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Reg.No.______________ Name:______________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST/SECOND SEMESTER B.TECH DEGREE SPECIAL EXAMINATION, AUGUST 2016

BE 100 - ENGINEERING MECHANICS

Time : 3 Hours Maximum Marks : 100

Part – A

Answer all the questions. Each question carries 5 Marks (8 X 5 = 40 Marks)

1. (a). State the principle of Transmissibility of forces.

   (b). Explain the concept of free body diagrams.

2. Find the support reactions in the beam shown in figure 1

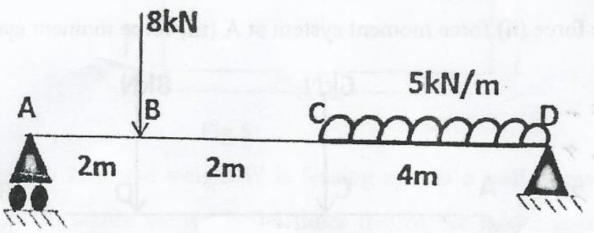


Fig.1

3. What is Polar Moment of Inertia? How does it differ from Product of Inertia?
4. Define virtual work and state principle of virtual work.
5. Explain Instantaneous centre of zero velocity. How can you locate it?
6. Calculate the velocity at a point $\frac{2}{3}$rd radius from thetop point of a rail car wheel of radius 250 mm, if the car moves without slipping on straight rails at 15 m/s.
7. Differentiate between free vibration and forced vibration of bodies
8. Explain the concept of single degree of freedom systems.

C

Part – B

Answer two questions from each set

SET 1 Answer any 2 questions. Each question carries 10 Marks (2 X 10 = 20 Marks)

9. Forces of 15N, 20N, 25N, 35N and 45N act at an angular point of a regular hexagon towards the other angular points as shown in figure 2. Calculate the magnitude and direction of the resultant force.

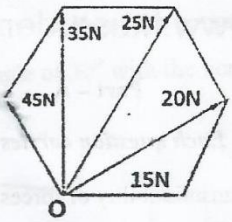


Fig.2

10. A rigid bar AB is acted upon by forces as shown in figure 3. Reduce the force system to (i) a single force (ii) force moment system at A (iii) force moment system at D

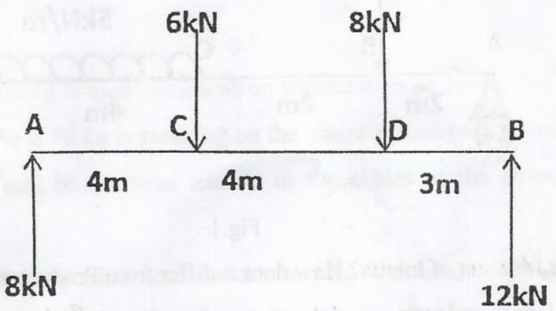


Fig.3

11. Determine the reactions at supports A and B of the beam as shown in the figure 4.

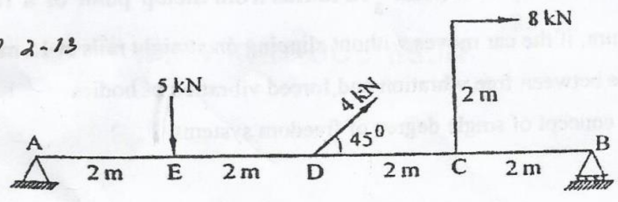


Fig.4

SET 2 Answer any 2 questions. Each question carries 10 Marks (2 X 10 = 20 Marks)

12. Z - section is shown in the below figure 5. Determine the centroid of the given section after selecting proper references axis.

   a) X coordinate

   b) Y coordinate

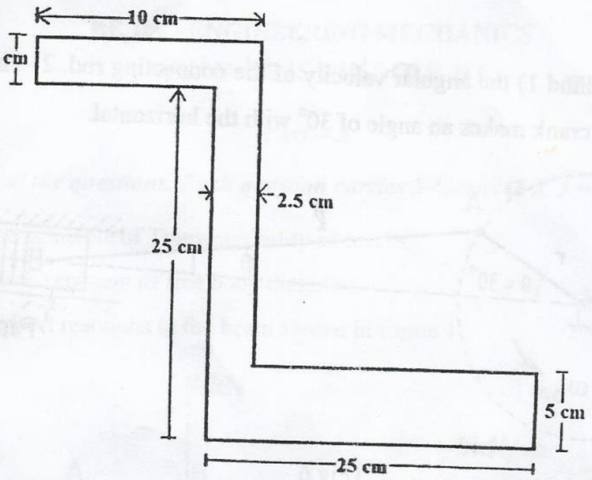


Fig.5

13. A uniform ladder of length 25m and weight W is leaning against a wall. It makes 45^0 with the horizontal. A man whose weight is 0.6 times that of the ladder goes up the ladder. Determine the maximum distance he can climb before the ladder slips. Assume coefficient of friction between the ladder and the wall to be 0.25 and that between the floor and the wall to be 0.3.

