$\cos 45 = \frac{\sqrt{2}}{2}$$

$$\cos 30 = \frac{\sqrt{3}}{2}$$

$$\frac{\cos 45}{\cos 30} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}}$$

$$\sum F_y = 0$$

$$R_A \sin 30 + P \sin 30 - R_B \sin 45 = 0$$

$$R_A + P - R_B \frac{\sin 45}{\sin 30} = 0$$

$$R_A + P - R_B \cdot 1.414 = 0$$

$$R_A = \frac{(R_A + P)}{1.414} = 0.7071 (R_A + P)$$

$$\frac{\sin 45}{\sin 30} = \frac{\frac{\sqrt{2}}{2}}{\frac{1}{2}} = \sqrt{2}$$

Substituting this value of R_B in eq (I)

$$R_A - P + 0.8165 \times 0.7071 (R_A + P) = 115.47$$

$$1.5774 R_A - 0.4226 P = 115.47$$

$$R_A - 0.27 P = \frac{115.47}{1.5774} = 73.2 \quad \text{--- II}$$

$$R_A - 0.27 P = 73.2 \quad \text{(17)}$$

For $\sum m = 0$, taking moment about B

$$R_A \cos 30 \times 5 - P \cos 30 \times 4 - 100 \times 1.5 = 0$$

$$5 R_A - 4 P = \frac{100 \times 1.5}{\cos 30} = \frac{150}{\frac{\sqrt{3}}{2}} = \frac{300}{\sqrt{3}} = 100\sqrt{3}$$

$$R_A - 0.8 P = \frac{100\sqrt{3}}{5} = 20\sqrt{3} = 34.64$$

$$R_A - 0.8P = 34.64 \quad \text{--- (1)}$$

$$R_A - 0.27P = 73.20 \quad \text{--- (2)}$$

From eq. 2 and 3

21-111

$$-0.27P - (-0.8P) = 73.20 - 34.64$$

$$0.53P = 38.56$$

$$\therefore P = \frac{38.56}{0.53} = 72.75$$

$$P = \underline{72.75 \text{ N}}$$

From eq. 1

52-2

$$R_A = 0.8P + 34.64 = 0.8 \times 72.75 + 34.64 = 92.84$$

$$R_A = \underline{92.84 \text{ N}}$$

From eq. 1

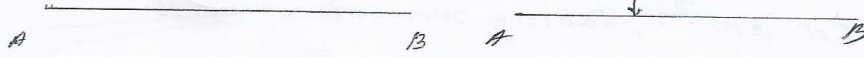
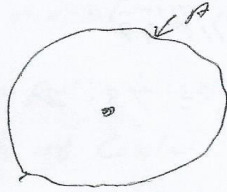
$$R_B - P + 0.8165 R_A = 115.47$$

$$R_B = \frac{115.47 - 92.84 + 72.75}{0.8165} = 116.816$$

$$R_B = \underline{116.816 \text{ N}}$$

(18)

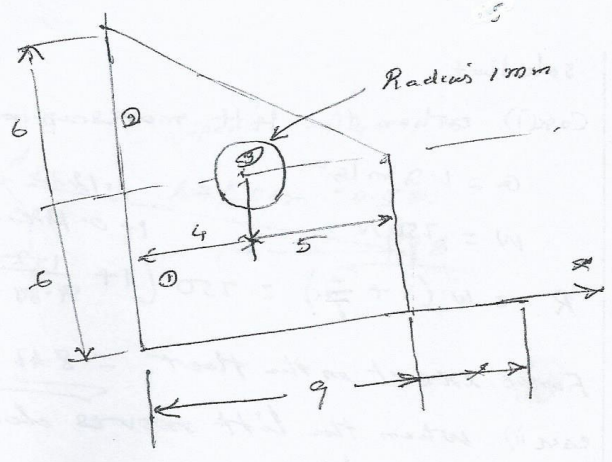
12-a Radius of gyration : Area A
moment of inertia I w.r.t.
reference axis AB



$I = AK^2$; $K = \sqrt{\frac{I}{A}}$ is called radius of gyration of the area w.r.t. the given axis AB

b)

- 1) Rectangl: 9×6
- 2) Triangle = $\frac{1}{2} b \times h$
= $\frac{1}{2} \times 9 \times 6$
- 3) Circle = πr^2
= $\pi \times 1^2$