14. A simply supported beam of length 4m has a concentrated load of 5kN at 1m from the left support. It also has a uniformly distributed load of 2kN/m over its right half. Find the support reactions using the principle of virtual work.

SET 3 Answer any 2 questions. Each question carries 10 Marks (2 X 10 = 20 Marks)

15. A reciprocating engine mechanism is shown in the figure 6. The crank OA has a constant angular velocity of 300 rpm. The crank OA is of length 12 cm and the connecting rod AB

C

is of length 60 cm. Find 1) the angular velocity of the connecting rod. 2) The velocity of the piston when the crank makes an angle of 30° with the horizontal.

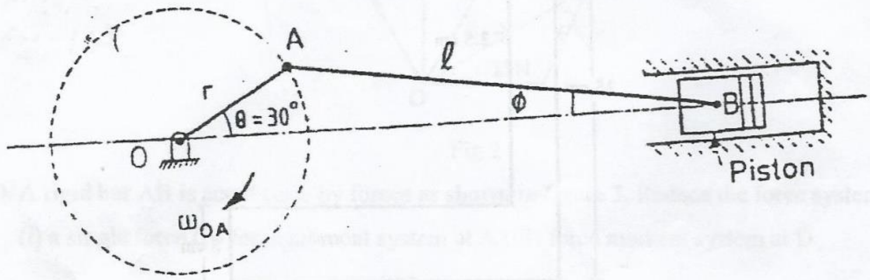


Fig.6

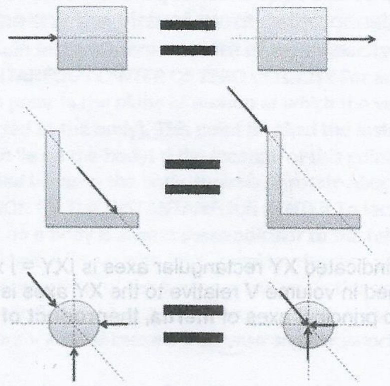
16. An elevator of mass 500 kg is ascending with an acceleration of $3m/s^2$. During this ascent its operator whose mass is 70 kg is standing on the scales placed on the floor. What is the scale reading? What will be the total tension in the cables of the elevator during his motion?

17. A body is moving with simple harmonic motion and has velocities of 8m/s and 3m/s at a distance of 1.5m and 2.5m respectively from the centre. Find the amplitude and time period of the body.

B. E. 100 E. M. Aug 2016

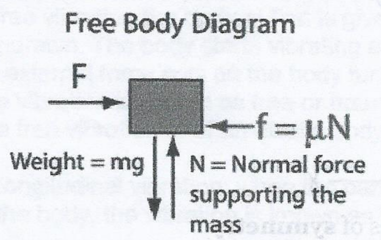
1. a) State the principle of transmissibility of forces

The **principle of transmissibility states** that the point of application of a **force** can be moved anywhere along its line of action without changing the external reaction **forces** on a rigid body



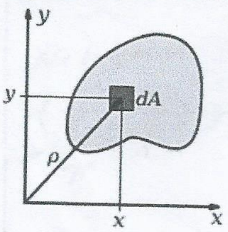
b) Explain the concept of free body diagram

A **free-body diagram** is a sketch of an object of interest with all the surrounding objects stripped away and all of the forces acting on the **body** shown. The drawing of a **free-body diagram** is an important step in the solving of mechanics problems since it helps to visualize all the forces acting on a single object.

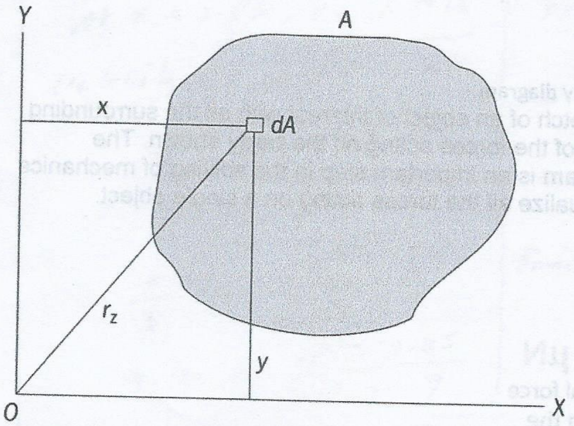


3. **What is polar moment of inertia? How does it differ from product of inertia?**

Polar moment of inertia is a quantity used to predict an object's ability to resist torsion, in objects (or segments of objects) with an invariant circular cross section and no significant warping or out-of-plane deformation. It is used to calculate the angular displacement of an object subjected to a torque



The **product of inertia** of area A relative to the indicated XY rectangular axes is IXY = ∫ xy dA . The **product of inertia** of the mass contained in volume V relative to the XY axes is IXY = ∫ xyρ dV—similarly for IYZ and IZX. Relative to principal axes of **inertia**, the**product of inertia** of a figure is zero.