All dimensions in mm

To find (\bar{x}, \bar{y}) of the given area

$$\bar{x} = \frac{\sum x_i A_i}{\sum A_i}$$

$$\bar{y} = \frac{\sum y_i A_i}{\sum A_i}$$

$$\bar{x} = \frac{x_1 A_1 + x_2 A_2 - x_3 A_3}{A_1 + A_2 - A_3}$$

$$\bar{y} = \frac{y_1 A_1 + y_2 A_2 - y_3 A_3}{A_1 + A_2 - A_3}$$

	x	y	A _i	x _i A _i	y _i A _i
1	4.5	3	54	243	162
2	3	8	27	81	216
3	4	6	3.14	-12.57	-18.85

$$\sum A_i = 77.86$$

$$\sum x_i A_i = 311.43$$

$$\sum y_i A_i = 359.15$$

$$\bar{x} = \frac{311.43}{77.86} = 3.9998 \text{ say } 4 \text{ mm}$$

$$\bar{y} = \frac{359.15}{77.86} = 4.613 \text{ mm}$$

$$\bar{x}, \bar{y} = (4, 4.613)$$

(19)

15

solution

Case i) when the lift moves up wards

$$a = 1.2 \text{ m/s}^2$$

$$W = 750 \text{ N}$$

$$1.1223$$

$$1 + 0.1223$$

$$R = W \left(1 + \frac{a}{g}\right) = 750 \left(1 + \frac{1.2}{9.81}\right) = 841.725 \text{ N}$$

Force exert on the floor = 841.725 N

Case ii) when the lift moves down wards

$$a = 1.2 \text{ m/s}^2$$

$$W = 750 \text{ N}$$

$$R = W \left(1 - \frac{a}{g}\right) = 750 \left(1 - \frac{1.2}{9.81}\right)$$

$$= 750 (1 - 0.1223) = 750 \times 0.8777 = 658.275 \text{ N}$$

$$R = \underline{658.275 \text{ N}}$$

$$\text{Case (iii)} \quad R = W \left(1 + \frac{a}{g}\right)$$

$$\therefore R = 900 \text{ N}$$

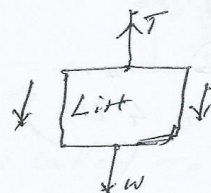
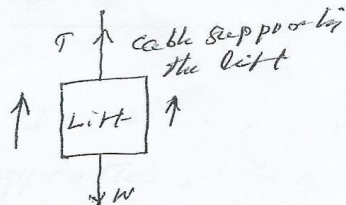
$$900 = 750 \left(1 + \frac{a}{9.81}\right)$$

$$1 + \frac{a}{9.81} = \frac{900}{750} = 1.2$$

$$\frac{a}{9.81} = 1.2 - 1 = 0.2$$

$$a = 0.2 \times 9.81 = 1.96 \text{ m/s}^2$$

$$a = 1.96 \frac{\text{m}}{\text{s}^2}$$

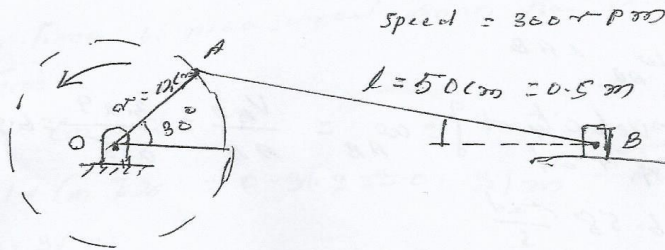


Lift is moving up wards

Lift is moving down wards.

16

AV
 Ex 5.15
 P 5.2.1
 5.2.2



(21)

$\omega = ?$
 ω_{OA}

$V = ?$

$\omega_{OA} = \frac{2\pi N}{60} = \frac{2\pi \times 360}{60} = 37.7 \text{ rad/s}$

$V_A = \omega \times OA = 37.7 \times 0.12 = 4.52 \text{ m/s}$

(perpendicular to OA, 60° inclined with horizontal)

$V_B = V_A + V_{BA}$

Let the inclination of AB with horizontal be ϕ

Then $OA \sin 30 = AB \sin \phi$

$\sin \phi = \frac{OA \sin 30}{AB} = \frac{12}{50} \times \frac{1}{2} = \frac{12}{100} = 0.12$

$\phi = \sin^{-1} 0.12 = 6.89^\circ$

$\phi = 6.89^\circ$

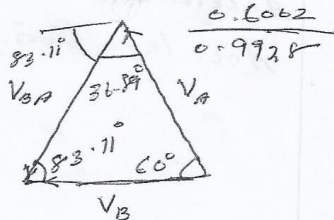
83.11	150
60.00	143.11
143.11	36.89

V_{BA} is perpendicular to AB or inclined $90 - 6.89 = 83.11^\circ$ with horizontal