- Products of inertia of a body are measures of **symmetry** .

If a particular plane is a plane of symmetry, then the products of inertia associated with any axis perpendicular to that plane are zero

4. Define virtual work and state principal of virtual work

The **virtual work** is the **work done** by the **virtual** displacements, which can be arbitrary, provided they are consistent with the constraints of the system. See Constraint. The principle of **virtual work** is equivalent to the conditions for static equilibrium of a rigid body expressed in terms of the total forces and torques . The **principle of virtual work states** that in equilibrium the **virtual work** of the forces applied to a system is zero. Newton's laws **state** that at equilibrium the applied forces are equal and opposite to the reaction, or constraint forces. This means the **virtual work** of the constraint forces must be zero as well

5. Explain instantaneous centre of zero velocity. How you can locate it

INSTANTANEOUS CENTER OF ZERO VELOCITY For any body undergoing planar motion, there always exists a point in the plane of motion at which the velocity is instantaneously zero (if it were rigidly connected to the body). This point is called the instantaneous center of zero velocity, or IC. It may or may not lie on the body! If the location of this point can be determined, the velocity analysis can be simplified because the body appears to rotate about this point at that instant.

LOCATION OF THE INSTANTANEOUS CENTER To locate the IC, we can use the fact that the velocity of a point on a body is always perpendicular to the relative position vector from the IC to the point. Several possibilities exist. First, consider the case when velocity v A of a point A on the body and the angular velocity ω of the body are known. In this case, the IC is located along the line drawn perpendicular to v A at A, a distance rA/IC = v A / ω from A. Note that the IC lies up and to the right of A since v A must cause a clockwise angular velocity ω about the IC.

7. Differentiate between free vibration and forced vibration of bodies

For a vibration to take place there is a need of application of an external force on elastic bodies such as a spring or a beam or a shaft and then after releasing the force, the body executes a vibratory motion. In a broader sense vibration is of two types:

1) Free or natural vibration:

In free vibration the body at first is given an initial displacement and the force is withdrawn. The body starts vibrating and continues the motion of its own accord. No external force acts on the body further to keep it in motion. The frequency of free vibration is known as free or natural frequency.

The free vibration of of an elastic body can further be of three types:

a)Longitudinal vibration: when the particles of the body move parallel to the axis of the body, the vibration is known as longitudinal vibration.

b)Transverse vibration: hen the particles of the body move nearly perpendicular to the axis of the body, the vibration is known as transverse vibration.

c)Torsional vibration: When the particles of the body move in a circle about the axis of the body, the vibration is known as torsional vibration.

2) Forced Vibration- In forced vibration, the object is under the influence of an outside force.

When a periodic disturbing force keeps the body in vibration throughout its entire period of motion, such vibration is said to be a forced vibration. The frequency of vibration of the body is same as the frequency of the applied force.

This can be understood more clearly by the following example:-

When a pendulum vibrates it is free vibration because it does not depend on any outside force to vibrate whereas when a drilling machine vibrates, it depends on a force from outside. Therefore, it is an example of forced vibration.

8. Explain the concept of single degree of freedom systemss

Single Degree of Freedom (SDOF) System

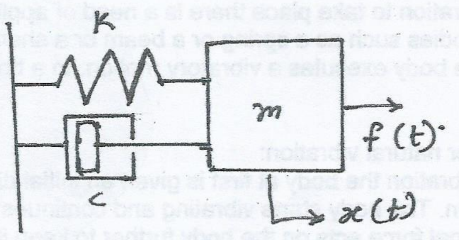
- One variable is required to describe the deformation
- For example: Mass-Spring-Dashpot System

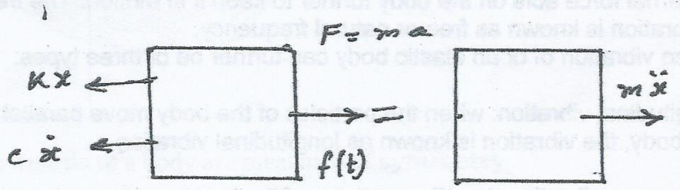
Newton's Law of Motion

- Resultant force = Mass X Acceleration
- Resultant Force Applied force , f (t)

,¬ Force offered by spring, , k x

¬ Force resisted by dashpot, f t( )⊢ 'cx&





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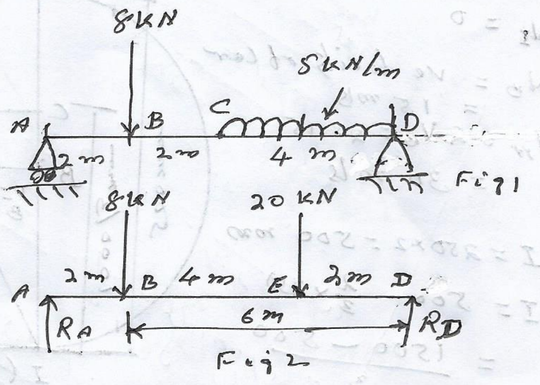


Solution: For calculating the support reactions, the UDL can be replaced by a concentrated load of magnitude $5\times4 = 20$ kN at the midpoint of CD as shown in fig 1

Consider the free body diagram of beam as shown in fig.

$\Sigma F_V = 0$

$R_A - 8 - 20 + R_D = 0$

$R_A + R_D = 28 \quad -(1)$

For $\Sigma M = 0$, taking moment about D

$\underset{40}{20\times2} + \underset{48}{8\times6} - R_A\times8 = 0$

$R_A = \frac{88}{8} = \underline{\underline{11\,kN}}$

$11 + R_D = 28$

$R_D = 28 - 11 = \underline{\underline{17\,kN}}$

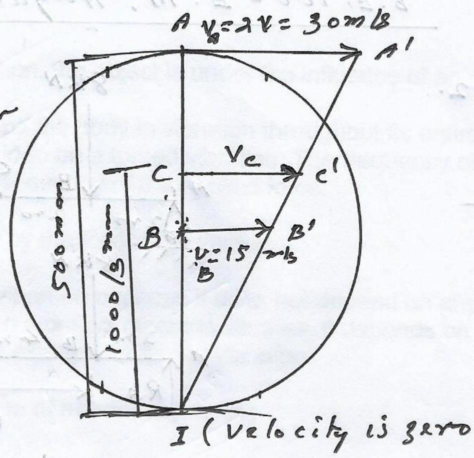
$R_A = 11\,kN$ & $R_D = 17\,kN$

6.

$V_I = 0$

V_B = velocity of car = 15 m/s

$V_A = 2V_B = 2 \times 15$ = 30 m/s



$AI = 250 \times 2 = 500$ mm

$CI = 500 - \frac{2}{3} \times 250$

$= \frac{1500 - 500}{3}$

$= \frac{1000}{3}$ mm

velocity at I = 0

velocity at B, $V_B = 15$ m/s (centre of the wheel)

velocity at A, $V_A = 2V_B = 2 \times 15 = 30$ m/s

To find velocity at C

Consider the triangles AA'I and CC'I

$$\frac{AA'}{CC'} = \frac{AI}{CI}$$

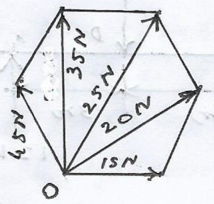
$$\frac{30}{CC'} = \frac{500}{1000} \times 3 \rightarrow CC' = \frac{30 \times 1000}{500 \times 3} = 20$$

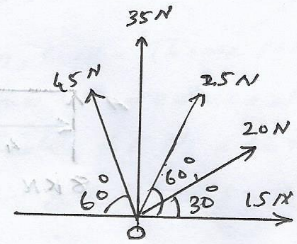
CC' = velocity at a point $\frac{2}{3}$ radius from the top point of rail car wheel.

CC' = 20 m/s

(6)

9.





Solution: Resolving the forces along x axis

$\Sigma F_x = 15 + 20\cos 30 + 25\cos 60 - 45\cos 60$

$= 15 + 20 \times \frac{\sqrt{3}}{2} + 25 \times \frac{1}{2} - 45 \times \frac{1}{2} =$

$= 15 + 17.32 + 12.5 - 22.5 = 22.32\,N$

Resolving the forces along y axis

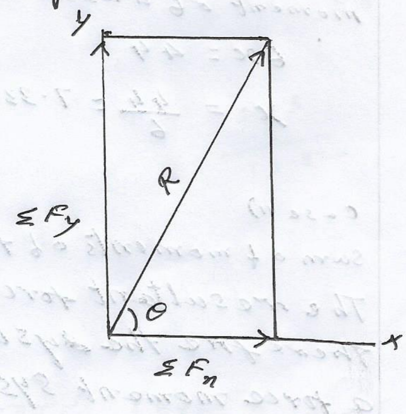
$\Sigma F_y = 0 + 20\sin 30 + 25\sin 60 + 35 + 45\sin 60 = 105.62\,N$

$\Sigma F_x = 22.32\,N$; $\Sigma F_y = 105.62\,N$

Resultant $R = \sqrt{(22.32)^2 + (105.62)^2} = \sqrt{498.18 + 11155.58}$

$R = 107.95\,N$

Inclination of resultant with horizontal



$\tan\theta = \frac{R_y}{R_x}$

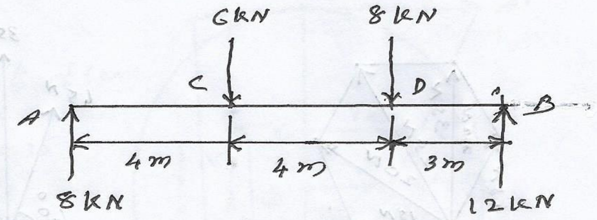
$\theta = \tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\frac{105.62}{22.32}$

$= 78.07°$

Inclination of resultant, $\theta = 78.07°$

1

10.