$\frac{V_A}{\sin 83.11} = \frac{V_B}{\sin 36.89} = \frac{V_{BA}}{\sin 60}$

Velocity of piston $V_B = V_A \frac{\sin 36.89}{\sin 83.11}$

$V_B = 4.52 \times \frac{0.6002}{0.9928} = 2.78 \text{ m/s}$



$$V_{BA} = \frac{V_A \times \sin 60}{\sin 83.11} = \frac{3.77 \times \sqrt{3}}{2 \times 0.9928} = 3.29 \frac{\text{m}}{\text{s}}$$

$$V_{BA} = \omega_{AB} \perp AB$$

$$\text{Angular velocity of connecting rod} = \omega_{AB} = \frac{V_{BA}}{AB} = \frac{3.29}{0.5} = 6.58 \frac{\text{rad}}{\text{s}}$$

$$\omega_{AB} = \underline{\underline{6.58 \frac{\text{rad}}{\text{s}}}}$$

17) case i) when time is measured from the mean position

n.b

26.13-14

n. 6.5

$$r = 1 \text{ m}$$

$$b_p = 2.5$$

$$t = 0.4 \text{ s}$$

$$\omega = \frac{2\pi}{b_p} = \frac{2\pi}{2} = \pi \text{ rad/s}$$

$$\omega t = \pi \times 0.4 \text{ rad}$$

$$= \frac{180}{\pi} \times 0.4\pi = 72^\circ$$

$$x = r \sin \omega t$$

$$x = 1 \times \sin 72^\circ = 0.95 \text{ m}$$

$$\text{velocity } V = \omega \sqrt{r^2 - x^2} = \pi \times \sqrt{1^2 - 0.95^2} = \pi \sqrt{0.095}$$

$$= \pi \times 0.309 = 0.97 \text{ m/s}$$

$$\text{Velocity } V = 0.98 \text{ m/s}$$

$$\text{Acceleration} = \omega^2 x = \pi^2 \times 0.95 = 9.376 \text{ m/s}^2$$

$$\text{Acceleration} = \underline{\underline{9.38 \text{ m/s}^2}}$$

(22)

case 2

when time is measured from the extreme position

$$x = r \cos \omega t$$

$$= 1 \times \cos 7\pi = 0.309 \approx 0.31 \text{ m}$$

$$\frac{1 - 0.0961}{0.9039}$$

$$x = 0.31 \text{ m}$$

$$v = \omega \sqrt{r^2 - x^2} = \pi \sqrt{1^2 - (0.31)^2} = \pi \sqrt{0.9039} = 0.95\pi$$

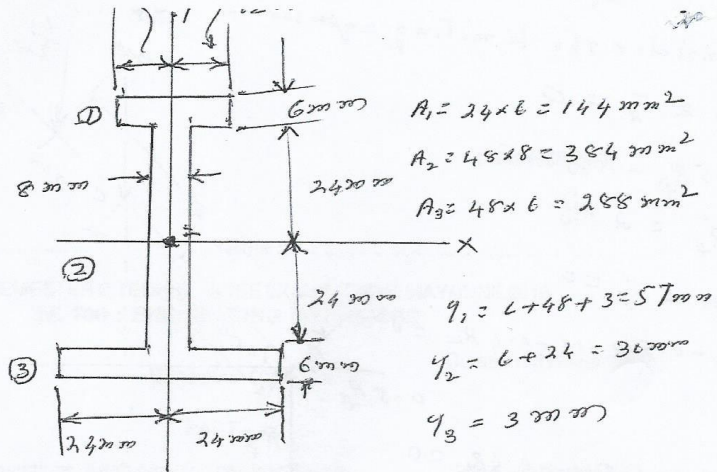
$$v = 2.9868 \text{ m/s} \approx 2.99 \text{ m/s}$$

$$v = 2.99 \text{ m/s}$$

$$\text{Acceleration } a = \omega^2 x = \pi^2 \times 0.31 = 3.0596$$

$$a = \underline{\underline{3.06 \text{ m/s}^2}}$$

(23)



As the figure is symmetric, centroid lies on y-axis, therefore $\bar{x} = 0$

Area A_i	\bar{x}_i	y_i	$A_i \bar{x}_i$	$A_i y_i^2$
144	0	57	0	8208
384	0	30	0	11520
288	0	3	0	864
				$\sum A_i y_i^2 = 20592$

$$\sum A_i = 816$$

$$y_c = \frac{\sum A_i y_i}{\sum A_i} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} = \frac{20592}{816}$$

$$y_c = 25.235 \text{ mm}$$

Thus the centroid is on the symmetric axis at a distance 25.235 mm from the bottom

(24)