Solution:

$\Sigma F_V = 8 - 6 - 8 + 12 = 6\ kN$

$\Sigma F_H = 0$

$R = \sqrt{(\Sigma F_H)^2 + (\Sigma F_V)^2} = \sqrt{0^2 + 6^2} = \sqrt{36} = 6\ kN$

Case i:: $R = 6\ kN$

Let the resultant be at a distance x from A

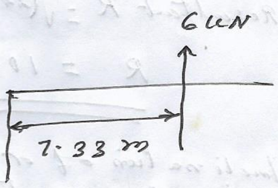
$\Sigma M_A = 6 \times 4 + 8 \times 8 - 12 \times 11 = 24 + 64 - 132 = -44\ kNm$

$= 44\ kNm$ c.c.w (counter clock wise)

moment of resultant about A $= 6 \times x$

$6x = 44$

$x = \frac{44}{6} = 7.33m$



Case ii)

Sum of moments of forces about A is 44 kN m

The resultant force is 6kN upwards.

Therefore the system can be reduced to a force moment system at A as shown in fig

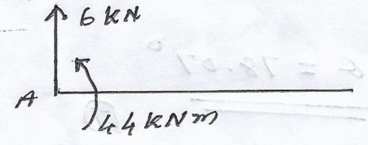


Fig 1.a

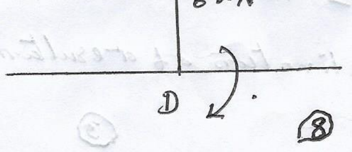


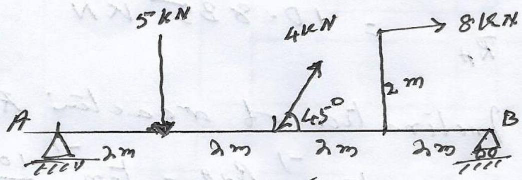
Fig 1-b

Case III

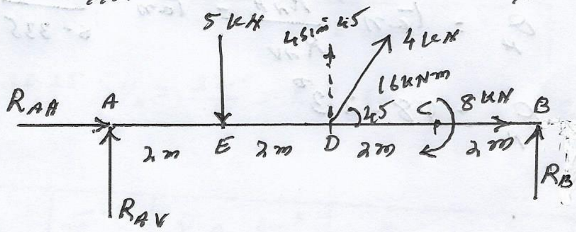
The sum of moments of forces about D is

$8 \times 8 - 6 \times 4 - 12 \times 3 = 4$ kNm, c.w. Therefore the given force system can be reduced to a single force of magnitude 6 kN along with a clockwise moment of 4 kN m at D as shown in fig 1.6.

11.



$\sin 45 = \frac{\sqrt{2}}{2}$



Solution : For $\Sigma F_V = 0$

$R_{AV} - 5 + 4 \overset{2.83}{\sin 45} + R_B = 0$

$R_{AV} + R_B = 2.17$ kN

For $\Sigma F_H = 0$

$R_{AH} + 4 \overset{2.83}{\cos 45} + 8 = 0$

$R_{AH} = -10.83$ kN

$R_{A} = 10.83$ kN, towards left

For $\Sigma M = 0$, taking moments about A

$5 \times 2 - 4 \overset{2.83 \times 4}{\sin 45 \times 4} + 16 - R_B \times 8 = 0$

$\frac{2.83 \times 4}{13.32}$

(9)

$$8 R_B = 10 - 11.32 + 16 = 14.68$$

$$R_B = \frac{14.68}{8} = 1.835 \text{ KN}$$

$$R_{AV} + 1.835 = 2.17$$

$$R_{AV} = 2.17 - 1.835 = 0.335 \text{ KN}$$

$$R_A = \sqrt{R_{AH}^2 + R_{AV}^2}$$

$$= \sqrt{10.83^2 + 0.335^2} = \sqrt{117.4}$$

$$R_A = 10.835 \text{ KN}$$

Inclination of reaction R_A with vertical

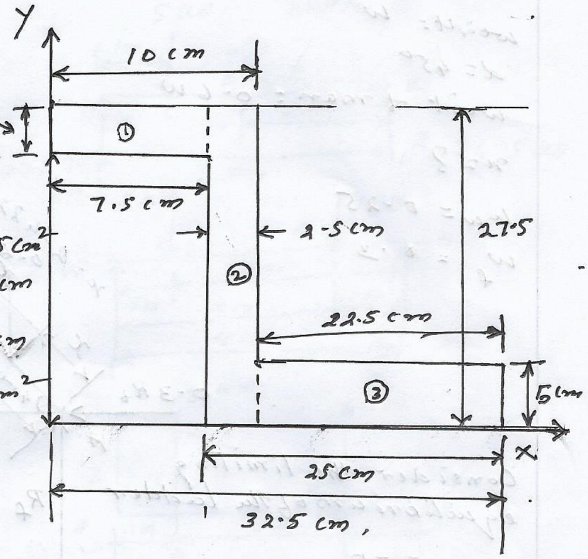
$$\theta_A = \tan^{-1}\frac{R_{AH}}{R_{AV}} = \tan^{-1}\frac{10.83}{0.335} = 88.23^0$$

$$\theta_A = 88.23^0$$

(10)

B.E100. E.M. August 2016

12.



$A_1 = 7.5 \times 2.5 = 18.75 cm^2$

$x_1 = 3.75$ cm

$A_2 = 27.5 \times 2.5 = 68.75 cm^2$

1.25   25 cm

$x_2 = 7.5 + \frac{2.5}{2} = 8.75$ cm

$A_3 = 22.5 \times 5 = 112.5 cm^2$

11.25

$x_3 = 7.5 + 2.5 + \frac{22.5}{2}$

$= 21.25$ cm

1.25

$y_1 = 25 + \frac{2.5}{2} = 26.25$; $y_2 = \frac{27.5}{2} = 13.75$ cm ; $y_3 = \frac{5}{2} = 2.5$ cm

Table method

Part	Area cm^2	$\bar{x}$ cm	$\bar{y}$ cm	$\bar{x}A$ cm^3	$\bar{y}A$ cm^3
1	18.75	3.75	26.25	70.3125	492.1875
2	68.75 cm^2	8.75	13.75	601.5625	945.3125
3	112.5	21.25	2.5	2390.625	281.25

$\Sigma A_i = 200$

$\Sigma \bar{x}_i A_i = 3062.5$ $\Sigma \bar{y}_i A_i = 1718.75$

$$\bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{3062.5}{200} = 15.3125 \text{ cm}$$

$$\bar{y} = \frac{\Sigma y_i A_i}{\Sigma A_i} = \frac{1718.75}{200} = 8.59375 \text{ cm}$$

Centroid (15.3125, 8.59375). Unit cm

13.

$l = 25\,m$

weight: W

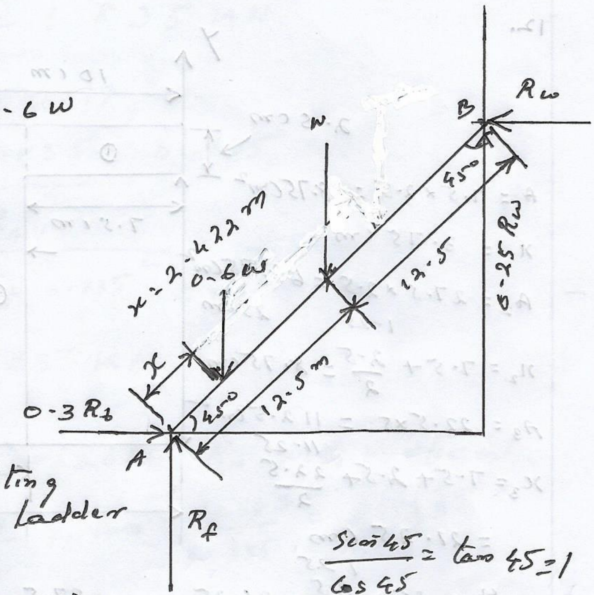
$\alpha = 45^\circ$

weight of man $= 0.6\,W$

$x = ?$

$\mu_w = 0.25$

$\mu_f = 0.3$



Consider the limiting equilibrium of the ladder

For $\Sigma F_H = 0$

$0.3 R_f - R_w = 0 \rightarrow R_f = \frac{10}{3} R_w$

$\Sigma F_V = 0$

$R_f - W - 0.6W + 0.25 R_w = 0$

$\frac{10}{3} R_w - 1.6W + 0.25 R_w = 0 \rightarrow \frac{43}{12} R_w = 1.6W$

$R_w = \frac{1.6 \times 12}{43} W = 0.4465 W$

$R_f = \frac{10}{3} \times 0.4465 = 1.488\,W$

$\frac{\sin 45}{\cos 45} = \tan 45 = 1$

For $\Sigma m = 0$, taking moment about A

$W \times 12.5 \cos 45 + 0.6 W \times x \cos 45 - 0.25 R_w \times 25 \cos 45$
$- R_w \times 25 \sin 45 = 0$ ← $\tan 45 = 1$

$W \times 12.5 + 0.6 Wx - 0.25 R_w \times 25 - R_w \times 25 \frac{\sin 45}{\cos 45} = 0$

$12.5W + 0.6xW - 6.25 R_w - 25 R_w = 0$

$(12.5 + 0.6x)W - 31.25 \times 0.4465 W = 0$

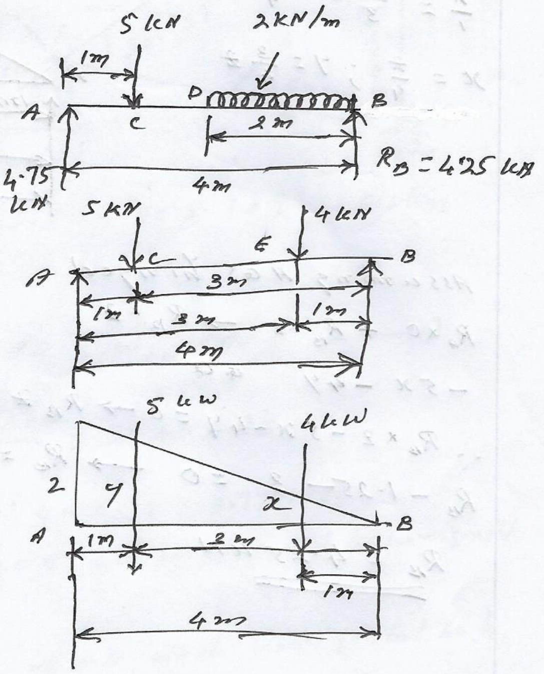
$12.5 + 0.6x = 13.953$

31.25
13.953

(12)

$x = \frac{13.953 - 12.5}{0.6} = \frac{1.43}{0.6} = 2.422\,m$

14.



$\frac{x}{1} = \frac{y}{3} = \frac{z}{4}$

$x = \frac{z}{4}$

$y = \frac{3}{4}z$

Assuming beam hinged at B, we will get reaction at A
From the principle of virtual work by the reactions

$R_B \times 0 + R_A \times z \rightarrow R_A \times z$

virtual work done by the point load and U.D.L
which is acting downward displacement upward direction

$\therefore -4x - 5y$

From the principal of virtual work algebraic
sum of the total work is zero

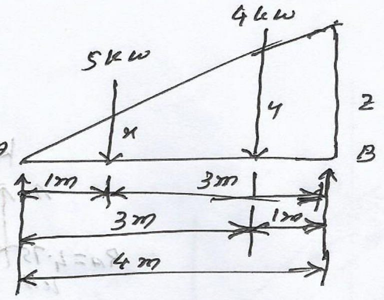
$R_A z - 4x - 5y = 0$ (13)

$R_A z - 4 \times \frac{z}{4} - 5 \times \frac{3z}{4} = 0 \rightarrow R_A - 1 - 3.75 = 0$

$R_A = 4.75$ kN

$\frac{x}{1} = \frac{y}{3} = \frac{z}{4}$

$x = \frac{z}{4}$; $y = \frac{3}{4} z$



Assuming A as hinged

$R_A \times 0 + R_B \times z \rightarrow R_B \times z$

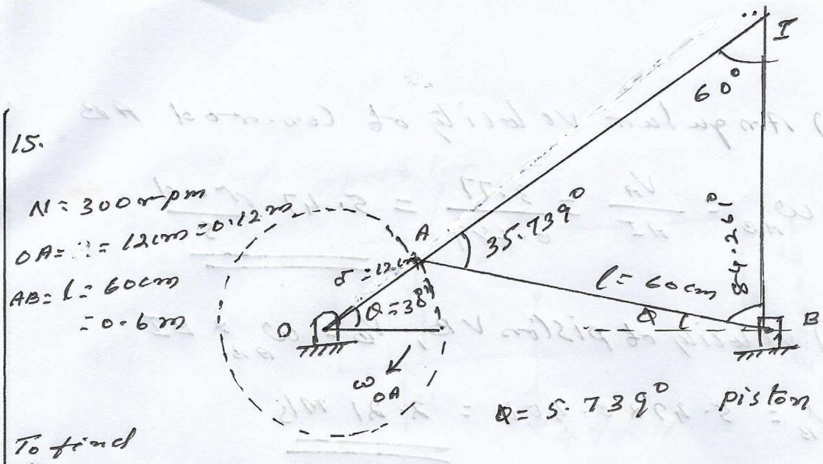
$-5x - 4y$

$\therefore R_B \times z - 5x - 4y = 0 \rightarrow R_B z - 5 \times \frac{z}{4} - 4 \frac{3z}{4} = 0$

$R_B - 1{\cdot}25 - 3 = 0 \longrightarrow R_B = 4{\cdot}25$

$R_B = 4{\cdot}25 \text{ kN}$

15.



To find

i) ω_{AB}

ii) $V_B = ?$

Solution : $V_a = \omega_{OA} \times OA = \omega_{AB} \times AI$ ——(1)

$\omega_{OA} = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60} = 31.4$ rad/s

$V_A = \omega_{OA} \times OA = 31.4 \times 0.12 = 3.77$ m/s

perpendicular to OA, 60° inclined to horizontal

$V_B = V_A + V_{BA}$

Let the inclination of AB with horizontal be ϕ

Then $OA \sin 30 = AB \sin \phi$

$\sin \phi = \frac{OA \sin 30}{AB} = \frac{12}{60} \sin 30 = 0.1$

$\phi = \sin^{-1} 0.1 = 5.739°$, $\angle ABI = 84.261°$

V_{BA} is perpendicular to AB or inclined

$90 - 5.739 = 84.261°$

$\frac{AB}{\sin 60} = \frac{AI}{\sin 84.261} = \frac{BI}{\sin 35.739}$

$BI = \frac{AB \sin 35.739}{\sin 60} = \frac{0.6 \times \sin 35.739}{\sin 60} = 0.404$ m

$AI = \frac{AB \sin 84.261}{\sin 60} = \frac{0.6 \times \sin 84.261}{\sin 60} = 0.689$ m

i) Angular velocity of Con-rod AB

$$\omega_{AB} = \frac{V_A}{AI} = \frac{3.77}{0.689} = 5.47 \ \frac{rad}{s}$$

ii) velocity of piston VB, $V_B = \omega_{AB} \times BI$

$$V_B = 5.47 \times 0.404 = 2.21 \ m/s$$

16. weight of the elevator, $W_1 = 500 \times 9.8 = 4900 N$

Acceleration of elevator, $a = 3 \ m/s^2$

weight of the operator $W_2 = 70 \times 9.8 = 686 N$

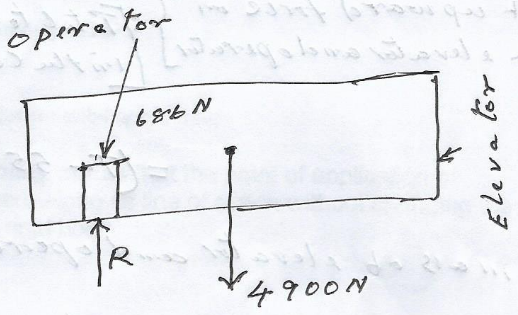
when the operator is standing on the scale placed on the floor of the elevator, the reading of the scale will be equal to the reaction (R) offered by the floor on the operator

Hence let R = Reaction offered by the floor on the operator. This is also equal to the reading of scale.

T = Total tension in the cables of elevator

Consider the motion of the operator. The operator is moving upwards along with the elevator with an acceleration $a = 3 \ m/s^2$

The net force on the operator is acting upwards



Net upward force on operator

= (Reaction offered by floor on operator) − weight of operator

= (R − 686)

mass of operator = 70 kg

Net force = mass × acceleration

(R − 686) = 70 × 3

R = 210 + 686 = 896 N Ans.

Total tension in the cables of elevator

Let T = total tension in the cables of elevator

W = total weight (ie weight of elevator + weight of operator

W = 4900 + 686 = 5586 N

(17)

As the elevator with the operator is moving upwards with an acceleration a = 3 m/s²

the net force will be acting on the elevator and operator in the upward direction

$$\left.\begin{array}{l}\text{Net upward force on}\\ \text{the elevator and operator}\end{array}\right\} = \left[\begin{array}{l}\text{Total tension}\\ \text{in the Cables}\end{array}\right] - \left[\begin{array}{l}\text{Total weight}\\ \text{of elevator}\\ \text{and operator}\end{array}\right]$$

$$= (T - 5586)$$

mass of elevator and operator $= 500 + 70$
$= 570$ kg

Net force = mass × acceleration

$(T - 5586) = 570 \times 3$

$T = 1710 + 5586 = \underline{\underline{7286\ N}}$

17. Solution:

At $x = 1.5$ m, $V = 8$ m/s

At $x = 2.5$ m, $V = 3$ m/s

Velocity $V = \omega\sqrt{a^2 - x^2}$

$8 = \omega\sqrt{a^2 - (1.5)^2}$

$3 = \omega\sqrt{a^2 - 3^2}$

$\frac{8}{3} = \frac{\sqrt{a^2 - 2.25}}{\sqrt{a^2 - 9}}$

$7.111 = \frac{a^2 - 2.25}{a^2 - 9}$

$7.111a^2 - 64 = a^2 - 2.25$

$6.111a^2 = 61.75$

$a = \sqrt{\frac{61.75}{6.111}} = 3.179$ m

Amplitude = $\underline{\underline{3.179\text{m}}}$

$8 = \omega\sqrt{a^2 - 2.25}$

$8 = \omega\sqrt{(3.179)^2 - 2.25}$

$= \omega\sqrt{10.1 - 2.25}$

$= \omega \times 2.8$

$\omega = \frac{8}{2.8} = 2.857$ rad/s

Time period t_p $= \frac{2\pi}{\omega} = \frac{2\pi}{2.857}$

$\underline{\underline{t_p = 2.199\text{s}}}$

(18)